Towards an Optimal Substructuring Method for Model Reduction

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Abstract. Substructuring methods have been studied since 1960s. The modes of subsystems associated with the lowest frequencies are typically retained. This mode selection rule is largely heuristic. In this paper, we use a moment-matching analysis tool to derive a new mode selection criterion, which is compatible to the one recently derived by Givoli *et al* using Dirichlet-to-Neumann (DtN) map as an analysis tool. The improvements of the new mode selection criterion are demonstrated by numerical examples from structural dynamics and MEMS simulation.

1 Introduction

Model-order reduction techniques play an indispensable role to meet the continual and compelling need for accurately and efficiently simulating dynamical behavior of large and complex physical systems. One popular method is the substructuring or the component mode synthesis (CMS), which was developed back to early 1960s [7,8,4]. CMS explicitly exploits underlying structures of a system and effectively avoids the expenses of processing the entire system at once. The model-order reduction of subsystems can be conducted in parallel. The subsystem structure is preserved.

Specifically, in this paper, we consider a lumped MIMO dynamical system of the form

$$\Sigma_N : \begin{cases} M\ddot{x}(t) + Kx(t) = Bu(t), \\ y(t) = L^T x(t), \end{cases} \tag{1.1}$$

with the initial conditions $x(0) = x_0$ and $\dot{x}(0) = v_0$. Here t is the time variable, $x(t) \in \mathcal{R}^N$ is a state vector, N is the degree of freedoms (DOFs), $u(t) \in \mathcal{R}^p$ the input excitation force vector, and $y(t) \in \mathcal{R}^m$ the output measurement vector. $B \in \mathcal{R}^{N\times p}$ and $L \in \mathcal{R}^{N\times m}$ are input and output distribution arrays, respectively. M and K are system matrices, such as mass and stiffness. Assume that M is symmetric positive definite and K is symmetric semidefinite. Furthermore, the state vector $x(t)$ and the system matrices M and K are posed of subsystem structure, namely, they are partitioned into the three blocks, representing subsystems I, II and interface:

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$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix},\ M = \begin{bmatrix} M_{11} & & M_{13} \\ & M_{22} & M_{23} \\ M_{13}^T & M_{23}^T & M_{33} \end{bmatrix},\ K = \begin{bmatrix} K_{11} & & K_{13} \\ & K_{22} & K_{23} \\ K_{13}^T & K_{23}^T & K_{33} \end{bmatrix}. \tag{1.2}$$

We denote the number of DOFs of subsystems I, II and the interface by N_1, N_2 and N_3, respectively. Thus the total number of DOFs of Σ_N is $N = N_1 + N_2 + N_3$.

By Laplace transform, the input-output behavior of Σ_N in the frequency domain is characterized by the transfer function

$$H(\omega) = L^T(-\omega^2 M + K)^{-1}B,$$

where ω is referred to as the frequency. For the simplicity of exposition, we have assumed that $x(0) = \dot{x}(0) = 0$.

A substructuring method replaces the system Σ_N with a system of the same form but (much) smaller dimension of the state-vector $z(t)$:

$$\Sigma_n : \begin{cases} M_n \ddot{z}(t) + K_n z(t) = B_n\, \mathbf{u}(t), \\ \widehat{y}(t) = L_n^T\, z(t), \end{cases} \tag{1.3}$$

such that the input-output behavior of Σ_n is an acceptable approximation of Σ_N. The number of DOFs of the new state-vector $z(t)$ is $n = n_1 + n_2 + N_3$ with $n_1 < N_1$ and $n_2 < N_2$. The DOFs of the interface block is unchanged. Furthermore, M_n and K_n preserve the block structures of M and K.

