Data, Logic, and Computing

ECS 17 (Winter 2024)

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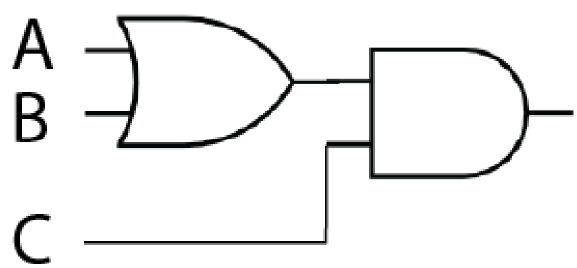
Discussion 3: logic gates

Exercise 1

Find the output/ truth table for this logic gate circuit. Convert it into a Boolean expression

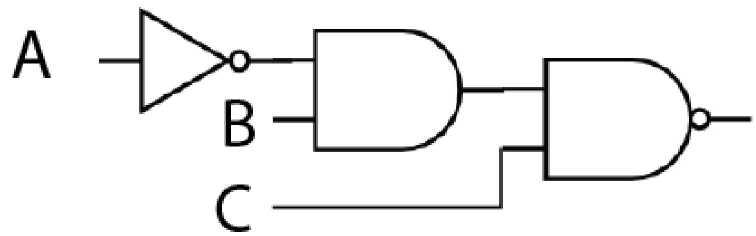


A	B	C	$A+B$	$(A+B)C$
1	1	1	1	1
1	1	0	1	0
1	0	1	1	1
1	0	0	1	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	0
0	0	0	0	0

The corresponding Boolean expression is $(A+B)C$.

Exercise 2

Find the output/ truth table for this logic gate circuit. Convert it into a Boolean expression



A	B	C	$\bar{A}$	$\bar{A}B$	$\bar{A}BC$	$\overline{\bar{A}BC}$
1	1	1	0	0	0	1
1	1	0	0	0	0	1
1	0	1	0	0	0	1
1	0	0	0	0	0	1
0	1	1	1	1	1	0
0	1	0	1	1	0	1
0	0	1	1	0	0	1
0	0	0	1	0	0	1

The corresponding Boolean expression is $\overline{\bar{A}BC}$.

Exercise 3

Build the truth table for the Boolean expressions:

a) $\bar{A}+B$

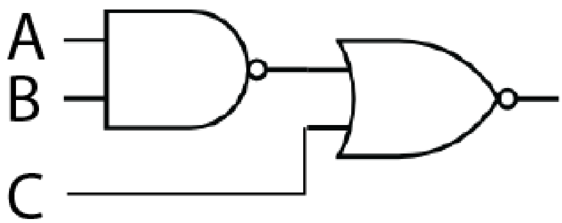
A	B	$\bar{A}$	$\bar{A}+B$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

b) $A+\bar{A}B$

A	B	$\bar{A}$	$\bar{A}B$	$A+\bar{A}B$
1	1	0	0	1
1	0	0	0	1
0	1	1	1	1
0	0	1	0	0

Exercise 4

Build a logic gate circuit for the Boolean expression $\overline{(\overline{AB}+C)}$



Exercise 5

Let us play a logical game. You find yourself in front of two rooms whose doors are closed. Behind each door, there could be a Lady or a Tiger. There is one sign on each door; you are told that if the first room contains a Lady, then the sign is true, but if a Tiger is in it, the sign is false. In room 2, the situation is the opposite: a Lady in room 2 means the sign is false, while a Tiger in room 2 means the sign is true. Here are the signs:

Room I	*Room II*
Both rooms contain Ladies	*Both rooms contain Ladies*

Can you find what is inside each room?

Let us build the table for the possible options for the two doors. We then check the validity of the two statements on the door, and finally check the consistency of the truth values for those statements with what we know about their validity.

Line	Room I	Room II	Sign on Room I	Sign on Room II	Compatibility
1	Lady	Lady	T	T	No
2	Lady	Tiger	F	F	No
3	Tiger	Lady	F	F	Yes
4	Tiger	Tiger	F	F	No

Justification:

Line 1 : There is a Lady in room II; the sign on that room should then be false, but it is true: incompatibility

Line 2 : There is a Lady in room I; the sign on that room should then be true, but it is false: incompatibility

Line 4 : There is a Tiger in room II; the sign on that room should then be true, but it is false: incompatibility

Therefore only line 3 is compatible: there is a Tiger in room 1 and a Lady in room 2!

Exercise 6

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter three people, John, Kari, and Tania. They make the following statements:

John says: "Tania is not a knave"
Kari says: "John and Tania are both knights"
Tania says: "John is a knight or Kari is a knave"

What are John, Kari, and Tania?

We check all possible "values" for John, Kari, and Tania, as well as the veracity of their statements:

Line number	John	Kari	Tania	John says	Kari says	Tania says
1	Knight	Knight	Knight	T	T	T
2	Knight	Knight	Knave	F	F	T
3	Knight	Knave	Knight	T	T	T
4	Knight	Knave	Knave	F	F	T
5	Knave	Knight	Knight	T	F	F
6	Knave	Knight	Knave	F	F	F
7	Knave	Knave	Knight	T	F	T
8	Knave	Knave	Knave	T	F	T

We can eliminate:

- Line 2, as John would be a knight but he lies
- Line 3, as Kari would be a knave but she tells the truth
- Line 4, as John would be a knight but he lies
- Line 5, as John would be a knave but he tells the truth
- Line 6, as Kari would be a knave but she tells the truth
- Line 7, as John would be a knave but he tells the truth
- Line 8, as John would be a knave but he tells the truth

Line 1 is valid, and it is the only one. Therefore,John, Kari, and Tania are all knights