## Syntactic Analysis

## Introduction

- Second phase of the compiler.
- Main task:
- Analyze syntactic structure of program and its components
- to check these for errors.
- Role of parser:

- Approach to constructing parser: similar to lexical analyzer
- Represent source language by a meta-language, Context Free Grammar
- Use algorithms to construct a recognizer that recognizes strings generated by the grammar.
This step can be automated for certain classes of grammars. One such tool: YACC.
- Parse strings of language using the recognizer.


## Context Free Grammar (CFG)

- Syntax analysis based on theory of automata and formal languages, specifically the equivalence of two mechanisms of context free grammars and pushdown automata.
- Context free grammars used to describe the syntactic structures of programs of a programming language. Describe what elementary constructs there are and how composite constructs can be built from other constructs.

```
Stmt }->\mathrm{ if (Expr) Stmt else Stmt
```

Note recursive nature of definition.

- Formally, a CFG has four components:
a) a set of tokens $V_{t}$, called terminal symbols, (token set produced by the scanner) examples: if, then, identifier, etc.
b) a set of different intermediate symbols, called non-terminals, syntactic categories, syntactic variables, $V_{n}$
c) a start symbol, $S \in V_{n}$, and
d) a set of productions $P$ of the form

$$
A \rightarrow X_{1} \cdots X_{n}
$$

$$
\text { where } A \in V_{n}, X_{i} \in\left(V_{n} \cup V_{t}\right), 1 \leq i \leq m, m \geq 0
$$

- Sentences generated by starting with $S$ and applying productions until left with nothing but terminals.
- Set of strings derivable from a CFG G comprises the context free language, denoted $L(G)$.


## CFG - example.

- Nonterminal start with uppercase letters. rest are non-terminals.
- If-then-else:

```
Stmt }->\quad\mathrm{ IfStmt | other
IfStmt }->\mathrm{ if ( Exp ) Stmt ElseStmt
ElseStmt }->\mathrm{ else Stmt | }
Exp -> 0 | 1
```

Example strings:
other
if (0) other
if (1) other else if (0) other else other
Derivation of if (1) other else if (0) other else other:
Stmt $\Rightarrow$ IfStmt $\Rightarrow$ if (Exp) Stmt ElseStmt
$\Rightarrow$ if (1) Stmt ElseStmt

- Grammar for sequence of statements:

StmtSeq $\rightarrow$ Stmt; StmtSeq \| Stmt
Stmt $\rightarrow \quad$ s
$\mathrm{L}(\mathrm{G})=\{\mathrm{s}, \mathrm{s} ; \mathrm{s}, \mathrm{s} ; \mathrm{s} ; \mathrm{s}, \ldots \mathrm{F}\}$

- What if statment sequence is empty?

StmtSeq $\rightarrow$ Stmt; StmtSeq $\mid \epsilon$
Stmt $\rightarrow \quad$ s
$\mathrm{L}(\mathrm{G})=\{\epsilon, \mathrm{s} ;, \mathrm{s} ; \mathbf{s} ;, \mathbf{s} ; \mathbf{s} ; \mathbf{s} ;, \ldots\}$
Note: Here ';' is not a statement separator, but a terminator.
What if we want a statement separator?

```
StmtSeq }->\quad\mathrm{ NonEmpStmtSeq | }
NonEmpStmtSeq }->\mathrm{ Stmt; NonEmpStmtSeq | Stmt
Stmt }->\mathrm{ s
```


## Context Free Grammar (CFG) - cont'd.

- Notations:

1. Nonterminals: Uppercase letters such as $A, B, C$
2. Terminals: lower case letters such as $a, b, c$, operators,+- , etc, punctuation, digits, and boldface strings such as id.
3. Nonterminals or terminals: Upper-case letters late in alphabet, such as $X, Y, Z$.
4. Strings of terminals: lower-case letters late in alphabet, such as $x, y$, $z$.
5. Strings of grammar symbols: lower-case greek letters $\alpha, \beta$, etc.
6. Write $A \rightarrow \alpha_{1}, A \rightarrow \alpha_{2}$, etc as

$$
A \rightarrow \alpha_{1}\left|\alpha_{2}\right| \cdots
$$

- Example:

$$
\begin{aligned}
& E \rightarrow E A E|(E)|-E \mid \text { id } \\
& A \rightarrow+|-|*| /| \uparrow
\end{aligned}
$$

- Derivation of strings: a production can be thought of as a rewrite rule in which nonterminal on left is replaced by string on right side. Notation: Write such a replacement as $\mathrm{E} \Rightarrow(\mathrm{E})$. Example:

$$
E \Rightarrow-E \Rightarrow-(E) \Rightarrow-(\mathbf{i d})
$$

## CFG - cont'd.

- Notation: Write $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if $A \rightarrow \gamma$.
- Notation: Write $\alpha \stackrel{*}{\Rightarrow} \beta$ to denote that $\beta$ can be derived from $\alpha$ in zero or more steps.