A key step in substructuring methods is to compute and retain the modes of subsystems. A standard mode selection practice is to retain the modes associated with few lowest frequencies. This is largely heuristic and does not guarantee to produce an optimal reduced system Σ_n as shown by the following simple example. Let

$$M = \left[\begin{array}{c|c|c} 1 & & 0.7 \\ \hline & \mathbf{1} & 10^{-3} \\ & & \mathbf{1}\ \ 0.3 \\ \hline 0.7 & 10^{-3}\ 0.3 & 1 \end{array}\right],\ K = \left[\begin{array}{c|c|c} 0.9 & & \\ \hline & \mathbf{1} & \\ & & \mathbf{2} \\ \hline & & 1 \end{array}\right],\ B = \begin{bmatrix} 1 \\ \hline 0 \\ \hline 0 \\ \hline 0 \end{bmatrix},\ L = \begin{bmatrix} 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \end{bmatrix}. \tag{1.4}$$

Suppose the subsystem II is reduced. Then by the standard lowest frequency mode selection criterion, the reduced system Σ_n is given by

$$M_n = \left[\begin{array}{c|c|c} 1 & & 0.7 \\ \hline & \mathbf{1} & 10^{-3} \\ \hline 0.7 & 10^{-3} & 1 \end{array}\right],\ K_n = \left[\begin{array}{c|c|c} 0.9 & & \\ \hline & \mathbf{1} & \\ \hline & & 1 \end{array}\right],\ B_n = \begin{bmatrix} 1 \\ \hline 0 \\ \hline 0 \end{bmatrix},\ L_n = \begin{bmatrix} 1 \\ \hline 1 \\ \hline 1 \end{bmatrix}. \tag{1.5}$$

However, if we retain the other mode in the system II, then the reduced system $\widehat{\Sigma}_n$ is given by

$$\widehat{M}_n = \left[\begin{array}{c|c|c} 1 & & 0.7 \\ \hline & \mathbf{1} & 0.3 \\ \hline 0.7 & 0.3 & 1 \end{array}\right],\ \widehat{K}_n = \left[\begin{array}{c|c|c} 0.9 & & \\ \hline & \mathbf{2} & \\ \hline & & 1 \end{array}\right],\ \widehat{B}_n = \begin{bmatrix} 1 \\ \hline 0 \\ \hline 0 \end{bmatrix},\ \widehat{L}_n = \begin{bmatrix} 1 \\ \hline 1 \\ \hline 1 \end{bmatrix}. \tag{1.6}$$

Figure 1 shows the magnitudes (in log of base 10) of the transfer function $H(\omega)$ of the original system Σ_N and the reduced ones $H_n(\omega)$ (called CMS line) and $\widehat{H}_n(\omega)$ (called CMS_χ line). It is clear that the low-frequency dominant mode selection criterion is not optimal.

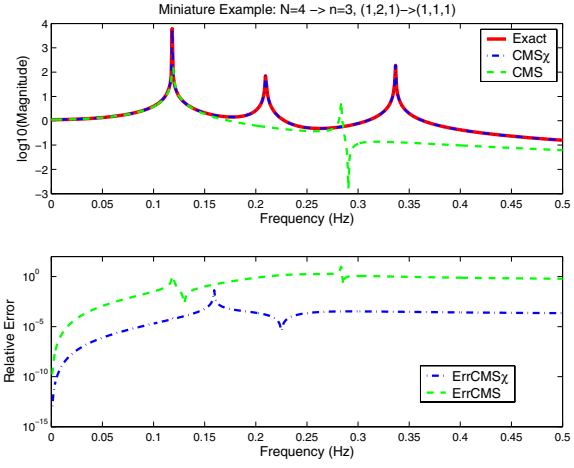

Fig. 1. The frequency response analysis (top) and relative error (bottom) for the miniature example

A question that arises naturally is "*which are the important modes of subsystems?*" In the recent work of Givoli *et al* [1,6], an optimal modal reduction (OMR) algorithm is proposed. In contrast to the low-frequency dominant mode selection rule, they introduce the concept of *coupling matrix*-based mode selection criterion. The concept is derived via the DtN map analysis tool, originally developed for solving partial differential equations with non-reflecting boundary conditions [9]. They show that the OMR method is better than the standard modal reduction (SMR) method. However, there are a number of limitations in the OMR method, such as the assumption of external force $Bu(t)$ only applied to one of the subsystems.

In this paper, we present an alternative mode selection criterion to the CMS method. The resulting method is called CMS_χ. The new mode selection criterion in CMS_χ is derived in an algebraic setting based on the concept of moment-matching in frequency domain. It coincides with the coupling matrix-based mode selection criterion used in the OMR method. However, mathematical derivation of moment-matching based mode selection criterion is much simpler than the DtN mapping based derivation used in OMR. Moreover, it does not need the assumption of the special form of external force $Bu(t)$ as used in OMR.

