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ECS 17: Data, Logic, and Computing
Midterm
February 28, 2023

Notes:

1) Midterm is open book, open notes…
2) You have 50 minutes, no more: I will strictly enforce this.
3) The midterm is graded over 70 points.
4) You can answer directly on these sheets (preferred), or on loose paper.
5) Please write your name at the top right of at least the first page that you turn in!
6) Please, check your work!

Exercise 1 (2 parts, each worth 10 points; total 20 points)

Let n be an integer. Give a direct proof and an indirect proof of the proposition, if n is odd then $2n^2+5n+2$ is odd.

Exercise 2 (10 points)

Let m and n be 2 integers. Using the method of proof of your choice, show that if mn is odd, then m is odd and n is odd.

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Exercise 3 (1 question, 10 points)

Let *n* be an integer. Use a proof by contradiction to show that $\frac{6n+1}{2n+4}$ is not an integer.

Exercise 4 (1 question, 10 points)

Let *n* be a natural number (i.e., *n* is a positive integer different from 0). Use a proof by contradiction to show that if *n* is a perfect square, then *2n* is not a perfect square. (*A natural number n is a perfect square if there exists an integer k such that* $n=k^2$).

Exercise 5 (1 question, 10 points)
Let x be a real number. Show that if $x^3 + x^2 - 2x < 0$ then $x < 1$.

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Exercise 6 (1 question, 10 points)

Prove or disprove that there exits an integer *n* such that $n^2 + 3n + 2$ is odd.

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Appendix

The ECS17 Potato Prayers

1) Thou shalt not say "there exists k" without mentioning the domain of k.

2) Thou shalt not say "it is obvious"

3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg q \to \neg p$. This is the basis for the proof by contrapositive.

3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.

4) An integer n is even if and only if there exists and integer k such that $n = 2k$. We say also that n is a multiple of 2.

5) An integer n is odd if and only if there exists and integer k such that $n = 2k + 1$.

6) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let n be an integer. Then:

    a) If n is even, then $n + 1$ is odd
    b) if n is odd, then $n + 1$ is even

2) Let n be an integer. Then:

    a) n is even, if and only if n^2 is even
    b) n is odd, if and only if n^2 is odd

3) $\forall n \in \mathbb{Z}, \quad n(n+1)$ is even.

4) $\sqrt{2}$ is irrational.