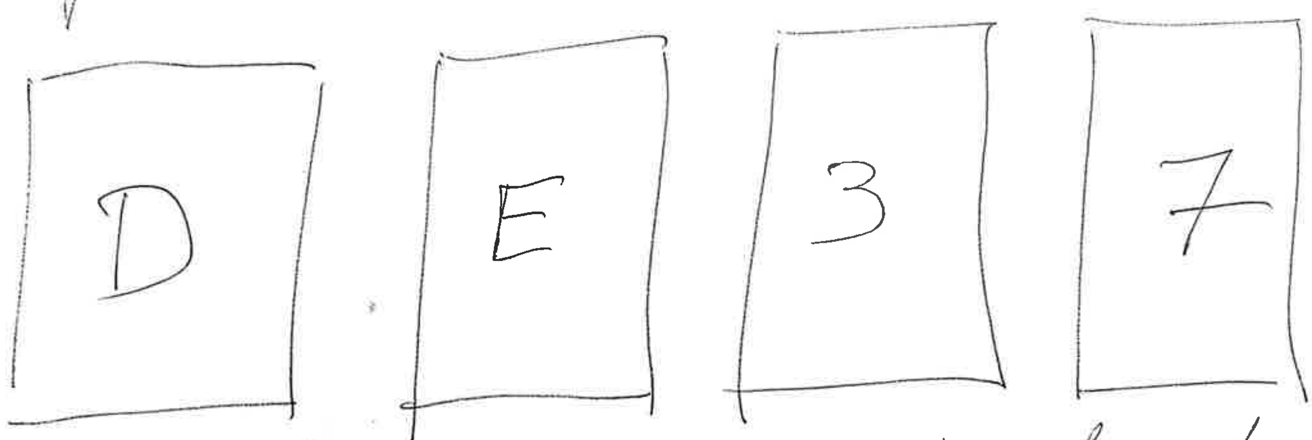
Discussion 2/12

Exercise 1

You are given a deck of cards.
Each card has a letter on its front and a digit (0 - 9) on its back.
Special rule. If there is a D on the front, there is a 3 on the back.



What is the minimum number of cards you need to turn to check the rule?

p : The front of the card is a D

q : the back of the card is a 3.

Rule: $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The rule is broken when p is true and q is false.

$\boxed{D}$ I need to turn this card.

$\boxed{E}$ No need to turn this one.

$\boxed{3}$ $(p \to q) \Leftrightarrow (\neg q \to \neg p)$

$\neg q$: the back is not a 3

$\neg p$: the front is not a D

$\neg q$ is false. I don't not need to return the 3

$\boxed{7}$ I need to return this card.

Exercise 2 Let n be an integer.

Show that

$$\text{if } \underbrace{2n^2 + n + 9 \text{ is odd,}}_{p} \text{ then } \underbrace{n \text{ is even.}}_{q}$$

p $2n^2 + n + 9$ is odd $\qquad \neg p$ $2n^2 + n + 9$ is even

$\downarrow \qquad\qquad\qquad\qquad \uparrow$

q n is even $\qquad \neg q$: n is odd

Indirect proof:

We assume $\neg q$ is true : n is odd.

There exists an integer k such that $n = 2k + 1$

$$\begin{aligned} 2n^2 + n + 9 &= 2(2k+1)^2 + 2k + 1 + 9 \\ &= 2(4k^2 + 4k + 1) + 2k + 1 + 9 \\ &= 8k^2 + 10k + 12 \\ &= 2(\underbrace{4k^2 + 5k + 6}_{\text{integer}}) \end{aligned}$$

Therefore $2n^2 + n + 9$ is even; $\neg p$ is true.

p: $2n^2 + n + 9$ is odd

q: n is even.

We assume p is true. There exists an integer k such that

$$2n^2 + n + 9 = 2k + 1$$

$$\begin{aligned} n &= 2k + 1 - 2n^2 - 9 \\ &= 2k - 2n^2 - 8 \\ &= 2\underbrace{(k - n^2 - 4)}_{\text{integer.}} \end{aligned}$$

n is even: q is true.

p: $2n^2 + 3n + 9$ is odd

q: n is even

Direct proof: I suppose p is true.
there exists an integer k such that

$$2n^2 + 3n + 9 = 2k + 1$$

$$2n^2 + 2n + n + 9 = 2k + 1$$

$$n = -2n^2 - 2n + 2k - 8$$

$$= 2 (\underbrace{-n^2 - n + k - 4}_{\text{integer}})$$

n is an even number.

Inspector Craig. A bank in London has been robbed. The notorious criminals A, B, and C have been called in. It has been established that.

- A is innocent
- if B is guilty, then A is guilty
- if C is innocent, then A is guilty.
- at least one of them is guilty.

P_A : A is guilty

P_B : B is guilty

P_C : C is guilty.

$$\neg P_A \qquad (1)$$

$$\neg P_A \to \neg P_B \Leftrightarrow P_B \to P_A \qquad (2)$$

$$\neg P_C \longrightarrow P_A \qquad (3)$$

$$P_A \vee P_B \vee P_C \qquad (4)$$