The joy of counting 3/8

I) Set theory

Set: unordered list of objects.

associated with a concept of domain.

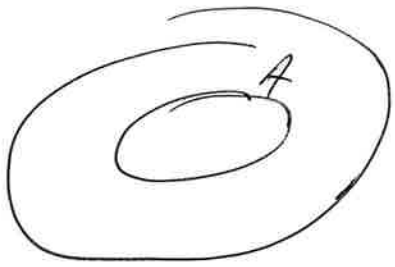
Terminology

$x \in A$ (x belongs)

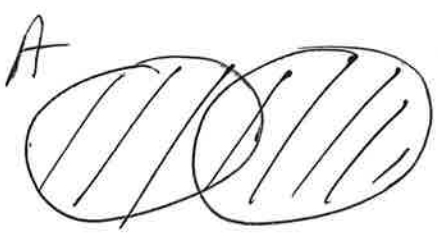
$x \notin A$ (x does not belong)

$\neg(x \in A)$

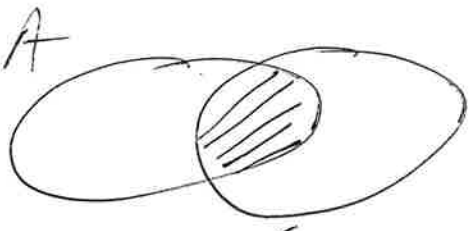
- inclusion $A \subset B$

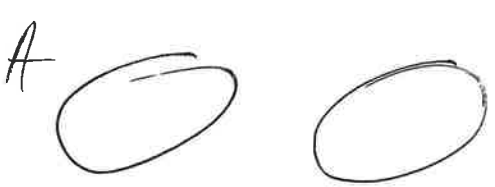


- Empty set: $\emptyset$
- union $A \cup B$



- intersection $A \cap B$





$A \cap B = \emptyset$

Cartesian product

$$A \times B$$

all pairs (a, b) where $a \in A$ and $b \in B$

Properties

- Complement of a set: $\overline{A}$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Domain D and a set A:

$$A \cap \overline{A} = \emptyset$$

$$A \cup \overline{A} = D$$

Counting is the concept of finding the number of elements in a set.

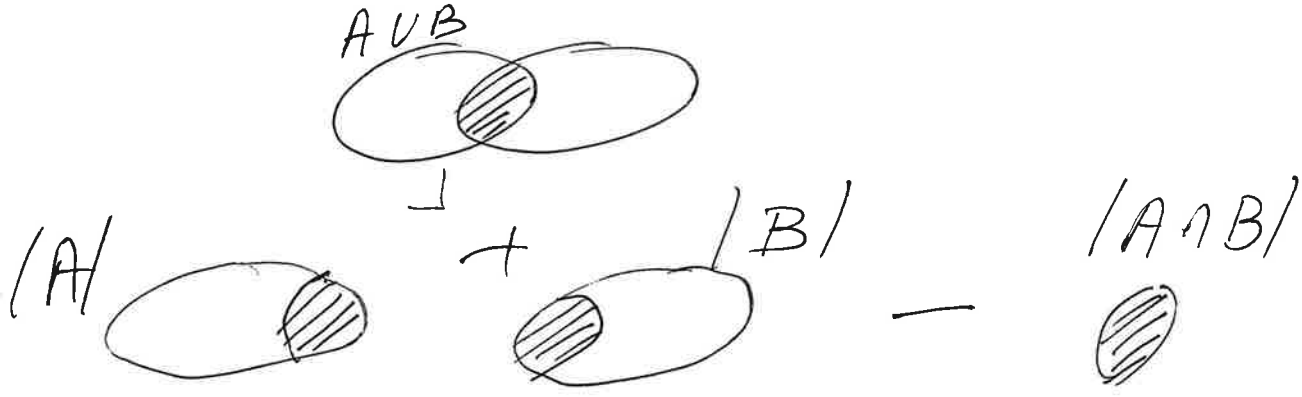
II) Cardinality

The cardinality of a finite set A, noted $|A|$, is the number of elements in A.

Properties :

Inclusion - exclusion principle :

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Inclusion - Exclusion:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Given set A in a domain D

$$B = \overline{A}$$

$$|A \cup \overline{A}| = |A| + |\overline{A}| - |A \cap \overline{A}|$$

$$|D| = |A| + |\overline{A}| - \emptyset$$

$$|\overline{A}| = |D| - |A|$$

Cardinality of a Cartesian product:

$$|A \times B| = |A|\,|B|$$

↳ $$|A \times B \times C \cdots \times Z| = |A|\,|B|\,|C| \cdots |Z|$$

Counting

I) The product rule

Let us suppose that we want to find the number of elements in a list (set) L whose elements are pairs of values.

If there are n_1 ways to choose the first value, and n_2 ways to choose the second value, after each option for the first value, then there are $n_1 \, n_2$ ways to build the list.

$$L = A_1 \times A_2$$

$$|L| = |A_1| \cdot |A_2|$$

$$n_1 \, n_2 \qquad n_1 \quad n_2$$

Knights and knaves.

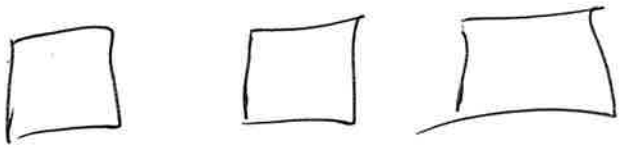
List: (A, B, C)

$2 \times 2 \times 2$

How many bit strings are there of length 8?

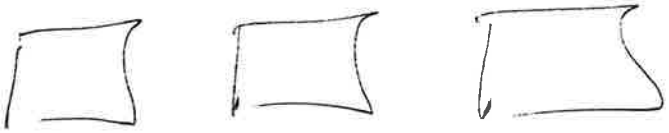
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

How many passwords with 3 digits, each digit being a number between 0 and 9, are there?



$10 \times 10 \times 10$

How many password with 3 <u>no repeating</u> digit, with each digit being a number between 0 and 9 are there?



$10 \times 9 \times 8$

How many bit strings of length 8 contain at least one 1?

A = { bit strings of length 8 that contains at least one 1 }

D = { bit strings of length 8 }

$\bar{A}$ = { bit strings of length 8 that do not contain any 1 }

$|\bar{A}| = 1$

$|D| = 256$

$|A| = |D| - |\bar{A}|$

$= 256 - 1 = 255$