Logic 1/31

I) Propositions (statements)

A sentence that has a value of true (T) or false (F)

II) Compound propositions.

a) Compound proposition involving a single proposition:

not: $\neg p$

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

b) Compound propositions that are based on 2 propositions.

p	q	AND $p \wedge q$	OR $p \vee q$	XOR $p \oplus q$
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Conditional: $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional: $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \rightarrow q$

If you cheat during the exam, then you get a $\emptyset$

p: "You cheat during the exam"

q: "You get a $\emptyset$"

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

II) Comparison of propositions

"Equal"

$A = B$

Proposition equivalence

$p \Longleftrightarrow q$

$>$

$<$

III) Special propositions

- A proposition p that is always true is called a tautology.
- A proposition p that is always false is called a contradiction.

Example:

Let p be a proposition. $p \vee \neg p$ is a tautology.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F
		tautology	contradiction

Properties:

$\neg(\neg p) \iff p$

$p \vee p \iff p$

$p \wedge p \iff p$

$p \vee q \iff q \vee p$

$\neg(p \wedge q) \iff \neg p \vee \neg q$ | De Morgan's Law.

$\neg(p \vee q) \iff \neg p \wedge \neg q$ | De Morgan's Law.

$p \vee \neg p \iff T$ tautology

$p \wedge \neg p \iff F$ contradiction.

Exercise :

Show that

$A = (p \wedge q) \vee (\neg p \vee \neg q)$ is a tautology

1) Safe method

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	A
T	T	T	F	F	F	T
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

2) $B = p \wedge q$

$\neg B = \neg (p \wedge q) = \neg p \vee \neg q$

$A = B \vee \neg B \Leftrightarrow T$

3.4 List of important logical equivalences

Let p, q, and r be two propositions. T is a tautology and F is a contradiction.

Logical equivalence	Name
$\neg(\neg p) \Leftrightarrow p$	Double negation
$p \vee p \Leftrightarrow p$	Idempotent 1
$p \wedge p \Leftrightarrow p$	Idempotent 2
$p \vee q \Leftrightarrow q \vee p$	Commutativity 1
$p \wedge q \Leftrightarrow q \wedge p$	Commutativity 2
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$	Associativity 1
$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	Associativity 2
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	Distributivity 1
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributivity 2
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	De Morgan's law 1
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan's law 2
$p \vee F \Leftrightarrow p$	Absorption law 1
$p \vee T \Leftrightarrow T$	Absorption law 2
$p \wedge F \Leftrightarrow F$	Absorption law 3
$p \wedge T \Leftrightarrow p$	Absorption law 4
$\neg T \Leftrightarrow F$	Complement law 1
$\neg F \Leftrightarrow T$	Complement law 2
$p \vee \neg p \Leftrightarrow T$	Complement law 3
$p \wedge \neg p \Leftrightarrow F$	Complement law 4
$p \rightarrow q \Leftrightarrow (\neg p) \vee q$	Implication law 1
$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$	Implication law 2
$(p \leftrightarrow q) \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$	Equivalence law