2 Substructuring Methods

In this section, we first discuss a generic CMS method, which is based on the original CMS developed by Hurty [7,8] and Craig and Bampton [4]. Then we specify the

difference between the standard CMS method and the new one we propose. We give a justification for the new method in the next section.

In a generic and compact form, the key step of the CMS method is on the construction of the transformation matrix V_n of the form

$$V_n = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \end{array} \begin{array}{c} \begin{array}{ccc} n_1 & n_2 & N_3 \end{array} \\ \begin{pmatrix} \Phi_1 & & \Psi_{13} \\ & \Phi_2 & \Psi_{23} \\ & & I_{N_3} \end{pmatrix} \end{array}, \tag{2.7}$$

where $\Psi_{i3} = -K_{ii}^{-1}K_{i3}$ for $i = 1, 2$, and Φ_i is an $N_i \times n_i$ matrix whose columns are the selected n_i eigenvectors $\boldsymbol{\phi}_j^{(i)}$ of the matrix pair (M_{ii}, K_{ii}):

$$K_{ii}\boldsymbol{\phi}_j^{(i)} = \lambda_j^{(i)} M_{ii}\, \boldsymbol{\phi}_j^{(i)} \quad \text{and} \quad (\boldsymbol{\phi}_j^{(i)})^T M_{ii} \boldsymbol{\phi}_k^{(i)} = \delta_{jk}, \tag{2.8}$$

where δ_{jk} is the Kronecker delta. In structural dynamics, Φ_i is the interior partition of the fixed-interface modal matrix and Ψ_{i3} is the interior partition of the constraint-mode matrix.

An orthogonal projection technique for model-order reduction seeks an approximation of $x(t)$ constrained to stay in the subspace spanned by the columns of V_n, namely

$$x(t) \approx V_n z(t)\ .$$

Then by imposing the so-called Galerkin orthogonal condition:

$$MV_n\ddot{z}(t) + KQ_n z(t) - Bu(t) \ \perp\ \text{span}\{V_n\}.$$

it yields a reduced-order system:

$$\Sigma_n : \begin{cases} M_n\ddot{z}(t) + K_n z(t) = B_n u(t) \\ \hat{y}(t) = L_n^T z(t) \end{cases}, \tag{2.9}$$

where $M_n = V_n^T M V_n$, $K_n = V_n^T K V_n$, $B_n = V_n^T B$ and $L_n = V_n^T L$. By the definition of V_n, the matrices M_n and K_n of the reduced system Σ_n are of the following forms

$$M_n = \begin{bmatrix} I & & M_{13}^{(n)} \\ & I & M_{23}^{(n)} \\ (M_{13}^{(n)})^T & (M_{13}^{(n)})^T & \widehat{M}_{33} \end{bmatrix} \text{ and } K_n = \begin{bmatrix} \Lambda_1^{(n)} & & \\ & \Lambda_2^{(n)} & \\ & & \widehat{K}_{33} \end{bmatrix},$$

where

$$M_{i3}^{(n)} = \Phi_i^T \widehat{M}_{i3} \quad \text{and} \quad \widehat{M}_{i3} = M_{i3} - M_{ii}K_{ii}^{-1}K_{i3} \quad \text{for } i = 1, 2,$$

$$\widehat{M}_{33} = M_{33} - \sum_{i=1}^{2} \left(K_{i3}^T K_{ii}^{-1} M_{i3} + M_{i3}^T K_{ii}^{-1} K_{i3} - K_{i3}^T K_{ii}^{-1} M_{ii} K_{ii}^{-1} K_{i3} \right),$$

and $\widehat{K}_{33}$ is the Schur complement of $\text{diag}(K_{11}, K_{22})$ in K of the form

$$\widehat{K}_{33} = K_{33} - K_{13}^T K_{11}^{-1} K_{13} - K_{23}^T K_{22}^{-1} K_{23},$$

and $\Lambda_i^{(n)} = \mathrm{diag}(\lambda_1^{(i)}, \lambda_2^{(i)}, \ldots, \lambda_{n_i}^{(i)})$.

A high-level description of a generic CMS method is as followings.

