# Data, Logic, and Computing

ECS 17 (Winter 2024)

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January 25, 2024

## Homework 3 - For 1/31/2024

## Exercise 1 *(10 points)*

*We are on the island of knights and knaves . You meet three residents, Alex, Bill, and Claudia who make the following statements:*

*Alex says: "Bill is a knave and Claudia is a knight"*
*Bill says: "Claudia is a knight and Alex is a knight"*
*Claudia says: "I like cookiet"*

*Does Claudia really like cookies?*

We check all possible "values" for Alex, Bill, and Claudia, as well as the veracity of their statements.

| Line number | Alex | Bill | Claudia | Alex says | Bill says |
|---|---|---|---|---|---|
| 1 | Knight | Knight | Knight | F | T |
| 2 | Knight | Knight | Knave | F | F |
| 3 | Knight | Knave | Knight | T | T |
| 4 | Knight | Knave | Knave | F | F |
| 5 | Knave | Knight | Knight | F | F |
| 6 | Knave | Knight | Knave | F | F |
| 7 | Knave | Knave | Knight | T | F |
| 8 | Knave | Knave | Knave | F | F |

We can eliminate:

- Line 1, as Alex would be a knight but he lies
- Line 2, as Alex would be a knight but he lies
- Line 3 as Bill would be a knight but he lies

- Line 4 as Alex would be a knight but he lies
- Line 5 as Bill would be a knight but he lies
- Line 6 as Bill would be a knight but he lies
- Line 7 as Alex would be a knave but he says the true

Therefore all three are knaves. Claudia then lies: she does not like cookies!

## Exercise 2 *(10 points)*

*A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, "At least one of the following is true: that Sally is a knave or that I am a knight." John says, "Alex could claim that I am a knave." Sally claims, "Neither Alex nor John are knights." Can you find who is a knight and who is a knave?*

We check all possible "values" for Alex, John and Sally, as well as the veracity of their statements.

| Line number | Alex | John | Sally | Alex says | John says | Sally says |
|---|---|---|---|---|---|---|
| 1 | Knight | Knight | Knight | T | F | F |
| 2 | Knight | Knight | Knave | T | F | F |
| 3 | Knight | Knave | Knight | T | T | F |
| 4 | Knight | Knave | Knave | T | T | F |
| 5 | Knave | Knight | Knight | F | T | F |
| 6 | Knave | Knight | Knave | T | T | F |
| 7 | Knave | Knave | Knight | F | F | T |
| 8 | Knave | Knave | Knave | T | F | T |

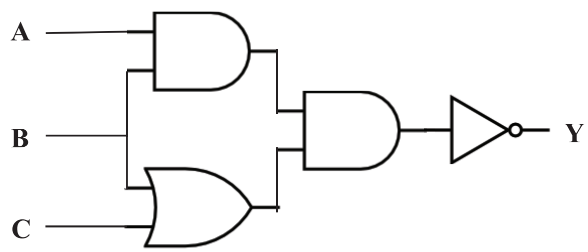
We can eliminate:

- Line 1, as John would be a knight but he lies
- Line 2, as John would be a knight but he lies
- Line 3 as John would be a knave but he says the true
- Line 4 as John would be a knave but he says the true
- Line 5 as Sally would be a knight but she lies
- Line 6 as Alex would be a knave but he says the true
- Line 8 as Alex would be a knave but he says the true

Line 7 is valid, and it is the only one. Therefore, both Alex and John are knaves, while Sally is a knight.

## Exercise 3 *(10 points)*

*Find the output/ truth table for this logic gate circuit. Convert it into a Boolean expression*

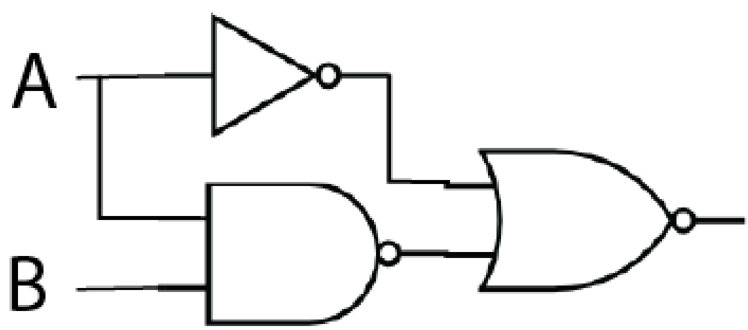



| $A$ | $B$ | $C$ | $AB$ | $B+C$ | $AB(B+C)$ | $Y$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |

The corresponding Boolean expression is $\overline{AB(B+C)}$.

## Exercise 4 *(10 points)*

*Find the output/ truth table for this logic gate circuit. Can you find a simpler logic gate that would perform the same operation on $A$ and $B$?*




| $A$ | $B$ | $\overline{A}$ | $\overline{AB}$ | $\overline{A}+\overline{AB}$ | $\overline{\overline{A}+\overline{AB}}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 |

The corresponding Boolean expression is $\overline{\overline{A}+\overline{AB}}$. Note however that the output of this logic gate is exactly the ouput of the AND logic gate.

## Exercise 5 *(10 points)*

*Let us play a logical game. You find yourself in front of three rooms whose doors are closed. Behind each door, there could be a Lady or a Tiger. There is only one Lady and two Tigers. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, and that at least one of the other two signs is false Here are the signs:*

| *Room I* | *Room II* | *Room III* |
|---|---|---|
| *A Tiger is in room II* | *A Tiger is in this room* | *A Tiger is in room I* |

*Can you find what is inside each room?*

Let us build the table for the possible options for the three doors. We then check the validity of the two statements on the door, and finally check the consistency of the truth values for those statements with what we know about their validity.

| Line | Room I | Room II | Room III | Sign I | Sign II | Sign III | Compatibility |
|---|---|---|---|---|---|---|---|
| 1 | Lady | Tiger | Tiger | T | T | F | Yes |
| 2 | Tiger | Lady | Tiger | F | F | T | No |
| 3 | Tiger | Tiger | Lady | T | T | T | No |

Justification:

Line 2 : There is a Lady in room II; the sign on that room should then be true, but it is false: incompatibility

Line 3 : There are in room I and room II; one of the signs on these rooms should then be false, but they are both true: incompatibility

Therefore only line 1 is compatible: there is a Tiger in room II and room III and a Lady in room I!

## Exercise 6 *(Extra credit: 5 points)*

*Which answer in the list below is the correct answer to this question?*

1) *All of the below*

2) *None of the below*

3) *All of the above*

4) *One of the above*

5) *None of the above*

6) *None of the above*

We check the validity of each answer:

Answer 1) If answer 1 is true, then both answer 4 and 5 would be true... but those two answers contradict each other. Therefore answer 1 is false.

Answer 2) If answer 2 is true, then answer 5 is false, which would mean that all answers from1 to 4 are true... but we know that answer is false. Therefore answer 2 is false.

Answer 3) We know that answers 1 and 2 are false; therefore answer 3 cannot be true; it is false

Answer 4) As answers 1, 2, and 3 are false, answer 4 is false.

Answer 5) As answer 1, 2, 3, and 4 are false, answer 5 is true

Answer 6) As answer 5 is true, answer 6 cannot be true.

Therefore, there is a single solution, Answer 5.