$$
L(G)=\{\alpha \mid S \stackrel{*}{\Rightarrow} \alpha\}
$$

- Sentential form: $\alpha$ is a sentential form, if $S \stackrel{*}{\Rightarrow} \alpha$ and $\alpha$ contains non-terminals.
Example: $E+E$
- Leftmost derivation: Derivation $\alpha \Rightarrow \beta$ is leftmost if the leftmost terminal in $\alpha$ is replaced.
Example:
$E \stackrel{*}{\Rightarrow} E A E \stackrel{*}{\Rightarrow} \mathbf{i d} A E \stackrel{*}{\Rightarrow} \mathbf{i d}+E \stackrel{*}{\Rightarrow} \mathbf{i d}+\mathbf{i d}$
Production sequence discovered by a large class of parsers (the top-down parsers) is a leftmost derivation; hence, these parsers are said to produce leftmost parse.
- Rightmost derivation: Derivation $\alpha \Rightarrow \beta$ is left most if the rightmost terminal in $\alpha$ is replaced.
Example:
$E \stackrel{*}{\Rightarrow} E A E \stackrel{*}{\Rightarrow} E A i d \stackrel{*}{\Rightarrow} E+\mathbf{i d} \stackrel{*}{\Rightarrow} \mathbf{i d}+\mathbf{i d}$
Also, called canonical derivation. Corresponds well to an important class of parsers (the bottom-up parsers). In particular, as a bottom up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in reverse order: last production applied is discovered first, while the first production is the last to be discovered.


## Representations of derivations

- Derivations represented graphically by a derivation of parse tree:
- Root: start symbol, leaves: grammar symbols or $\epsilon$
- Interior nodes: nonterminals; Offsprings of a nonterminal represent application of a rule.
- Example: Parse tree for leftmost and rightmost derivations of string id +id *id:

- Abstract syntax tree: A more abstract representation of the input string.

- Parse tree may contain information that may not be needed in later phases of compiler. AST does not include intermediate nodes primary used for derivation purposes.
- In general, during the semantic analysis phase, the parse tree of a string may be converted into an abstract syntax tree.


## Parse Tree - Examples

- Parse tree for string: if (o) other else other

- Parse tree for string: $\mathrm{s} ; \mathrm{s} ; \mathrm{s}$



## Properties of Context Free Grammars

- Context free grammars that are limited to productions of the form $A \rightarrow a$ B and $\mathrm{C} \rightarrow \epsilon$ form the class of regular grammars. Languages defined by regular grammars are a proper subset of the context-free languages.
- Why not use lexical analysis during parsing?
- Lexical rules are in general simple.
- RE are more concise and easier to understand.
- Domain specific language so that efficient lexical analyzer can be constructed.
- Separate into two manageable parts. Useful for multi-lingual programming.
- Non-reduced CFGs: A CFG containing nonterminals that are unreachable or derive no terminal string.
Example:

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{~A} \mid \mathrm{B} \\
& \mathrm{~A} \rightarrow \mathrm{a} \\
& \mathrm{~B} \rightarrow \mathrm{Bb} \\
& \mathrm{C} \rightarrow \mathrm{C}
\end{aligned}
$$

Nonterminal C cannot be reached from S. B does not derive any strings. Useless terminals can be safely removed from a CFG without affecting the language. Reduced grammar:

$$
S \rightarrow A
$$

$$
\mathrm{A} \rightarrow \mathrm{a}
$$

Algorithms exist that check for useless nonterminals.

## Properties of Context Free Grammars - Ambiguity

- Ambiguity: A context free grammar is ambiguous if it allows different derivation trees for a single tree.


Each tree defines a different semantics for -

- No algorithm exists for automatically checking if a grammar is ambiguous (impossibility result). However, for certain grammar classes (including those that generate parsers), one can prove that grammars are unambiguous.
- How to eliminate ambiguity: one way is to rewrite the grammar: Example:

$$
S \rightarrow \text { if } E \text { then } S \mid \text { if } E \text { then } S \text { else } S
$$

$$
\mathrm{S} \rightarrow \mathrm{M} \mid \mathrm{U}
$$

$$
M \rightarrow \text { if } E \text { then } M \text { else } M
$$

$$
U \rightarrow \text { if } E \text { then } S \mid \text { if } E \text { then } M \text { else } U
$$

Represents semantics:Match each else with the closet previous unmatched then. The above transformation makes the grammar unnecessarily complex.

- Another approach: Disambiguate by defining additional tokens end. $S \rightarrow$ if $E$ then $S$ end $\mid$ if $E$ then $S$ else $S$ end
- Provide information to the parser so that it can handle it in a certain way.


