Homework 3: due 1/29/2019

ECS 20 (Winter 2019)

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Exercise 1 *(5 points)*

Let a, b, and c be three propositions. Show that this implication is a tautology, using a truth table:

$$(a \vee b) \wedge (\neg a \vee c) \rightarrow (b \vee c)$$

Exercise 2 *(5 points)*

Let p, q, and r be three propositions. Show that $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \vee (q \rightarrow r)$ are not logically equivalent.

Exercise 3 *(5 points each; total 20 points)*

Determine the truth values of the following statements; justify your answers:

a) $\forall n \in \mathbb{N}, n < (n+2)$

b) $\exists n \in \mathbb{N}, 4n = 7n$

c) $\forall n \in \mathbb{Z}, 2n \leq 3n$

d) $\exists x \in \mathbb{R}, x^3 < x^2$

Exercise 4 *(5 points each; total 25 points)*

Solve the following proof problems.

a) Let x be a real number. Prove that if x^2 is irrational, then x is irrational.

b) Let x be a positive real number. Prove that if x is irrational, then $\sqrt{x}$ is irrational.

c) Prove or disprove that if a and b are two rational numbers, then a^b is also a rational number.

d) let n be a natural number. Show that n is even if and only if $5n + 12$ is even.

e) Prove that either $4 \times 10^{769} + 22$ or $4 \times 10^{769} + 23$ is not a perfect square. Is your prove constructive, or non-constructive?

Note: for question e), a natural number n is a perfect square if there exists a natural number q such that $n = q^2$. For example, 4, 9, 16, 25, ... are all perfect squares while 2, 3, 5, 6,.... are not.

Exercise 5 *(10 points)*

Let n be a natural number and let $a_1, a_2, \ldots, a_n$ be a set of n real numbers. Prove that at least one of these numbers is less than, or equal to the average of these numbers. What kind of proof did you use?

Exercise 6 *(5 points each; total 10 points)*

Let n be an integer. Show that if $n^3 + 9$ is even, then n is odd, using:

a) a proof by contraposition

b) a proof by contradiction

Extra Credit *(5 points)*

Use Exercise 5 to show that if the first 12 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum smaller than or equal to 19.

(+ 2 points for submitting online)