Data, Logic, and Computing

ECS 17 (Winter 2024)

Patrice Koehl
koehl@cs.ucdavis.edu

January 2, 2024

Homework 1 - For 1/17/2024

Exercise 1 *(10 points)*

Seventy biscuits are to be fed to twelve pets; each pet is either a cat or a dog. Each dog is to get seven biscuits, and each cat is to get five. How many dogs and how many cats are there?

It is best to formalize the problem. Let C be the number of cats, and D the number of dogs. We translate the information we have as relationships on C and D:

- There are 12 pets total: $C + D = 12$
- Each dog eats 7 biscuits, each cat eats 5 biscuits, and there are 70 biscuits: $7D + 5C = 70$

The corresponding system of equation

$$\begin{aligned} D + C &= 12 \\ 7D + 5C &= 70 \end{aligned}$$

leads to $D = 5$ and $C = 7$.

Interestingly, there is another way to solve this problem. Each pet eats at least 5 biscuits. As there are 12 pets, this leads to 60 biscuits, and therefore there are only 10 biscuits left. Those biscuits are then given to dogs, two per dog... and therefore there are 5 dogs, and consequently 5 cats.

Exercise 2 *(10 points)*

A baseball and a baseball bat cost \$20.10. The bat costs \$20.0 more than the ball. How much does the ball cost?

Intuitively, we would say the bat costs \$20.0 and the ball costs \$0.10 but we would be wrong!. We need a more systematic approach to solve this problem. Let x be the cost of the ball, and y the cost of the bat. We translate the two sentences given to us as:

$$\begin{aligned} x + y &= 20.10 \\ y &= x + 20.0 \end{aligned}$$

The solution to this system of equations is $x = \$0.05$ and $y = \$20.05$.

Exercise 3 *(10 points)*

There are 50 cats living in a barn, all are either brown or orange. There is at least one orange cat. For every two cats, at least one is brown. How many orange cats and how many brown cats are there?

We know that there is at least one orange cat. Let us call this orange cat OR. Consider any other of the 49 cats, C. The pair (OR, C) is a pair of cats. Based on the setting of the problem, this pair contains at least one brown cat. Since OR is orange, C is then brown, and this is true for all the 49 cats besides OR.

Therefore, there is one orange cat and there are 49 brown cats.

Exercise 4 *(10 points)*

You are on an island inhabited by three types of people: knights (always make true statements), knaves (always make false statements) and spies (sometimes make true statements and sometimes make false statements). You come across 3 people Adam, Ben and Carl. You know that one is a knight, one is a knave and one is a spy. They say the following:

- *Adam: "I am not a knight"*
- *Ben: "I am not a knave"*
- *Carl: "I am not a spy"*

Determine which person is what or whether you do not have enough information.

The easiest approach to solve this problem is to use a table: T means that the person said the truth, and F that the person said a lie. Note that when we fill up the table, we need to consider that we have one Knight, one Knave, and one Spy.

Line	Adam	Ben	Carl	Adam says	Ben says	Carl says
1	Knight	Knave	Spy	F	F	F
2	Knight	Spy	Knave	F	T	T
3	Knave	Knight	Spy	T	T	F
4	Knave	Spy	Knight	T	T	T
5	Spy	Knight	Knave	T	T	T
6	Spy	Knave	Knight	T	F	T

Now we note that:

- lines 1 and 2 cannot be correct: Adam would be a knight that lies.
- line 3 and 4 cannot be correct: Adam would be a knave that tells the truth.
- line 5 cannot be correct: Carl would be a knave that tells the truth

Line 6 is the only possibility, i.e. Adam is a Spy, Ben is a Knave, and Carl is a Knight.