Generic CMS Method

1. Compute the *selected* eigenpairs $(\lambda_j^{(i)}, \boldsymbol{\phi}_j^{(i)})$ of the generalized eigenproblems $K_{ii}\boldsymbol{\phi}_j^{(i)} = \lambda_j^{(i)} M_{ii}\boldsymbol{\phi}_j^{(i)}$ for $i = 1, 2$,
2. Retain some eigenpairs $(\lambda_j^{(i)}, \boldsymbol{\phi}_j^{(i)})$ to define transformation matrix V_n,
3. Form M_n, K_n, B_n, L_n to define the reduced system Σ_n as in (2.9).

In the standard CMS method, the n_i modes $\boldsymbol{\phi}_j^{(i)}$ associated with smallest eigenvalues $\lambda_j^{(i)}$ are retained to define the projection matrix V_n. V_n is called the *Craig-Bampton transformation matrix* in structure dynamics [3].

In an alternative method, which we call the CMS$_\chi$, the n_i modes $\boldsymbol{\phi}_j^{(i)}$ in V_n are selected according to the highest norm of the rank-one *coupling matrices* $S_j^{(i)}$:

$$S_j^{(i)} = \frac{1}{\lambda_j^{(i)}} \widehat{M}_{i3}^T \boldsymbol{\phi}_j^{(i)} (\boldsymbol{\phi}_j^{(i)})^T \widehat{M}_{i3}. \tag{2.10}$$

Therefore, the selected modes $\boldsymbol{\phi}_j^{(i)}$ in CMS$_\chi$ may not be in the natural order as in CMS. As a result, to find such n_i modes, we may have to find more than n_i smallest eigenpairs of the matrix pairs (M_{ii}, K_{ii}). This will be shown by numerical examples in section 4. But first we give a justification for the CMS$_\chi$ method in the next section.

3 Derivation of CMS$_\chi$

Let us assume that Φ_i contains all N_i modes of the submatrix pairs (M_{ii}, K_{ii}) for $i = 1, 2$. Then the system Σ_N in its modal coordinate in frequency domain is of the form

$$\left(-\omega^2 \begin{bmatrix} I & & M_{13}^{(N)} \\ & I & M_{23}^{(N)} \\ (M_{13}^{(N)})^T & (M_{23}^{(N)})^T & \widehat{M}_{33} \end{bmatrix} + \begin{bmatrix} \Lambda_1^{(N)} & & \\ & \Lambda_2^{(N)} & \\ & & \widehat{K}_{33} \end{bmatrix} \right) \begin{bmatrix} X_1(\omega) \\ X_2(\omega) \\ X_3(\omega) \end{bmatrix} = \begin{bmatrix} B_1^{(N)} \\ B_2^{(N)} \\ \widehat{B}_3 \end{bmatrix} U(\omega). \tag{3.11}$$

For the sake of notation, we will drop the superscript $\cdot^{(N)}$ in the rest of section. By solving $X_1(\omega)$ and $X_2(\omega)$ from the first and second equations of (3.11) and then substituting into the third interface equation of the (3.11), it yields

$$\left(\omega^4 \sum_{i=1}^{2} \left[-M_{i3}^T (-\omega I + \Lambda_i)^{-1} M_{i3} \right] - \omega^2 \widehat{M}_{33} + \widehat{K}_{33} \right) X_3(\omega)$$
$$= \left(\omega^2 \sum_{i=1}^{2} \left[M_{i3}^T (-\omega I + \Lambda_i)^{-1} B_i \right] + \widehat{B}_3 \right) U(\omega). \tag{3.12}$$

In the context of structural dynamics, the equation (3.12) presents the force applied to the interface and applied to it by the subsystems.

Instead of solving equation (3.12) for $X_3(\omega)$ directly, we simplify the equation first, since we are only interested in looking for "important modes". An approximation of (3.12) is taking the first three terms of the power expansion in ω^2 of the coefficient matrix on the left hand side, and taking the constant term on the right hand side. This yields an approximate equation of (3.12):

$$\left[-\omega^4\left(M_{13}^T\Lambda_1^{-1}M_{13}+M_{23}^T\Lambda_2^{-1}M_{23}\right)-\omega^2\widehat{M}_{33}+\widehat{K}_{33}\right]\widetilde{X}_3(\omega)=\widehat{B}_3U(\omega), \quad (3.13)$$

