Data, Logic, and Computing

ECS 17 (Winter 2024)

Patrice Koehl
koehl@cs.ucdavis.edu

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Homework 7 - For 2/28/2024

Exercise 1 *(5 points each; total 20 points)*

Determine the truth values of the following statements; justify your answers:

a) $\forall n \in \mathbb{N}, (n+2) > n$

b) $\exists n \in \mathbb{N}, 2n = 3n$

c) $\forall n \in \mathbb{Z}, 3n \leq 4n$

d) $\exists x \in \mathbb{R}, x^4 < x^2$

Exercise 2 *(10 points each; total 50 points)*

Show that the following statements are true.

a) Let x be a real number. Prove that if x^3 is irrational, then x is irrational.

b) Let x be a positive real number. Prove that if x is irrational, then $\sqrt{x}$ is irrational.

c) Prove or disprove that if a and b are two rational numbers, then a^b is also a rational number.

d) let n be a natural number. Show that n is even if and only if $3n + 8$ is even.

e) Prove that either $4 \times 10^{769} + 22$ or $4 \times 10^{769} + 23$ is not a perfect square. Is your proof constructive, or non-constructive?

Note: for question e), a natural number n is a perfect square if there exists a natural number q such that $n = q^2$. For example, 4, 9, 16, 25, ... are all perfect squares while 2, 3, 5, 6,.... are not.

Exercise 3 *(10 points)*

Let n be a natural number and let $a_1, a_2, \ldots, a_n$ be a set of n real numbers. Prove that at least one of these numbers is greater than, or equal to the average of these numbers. What kind of proof did you use?

Extra Credit *(5 points)*

Use Exercise 3 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.