Proofs 2/5

I) What is a proof?

Premises

proof $\left\{ \begin{array}{l} \text{Existing knowledge} \\ \text{Logic to put together what is known} \end{array} \right.$

Conclusions

$p \to q$ is true

p true

is q true?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow$ true

$p \rightarrow q$ true

$\therefore$ q is true

Modus ponens

<u>Examples:</u> Inspector Craig at Scotland Yard.

Shipments of expensive items have been going missing from the docks. The notorious criminals A, B, and C are brought in for questioning. The following evidence was established:

- B is guilty
- if B is guilty, then C is innocent
- if C is innocent, then A is guilty
- At least one of A, B, or C is guilty.

A proof that B is guilty, C is innocent, and A is guilty.

I define three propositions:

A: A is guilty

B: B is guilty

C: C is guilty.

Translate the evidence into propositions:

$$B \quad (1)$$

$$B \to \neg C \quad (2)$$

$$\neg C \to A \quad (3)$$

$$A \lor B \lor C \quad (4)$$

B is guilty → based on (1)

B (1)

$B \to \neg C$ (2)

$\therefore$ $\neg C$ is true → C is innocent

$$\neg C$$
$$\neg C \to A \quad (3)$$

$\therefore \to A$ is true

Second excemple:

Mr Mc Gregor, a London shopkeeper, calls Scotland Yard to report a robbery. The notorious criminals A, B, and C were brought in for questioning. The following evidence was established:

- B is innocent.
- if A is innocent, then B is guilty.
- if C is innocent, then A is innocent.
- At least one of them is guilty.

We define 3 propositions:

A: A is guilty

B: B is guilty

C: C is guilty.

Translate the evidence:

- $\neg B$ (1)
- $\neg A \to B$ (2)
- $\neg C \to \neg A$
- $A \vee B \vee C$

Proof:

B is innocent ($\neg B$ is true) (1)

We haved shown in class that

$$p \to q \iff \neg q \to \neg p$$

Evidence (2) is equivalent to $\neg B \to A$.

$$\frac{\begin{array}{l}\neg B \\ \neg B \to A\end{array}}{}$$

Therefore A is true A is guilty.

$$\neg C \to \neg A \iff A \to C$$

$$\frac{\begin{array}{l}A \\ A \to C\end{array}}{}$$

Therefore C is true: C is guilty.

III How do we show that an implication is true?

<u>Definition 1:</u>

An integer number n is even if and only if it is a multiple of 2.

equivalent to:

An integer number n is even if and only if there exists an <u>integer number k</u> such that $n = 2k$

An integer number n is odd if and only if there exists an integer number k such that $n = 2k + 1$