Let the power series expansion of $\widetilde{X}_3(\omega)$ be formally denoted by

$$\widetilde{X}_3(\omega)=\left(\sum_{\ell=0}^{\infty}r_\ell\omega^{2\ell}\right)U(\omega),$$

where r_ℓ are called the ℓ-th moment (vector) of $\widetilde{X}_3(\omega)$. Then by comparing the two sides of equation (3.13) in the power of ω^2, the moments r_ℓ are given by

$$\begin{aligned}
r_0 &= \widehat{K}_{33}^{-1}\widehat{B}_3,\\
r_1 &= \widehat{K}_{33}^{-1}\widehat{M}_{33}r_0,\\
r_\ell &= \widehat{K}_{33}^{-1}(\widehat{M}_{33}r_{\ell-1}+(\sum_{i=1}^{2}M_{i3}^T\Lambda_i^{-1}M_{i3})r_{\ell-2}) \quad \text{for } \ell\geq 2.
\end{aligned}$$

By an exactly analogous calculation, for the reduced-order system Σ_n in its modal coordinates form, namely

$$M_n=\begin{bmatrix} I & & M_{13}^{(n)}\\ & I & M_{23}^{(n)}\\ (M_{13}^{(n)})^T & (M_{13}^{(n)})^T & M_{33}^{(n)}\end{bmatrix},\ K_n=\begin{bmatrix}\Lambda_1^{(n)} & & \\ & \Lambda_2^{(n)} & \\ & & K_{33}^{(n)}\end{bmatrix}$$

and

$$B_n=\begin{bmatrix}B_1^{(n)}\\ B_2^{(n)}\\ B_{33}^{(n)}\end{bmatrix},\ L_n=\begin{bmatrix}L_1^{(n)}\\ L_2^{(n)}\\ L_{33}^{(n)}\end{bmatrix}.$$

The moment vectors $r_\ell^{(n)}$ for the solution $\widetilde{X}_3^{(n)}(\omega)$ of the approximate interface equation are given by

$$\begin{aligned}
r_0^{(n)} &= (K_{33}^{(n)})^{-1}B_3^{(n)},\\
r_1^{(n)} &= (K_{33}^{(n)})^{-1}M_{33}^{(n)}r_0^{(n)},\\
r_\ell^{(n)} &= (K_{33}^{(n)})^{-1}(M_{33}^{(n)}r_{\ell-1}^{(n)}+(\sum_{i=1}^{2}(M_{i3}^{(n)})^T(\Lambda_i^{(n)})^{-1}M_{i3}^{(n)})r_{\ell-2}^{(n)}) \quad \text{for } \ell\geq 2.
\end{aligned}$$

Note that the dimensions of the moment vectors $\{r_\ell\}$ and $\{r_\ell^{(n)}\}$ are the same since we assume that the DOFs of the interface block is unchanged.

A natural optimal strategy to define a reduced-order system Σ_n is to match or approximate as many moments as possible. To match the first moment $r_0 = r_0^{(n)}$, it suggests that

$$K_{33}^{(n)} = \widehat{K}_{33} \quad \text{and} \quad B_3^{(n)} = \widehat{B}_3.$$

To match the second moment $r_1 = r_1^{(n)}$, it derives that

$$M_{33}^{(n)} = \widehat{M}_{33}.$$

Unfortunately, there is no easy way to match the third moment r_2 exactly. Instead, we try to minimize the difference between r_2 and $r_2^{(n)}$:

$$\begin{aligned}\|r_2 - r_2^{(n)}\|_2 &= \|\widehat{K}_{33}^{-1}\left(\sum_{i=1}^{2} M_{i3}^T \Lambda_i^{-1} M_{i3} - (M_{i3}^{(n)})^T (\Lambda_i^{(n)})^{-1} M_{i3}^{(n)}\right)\widehat{K}_{33}^{-1}\widehat{B}_3\|_2 \\ &\le c\|\underbrace{\sum_{j=1}^{N_1} S_j^{(1)} - \sum_{j=1}^{n_1}(S_j^{(1)})^{(n)}}_{1} + \underbrace{\sum_{j=1}^{N_2} S_j^{(2)} - \sum_{j=1}^{n_2}(S_j^{(2)})^{(n)}}_{2}\|_2, \qquad (3.14)\end{aligned}$$