## Properties of Context Free Grammars - cont'd.

- Left recursion: $G$ is left recursive if for a nonterminal $A$, there is a derivation $A \stackrel{+}{\Rightarrow} A \alpha$
Top-down parsing methods cannot handle left-recursive grammars. So eliminate left recursion.
- Left factoring: Factor out the common left prefixes of grammars: Replace grammar $A \rightarrow \alpha \beta_{1} \mid \alpha \beta_{2}$ by the rule:
$A \rightarrow \alpha A^{\prime}$
$A^{\prime} \rightarrow \beta_{1} \mid \beta_{2}$
- Context free grammars are not powerful enough to represent all constructs of programming languages.
Cannot distinguish the following:
- $L_{1}=\left\{w c w \mid w \in(a \mid b)^{*}\right\}$ : Conceptually represents problem of verifying that an identifier is declared before used. Such checkings are done during the semantic analysis phase.
- $L_{2}=\left\{a^{n} b^{m} c^{n} c^{m} \mid n \geq 1 \wedge m \geq 1\right\}$. Abstracts the problem of checking that number of formal parameters agrees with the number of actual parameters.
- $L_{3}=\left\{a^{n} b^{n} c^{n} \mid n \geq 0\right\}$.

CFG's can keep count of two items but not three.

## Properties of Context Free Grammars - cont'd.

- Context free grammar can capture some of language semantics as well.
- Example grammar:

```
<exp> ::=<exp> + <term> | <term>
<term> ::=<term> * <term>
    | '('<exp>')'
    | <number>
<number> ::= 0 | 1 | ... | 9
```

- Precedence of * over +: by deriving * lower in the parse tree.
- Left recursion <exp> ::= <exp> + <term> left associativity of +
- Right recursion:
<exp> ::= <term> + <exp>
right associativity of +


## Backus-Naur Form(BNF)

- BNF: a kind of CFG.
- First used in Algol60 report. Many extensions since, but all similar and most give power of context-free grammar.
- Has four parts: (i) terminals (atomic symbols), (ii) non-terminals (representing constructs), called syntactic categories, iii) productions and iv) a starting nonterminal.
- Each nonterminal denotes a set of strings. Set of strings associated with starting nonterminal represents language.
- BNF uses following notations:
(i) Non-terminals enclosed in $<$ and $>$.
(ii) Rules written as

$$
X::=Y
$$

(a) $X$ is LHS of rule and can only be a NT.
(b) $Y$ can be a string, which is a terminal, nonterminal, or concatenation of terminal and nonterminals, or a set of strings separated by alternation symbol |.

- Example: Terminals: A, B, $\cdots$ Z; $0,1, \cdots 9$ Nonterminals: <id>, <rest>, <alpha>, <alphanum>, <digit>
Starting NT: <id>
Productions/rules:

```
<id> ::= <alpha> | <alpha><rest>
```

<rest> ::= <rest><alphanum> | <alphanum>
<alphanum> ::= <alpha> | <digit>
<alpha> $::=\mathrm{A}|\mathrm{B}| \cdots$ | Z
<digit> $::=0|1| \cdots \mid 9$

## Extended BNF (EBNF)

- Extend BNF by adding more meta-notation $\Longrightarrow$ shorter productions
- Nonterminals begin with uppercase letters (discard <>)
- Terminals that are grammar symbols ('[' for instance) are enclosed in ".
- Repetitions (zero or more) are enclosed in \{\}
- Options are enclosed in []:
- Use () to group items together: Exp : : = Item \{+ Item $\}$ I Item \{- Item $\}$
$\qquad$
Exp ::= Item $\{(+\mid-)$ Item $\}$
Conversion from EBNF to BNF and Vice Versa
- BNF to EBNF:
i) Look for recursion in grammar:

$$
A::=a \operatorname{A} \mid B \Longrightarrow\{a\} B
$$

ii) Look for common string that can be factored out with grouping and options.
$\mathrm{A}::=\mathrm{a} \mid \mathrm{a} \Longrightarrow \mathrm{A}:=\mathrm{a}$ [B]

- EBNF to BNF:
i) Options []:

$$
\begin{gathered}
\mathrm{A}::=\mathrm{a}[\mathrm{~B}] \mathrm{C} \Longrightarrow \\
\mathrm{~A},::=\mathrm{a} N \mathrm{C} \\
\mathrm{~N}::=\mathrm{B} \mid \epsilon
\end{gathered}
$$

ii) Repetition $\}$ :

$$
\begin{aligned}
& \mathrm{A}::=\mathrm{a} \text { B1 B2 } \ldots \mathrm{Bn} \mathrm{C} \Longrightarrow \\
& \text { A' : := a N C } \\
& \mathrm{N}::=\mathrm{B} 1 \mathrm{~B} 2 \ldots \mathrm{Bn} \mathrm{~N} \mid \epsilon
\end{aligned}
$$

