

# Homework 6: due 2/19/2019

ECS 20 (Winter 2019)

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## Exercise 1: total 20 points (5 points for each of a to d)

- a) Show that  $2x - 10$  is  $\Theta(x)$ .
- b) Show that  $4x^2 + 8x - 6$  is  $\Theta(x^2)$ .
- c) Show that  $\lfloor x + \frac{2}{7} \rfloor$  is  $\Theta(x)$ .
- d) Show that  $\log_4(x)$  is  $\Theta(\log_7(x))$ .

## Exercise 2: 10 points

Show that  $x^2$  is  $\mathcal{O}(x^4)$  but that  $x^4$  is not  $\mathcal{O}(x^2)$ .

## Exercise 3: 10 points

Let  $a$ , and  $b$  be two strictly positive integers and let  $x$  be a real number. Show that:

$$\left\lfloor \frac{\lfloor \frac{x}{a} \rfloor}{b} \right\rfloor = \left\lfloor \frac{x}{ab} \right\rfloor$$

## Exercise 4: 10 points

Let  $x$  be a positive real number. Solve  $\lfloor x \lfloor x \rfloor \rfloor = 5$ .

## Exercise 5: 10 points

Let  $n$  be a natural number. Show that if  $n$  is a perfect square, then  $2n$  is not a perfect square.  
(Reminder: a natural number  $a$  is a perfect square if there exists a natural number  $k$  such that  $n = k^2$ .)

## Extra Credit: 5 points

Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that satisfy:

$$\forall (x, y) \in \mathbb{R}^2, f(x)f(y) + f(x + y) = xy$$