where $c = \|\widehat{K}_{33}^{-1}\|_2\|\widehat{K}_{33}^{-1}\widehat{B}_3\|_2$, a constant independent of the modes $\phi_j^{(i)}$. $S_j^{(i)}$ and $(S_j^{(i)})^{(n)}$ are the coupling matrices for the j-th mode of the subsystem i as defined in (2.10). Assume that $S_j^{(i)}$ and $(S_j^{(i)})^{(n)}$ are in descending order according to their norms, respectively,

$$\|S_1^{(i)}\| \ge \|S_2^{(i)}\| \ge \cdots \ge \|S_{N_i}^{(i)}\|, \quad \|(S_1^{(i)})^{(n)}\| \ge \|(S_2^{(i)})^{(n)}\| \ge \cdots \ge \|(S_{n_i}^{(i)})^{(n)}\|.$$

The best we can do is to set

$$(S_j^{(i)})^{(n)} = S_j^{(i)} \quad \text{for} \quad j = 1, 2, \ldots, n_i.$$

This cancels out the first n_i terms of the differences labled as 1 and 2 of the upper bound in (3.14), and leaves the sums of the remaining terms smallest possible. This yields the CMS$_\chi$-mode selection rule as we described in section 2: *retain the first n_i modes of the subsystem i according to the largest norms of the coupling matrices $S_j^{(i)}$.*

Note that the matrices $\widehat{M}_{i3}$ which couples subsystems and the interface are included in the coupling matrices $S_j^{(i)}$. Therefore, they are reflected for the retention of modes of importance. These coupling effects are essentially ignored by the CMS mode selection. To this end, we also note that CMS$_\chi$-mode selection criterion is essentially the same as the one in the OMR method derived by the DtN mapping [1,6], but without the assumption of the special form of the external force term $Bu(t)$ in the original system Σ_N (1.1).

4 Numerical Experiments

In this section, we present two numerical examples to compare the two mode selection criteria discussed in this paper. All numerical experiments were run in MATLAB on a Linux Server with Dual 1.2Ghz CPUs and 2GB of memory.

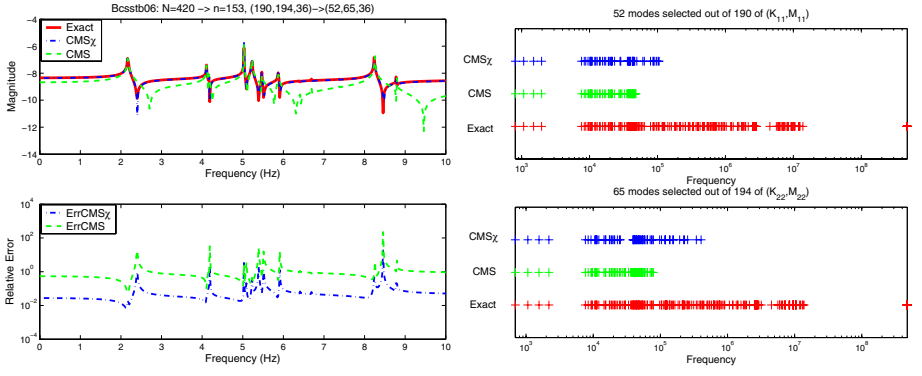

Fig. 2. Left: magnitudes (in log of base 10) of the transfer functions (top) and relative errors (bottom). Right: retained modes of subsystems by CMS and CMS_χ

Example 1. In this example, the mass and stiffness matrices M and K are from Harwell-Boeing BCS sparse matrix collection [5]. The number of DOFs of Σ_N is $N = 420$, and that of two subsystems are $N_1 = 190$ and $N_2 = 194$, respectively. The top left plot of Fig. 2 shows the magnitude (in log of base 10) of the transfer function $H(\omega)$ of the SISO system Σ_N with $B = L = [\ 1\ 0\ \ldots\ 0\ ]^T$. The transfer functions $H_n^{\mathrm{CMS}}(\omega)$ and $H_n^{\mathrm{CMS}_\chi}(\omega)$ of the reduced systems Σ_n, computed by CMS and CMS_χ, are shown in the same plot. The number of DOFs of reduced-order systems Σ_n is $n = 153$ with $n_1 = 52$ and $n_2 = 65$, respectively. The relative errors $|H(\omega) - H_n^{\mathrm{CMS}}(\omega)|/|H(\omega)|$ and $|H(\omega) - H_n^{\mathrm{CMS}_\chi}(\omega)|/|H(\omega)|$ shown in the lower left plot of Fig. 2 indicate that $H_n^{\mathrm{CMS}_\chi}(\omega)$ is a much accurate approximation of $H(\omega)$ than $H_n^{\mathrm{CMS}}(\omega)$, under the same order of reduced DOFs.

