Midterm

Instructions: Please answer the questions succinctly, thoughtfully and legibly. Good luck.

Name:

ID:

On section	you got	out of
1		10
2		15
3		30
4		45
5		45
6		30
Σ		175

1 Definitions [10 points]

1. (4 pts.) Define formally the star (*) operation on languages, i.e. $A^* = \ldots$

2. (6 pts.) State the Pumping Lemma for regular languages (formally and completely).

2 A Decision Procedure [15 pts.]

You are given a DFA D and want to determine if it accepts any words of which your name is a substring. Describe a decision procedure to do that.

3 Justified True or False [30 points]

Put an **X** through the **correct** box. Provide a **brief** (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, **make your justification a counter-example**. Each question in this section is worth 5 points.

---

1. $(\emptyset \circ \emptyset)^* = \emptyset^0$

True **False**

Explain:

2. If L is not a regular language then L is context-free.

True **False**

Explain:

3. If CFG G_1 has fewer rules than CFG G_2 then $|L(G_1)| \leq |L(G_2)|$.

True **False**

Explain:

4. $L = \{a^i b^j | i \geq j \geq 65536\}$ is a regular language.

True **False**

Explain:

5. Any Context Free Grammar can be transformed into an equivalent grammar having one rule of the form $S \rightarrow V_1 V_2 \ldots V_n$ and n rules of the form $V_i \rightarrow a_i, i = 1, \ldots, n$, where V_i are variables and a_i are terminals.

True **False**

Explain:

6. If $\overline{L}$ is finite then L is regular.

True **False**

Explain:

4 Finite Automata Problems [45 points]

1. (15 pts.) Give a DFA for the language of $(0 \cup 01)^*$.

2. (30 pts.) Determine if the following languages are regular or not and prove your claim in each case.

a) $\{a^n bc^{3n} | n \geq 0\}$

b) $\{a^{5n} | n \geq 0\}$

5 CFG Problems [45 pts.]

Write Context Free Grammars for the following languages:

a) (14 pts.) $A = \{ab^n c^m a^{n+m} c | n, m \geq 0\}$

b) (12 pts.) $B = \{ab^n c^{n+m} a^m c | n, m \geq 0\}$

c) (7 pts.) $A \cup B$

d) (12 pts.) $A \cap B$

(Hint: Consider the n's and the m's separately)

6 A Minimality Proof [30 points]

a) (10 pts.) Design a DFA for the language $\{w \in \{a,b\}^* | w$ starts with an a and contains an even number of b's$\}$.

b) (20 pts.) Find the smallest DFA for the language above and prove its minimality. (smaller=fewer states.)