ECS 17: Data, Logic, and Computing
Final
March 14, 2022

Notes:
1) The final is open book, open notes.
2) You have 2 hours minutes, no more: I will strictly enforce this.
3) The final is graded over 100 points
4) You can answer directly on these sheets (preferred), or on loose paper.
5) Please write your name at the top right of each page you turn in!
6) Please, check your work! **Also, do show your work**

Part I Data (10 questions, each 3 points; total 30 points)
(These questions are multiple choices; in each case, find the most **plausible** answer)

1) *CPUs in computers are covered with a heat sink to:*
   a. Dissipate the heat they release
   b. Make them look pretty
   c. Protect them from shocks (for example if the computer falls)
   d. Isolate them from the other devices to avoid interferences

2) *Let A be the number with the binary representation 100 and B the number whose hexadecimal representation is 1; which of these numbers X (in hexadecimal form) satisfies $BX^2-AX+A=0$?*
   a. A
   b. 1
   c. 2
   d. D

3) *Which word is encoded in the ASCII code 01100101 01100011 01110011?*
   a. ect
   b. ess
   c. ECS
   d. ecs

4) *A continuous-time signal is uniformly sampled over 5001 samples, for a total of 2.5 seconds. Components with which of those frequencies could be correctly detected in the sampled signal? (circle all that apply)*
   a. 400 Hz,
   b. 800 Hz,
   c. 1200 Hz,
   d. 1600 Hz.

5) *You take a picture with a digital camera, and you know that this picture requires 32 Mbytes of storage (without compression). Assuming that each pixel is stored on 32 bits, what is the resolution of your camera, in megapixels:*
   a. 16
   b. 1
   c. 8
   d. 32

6) *Which binary number comes right after the binary number 110111?*
   a. 111000
   b. 110111
   c. 111111
   d. 110112

7) *The binary representation of the hexadecimal 95 is*
   a. 1011001
   b. 1011111
   c. 10010101
   d. 149

8) *You want to store a movie on your computer. You know that your movie is 2 hour long. It was filmed at a rate of 25 frames per second and each frame requires 10 kilobytes of storage. The soundtrack was stored in stereo, recorded at 50KHz, with 2 bytes per point. How much space is needed to store the whole movie and its soundtrack, in megabytes (assuming that 1 megabyte = 1000 kilobytes)?*
   a. 2520 megabytes (=2.52 GB)
   b. 3240 megabytes (=3.24 GB)
   c. 1440 megabytes (=1.44 GB)
   d. 1800 megabytes (=1.8 GB)

9) *One of these four sentences would NOT be considered as a proposition in Logic:*
   a. All cats are white
   b. An apple is not a fruit
   c. 2+4 = 8
   d. *X+2=8*

10) *What is the largest number (unsigned integer) that can be stored on one bit?*
   a. 255
   b. 256
   c. 128
   d. 1

Part II Logic (three problems; total 30 points)

1) Complete the logic table corresponding to the logic gate shown below. Convert it into a Boolean expression
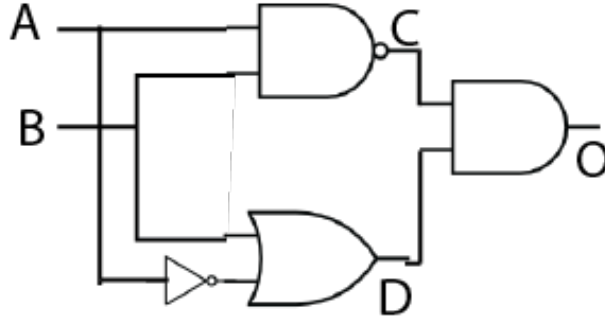
A	B	C	D	O
1	1			
1	0			
0	1			
0	0			

2) A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, `If Sally is a knight, I am a knave.' John says, `Alex and Sally are of the same type.' Sally claims, `I like chocolate.' *(Note that "to be of the same type" means that they are both knights or they are both knaves.)* Does Sally really like chocolate? Justify your answer.

3) Let *p, q,* and *r* be three propositions. Use a truth table or logical equivalence to indicate if the proposition $(p \wedge q) \vee r \vee (\neg q \wedge \neg r) \vee (\neg p \wedge \neg r)$ is a tautology, a contradiction, or neither

Part III. Proofs (4 questions; each 10 points; total 40 points)

1) Let *n* be an integer. Show that *n* is even if and only if $n + n^2 - n^3$ is even.

2) Let *a* and *b* be two integers. Show that $a^2 - 4b \neq 3$ *(hint: you may assume true the fact that when n is an integer, if n^2 is odd, then n is odd).*

3) Show by induction that $\sum_{i=1}^{n} \frac{2}{3^i} = 1 - \frac{1}{3^n}$ for all $n \geq 1$

3) Let a_k be the sequence defined by $a_k = a_{k-1} + k + 4$ for $k \geq 2$, with $a_1 = 5$. Show by induction that $a_n = \dfrac{n(n+9)}{2} \qquad \forall n \geq 1$

Part IV. Counting (Extra credit: 5 points**)**

How many bitstrings of length $n>2$ can we form that contain at least one 0 and at least one 1?

Appendix A: ASCII table

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	−	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C		
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

Appendix B: Binary to Hexadecimal

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F