Two right plots of Fig. 2 show the eigenvalues of original systems and the ones retained by CMS and CMS_χ. Note again that the numbers of eigenvalues of subsystems retained by the two methods are the same. CMS_χ skips some of lower frequency eigenvalues, and uses some higher frequency eigenvalues to take into the account of coupling effects between the subsystems and the interface. On the other, CMS simply takes the lowest frequency eigenvalues in order.

Example 2. This is a SISO system Σ_N arised from simulation of a prototype radio-frequency MEMS filter [2]. The DOFs of Σ_N is $N = 490$ and that of two subsystems are $N_1 = N_2 = 238$. The DOFs of interface is $N_3 = 14$. Fig. 3 shows the transfer functions $H(\omega)$, $H_n^{\mathrm{CMS}}(\omega)$ and $H_n^{\mathrm{CMS}_\chi}(\omega)$. The DOFs of reduced subsystems by the

both methods are $n_1 = n_2 = 85$. The relative errors $|H(\omega) - H_n^{\mathrm{CMS}}(\omega)|/|H(\omega)|$ and $|H(\omega) - H_n^{\mathrm{CMS}_\chi}(\omega)|/|H(\omega)|$ in the lower left plot of Fig. 3 show the improvement made by the new CMS_χ method. Two right plots of Fig. 3 show the differences in the retention of the same number of modes of subsystems.

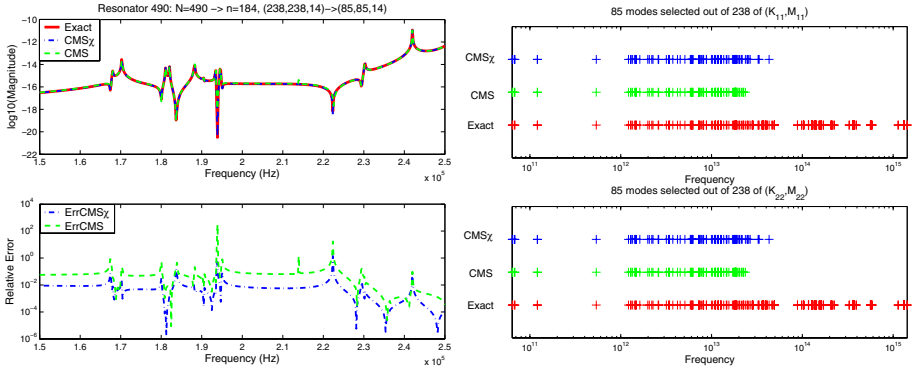

Fig. 3. Left: magnitudes (in log of base 10) of the transfer functions (top) and relative errors (bottom). Right: retained modes of subsystems by CMS and CMS_χ

5 Conclusion Remarks

A new coupling matrix-based mode selection criterion for the popular CMS method is presented in this paper. It is derived based on moment-matching property for the interface solution. Our work is motivated by the recent work of Givoli *et al* [1,6], in which the term "coupling matrix" is coined. Our mode selection criterion is compatible to the one proposed by Givoli *et al*, which uses Dirichlet-to-Neumann (DtN) mapping as an analysis tool. The performance improvement of the new mode selection criterion is demonstrated by numerical examples.

The coupling matrix-based mode selection costs more than the standard one, since some extra eigenpairs of the subsystems are typically required. If the sizes of subsystems are moderate, the extra cost may not be significant measured by the CPU time. Multilevel substructuring with an optimal mode selection is a subject of future study. It is worth to note that modal reduction methods as discussed in this paper are generally less accurate and efficient than Krylov subspace-based reduction methods. A Krylov subspace-based substructuring method is in progress.

Acknowledgments

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