I) Strong induction

Induction:

Given a property $P(n)$

If we want to show that $P(n)$ is true for all $n \geqslant n_0$

A proof by induction proceeds in 2 steps:

Basis step: $P(n_0)$ is true.

Inductive step:

$$P(n) \rightarrow P(n+1) \text{ is true}, \forall n \geqslant n_0$$

The method of proof by induction allows us to conclude that $P(n)$ is true for all $n \geqslant n_0$.

Strong inductive step

$P(n_0) \wedge P(n_0+1) \wedge \cdots \wedge P(n) \rightarrow P(n+1)$ for $n \geq 0$

Example of strong induction:

To ~~break~~ a chocolate bar with n squares, I need $n-1$ cuts.

Let us define a property $P(n)$:

$P(n)$: To cut a chocolate bar with n squares, I need $n-1$ cuts.

I prove $P(n)$ is true for all $n \geq 1$
I use a proof by (strong) induction.

Basis step: is $P(1)$ true? Yes.
Clearly $P(2)$ is also true.

(strong) inductive step:

$P(1) \wedge P(2) \ldots \ldots \wedge P(n) \rightarrow P(n+1)$

We use a direct proof.

We assume

$P(1)$ and $P(2)$ - $P(i)$ . . and $P(n)$ are true.

Now we consider a chocolate bar

with $n+1$ squares.

I break the chocolate bar into

2 pieces $\longrightarrow$ 1 cut

One piece will have p squares,

the other will have q squares.

$1 \leq p \leq n$ $\quad$ $P(p)$ is true $\rightarrow$ $p-1$ cuts

$1 \leq q \leq n$ $\quad$ $P(q)$ is true $\rightarrow$ $q-1$ cuts

$p+q = n+1$

Total number of cuts:

$$1 + p - 1 + q - 1$$

$$p + q - 1 = n + 1 - 1$$

$$= n \text{ cuts.}$$

$P(n+1)$ is true.

The method of proof by strong induction allows us to conclude that $P(n)$ is true for all $n \geq 1$