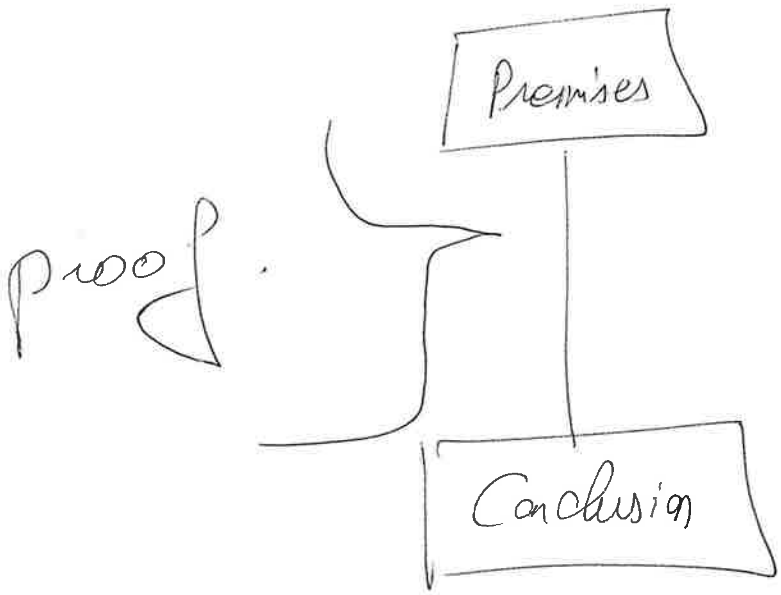
Proofs 2/9

Recap



One principle:

$$\frac{\begin{array}{l} p \text{ is true} \\ p \to q \text{ is true} \end{array}}{q \text{ is true}}$$

(2) How to prove that $p \to q$ is true?

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

if p is false, necessarily, $p \to q$ is true.

Let x be a real number. Show that if $x^2 = -1$, then x^3 is irrational

p: $x^2 = -1$ is false

q: x^3 is irrational

$p \to q$ is true

If $2+2=5$, then Trump will be president in 2025

Example 2

Let n be an integer.

Show that if n is even, then n^2 is even

p: n is even

q: n^2 is even

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

I assume that p is true.

p is true means that
n is an integer that is even.

A number n is even if there exists an $\boxed{\text{integer}}$ k such that $n = 2k$

What about q?

$$n^2 = (2k)^2$$
$$= 2k \times 2k$$
$$= 2\underbrace{(2k^2)}_{\text{integer}}$$

Therefore n^2 is even.

Therefore, if I assume that p is true, necessarily q is true.

Therefore $p \to q$ is true.

To show that $p \to q$ is true, I can assume that p is true and show that then q is true: direct proof of $p \to q$

Example 3: Let n be an integer. Show that if n is even, $(n+1)^2$ is odd.

p: n is even

q: $(n+1)^2$ is odd

Assumption: p is true.

n is an even integer. There exists an integer k such that $n = 2k$.

$$(n+1)^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2\underbrace{(2k^2+2k)}_{\text{integer}} + 1$$

$(n+1)^2$ is odd: q is true.

When p is true, necessarily q is true.

$p \to q$ is therefore true.

Example 4 Let n be an integer.

Show that if n^2 is even, then n is even.

p: n^2 is even

q: n is even

Let us assume p is true.

There exists an integer k such that $n^2 = 2k$

This is a dead end because I cannot safely ~~take~~ the the square root of an integer.

I need another route.

I want to show $p \to q$ is true.

$$p \to q \iff \underbrace{\neg q \to \neg p}_{\text{contrapositive}}$$

p : n^2 is even $\qquad \neg p$ n^2 is odd

$\downarrow$ direct $\qquad \uparrow$ indirect

q : n is even $\qquad \neg q$: n is odd

We assume $\neg q$ is true. n is odd.

$n = 2k+1$, where k is an integer.

$$n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(\underbrace{2k^2+2k}_{\text{integer}})+1$$

n^2 is odd. therefore $\neg p$ is true.

I have showed that $\neg q$ implies $\neg p$

therefore $p \to q$.