ECS20
Homework 1
Due January 15, 2019

Exercise 1 ***(5 points)***

Let A and B be two natural numbers. Follow the proof given below and identify which step(s) is (are) not valid

Step #	Equation	Justification
1	$A = B$	We start with this assumption
2	$AxA=BxA$	Multiply by A on each side
3	$A^2-B^2=AB-B^2$	$AxA=A^2$; $BxA=AxB$ (commutativity); substract B^2 on both side
4	$(A-B)(A+B)=(A-B)B$	Identity: $A^2-B^2=(A-B)(A+B)$; factor B on right side
5	$A+B = B$	Simplify: divide by $A-B$ on each side
6	$B+B = B$	From step 1, $A = B$, therefore $A+B=B+B$
7	$2B = B$	By definition, $B+B = 2B$
8	$2 = 1$	Simplify by B

Exercise 2 ***(15 points total – 5 points each for a, b, and c)***
Hints:

- An integer number N is odd if it can be written in the form N = 2q +1, where q is an integer number
- An integer number N is even if it can be written in the form N = 2q, where q is an integer number
- An integer number N is a multiple of an integer number k if there exists an integer number q such that N = kq
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Prove the following statements:
a) The sum of any three consecutive odd numbers is always a multiple of 3
b) The sum of any four consecutive odd numbers is always a multiple of 8
c) Prove that if you add the squares of two consecutive integer numbers and then add one, you always get an even number.

Exercise 3 ***(5 points)***

Let x be a real number. Solve the equation $5^{2x}-2(5^x)+1=0$.

Exercise 4 ***(20 points total – 5 points each for a, b, c, and d)***

Prove the following identities, where *p, q, x, y, m*, and *n* are real numbers:

a) $8(p-q)+4(p+q)=2(p+3q)+10(p-q)$
b) $x(m-n)+y(n+m)=m(x+y)+n(y-x)$
c) $(x+3)(x+8)-(x-6)(x-4)=21x$
d) $m^8-1=(m^2+1)(m^2-1)(m^4+1)$

Extra credit: ***(5 points)***

Prove that if you add the cubes of two consecutive integer numbers and then add one, you always get an even number.

+ 3 points for submitting online!