Logic 2/2/26

I) Compound propositions

Let p, q, r be 3 propositions.

- negation: $\neg p$
- conjunction: AND $p \wedge q$
- disjunction OR: $p \vee q$

  exclusive OR: XOR $p \oplus q$
- implication: $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Let x be a real number.

Show that if $x^2 = -4$ then Trump will be president in 2029.

p: $x^2 = -4$

q: Trump will be president in 2029.

p is always false, therefore $p \to q$ is true.

biconditional: $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

We need to distinguish the bicanditional $(p \leftrightarrow q)$ from the logically equivalent to $(p \Longleftrightarrow q)$

Properties:

$$\neg(\neg p) \Longleftrightarrow p$$

$$p \vee T \Longleftrightarrow T$$

$$p \wedge T \Longleftrightarrow p$$

De Morgan's laws: $\neg(p \wedge q) \Longleftrightarrow \neg p \vee \neg q$

$$\neg(p \vee q) \Longleftrightarrow \neg p \wedge \neg q$$

$$p \wedge \neg p \Longleftrightarrow F$$

Distributivity:

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

Implication:

$$p \to q \Leftrightarrow \neg q \to \neg p \quad (P)$$

implication — contrapositive.

$p \to q$ is not equivalent to $q \to p$

$p \to q$ is not equivalent to $\neg p \to \neg q$

Applying (P): $\neg p \to \neg q \Leftrightarrow (\neg\neg q) \to (\neg\neg p)$

$$\Leftrightarrow q \to p$$

$$p \to q \Leftrightarrow \neg p \vee q \quad (Q)$$

Exercise 1 :

Show that $p \vee \neg p \iff T$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Exercise 2:

Show that $(p \wedge q) \vee (\neg p \vee \neg q) = A$ is a tautology.

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	A
T	T	T	F	F	F	T
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

A is a tautology.

(6)

$$A \iff (p \wedge q) \vee (\neg p \vee \neg q)$$

$$\iff (p \wedge q) \vee (\neg (p \wedge q))$$

(De Morgan's law)

Let us define $B = p \wedge q$

$$A \iff B \vee \neg B$$

$$\iff \top$$