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5. Strongly connected components of a drected graph (see Sec.22.5 of [CLRS,3rd ed.])

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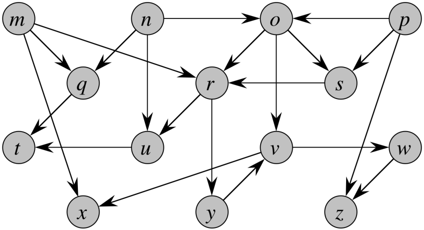

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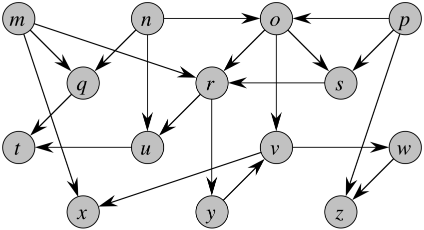

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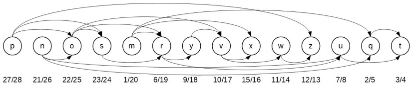

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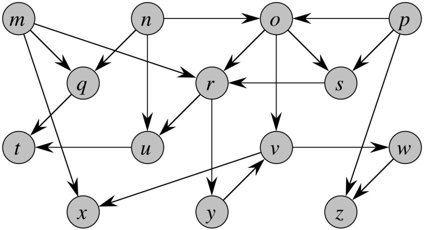

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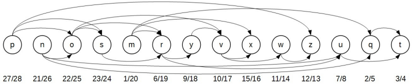

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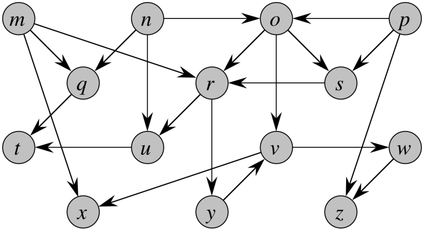

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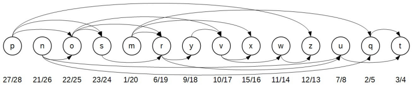
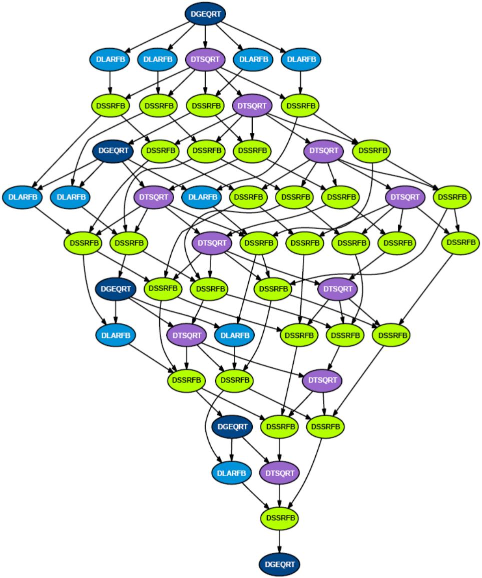


- A TS is not possible if G has a cycle.
- The ordering is not necessarily unique.

Topological sort

Applications: call-graph



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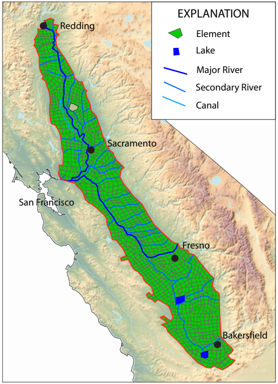

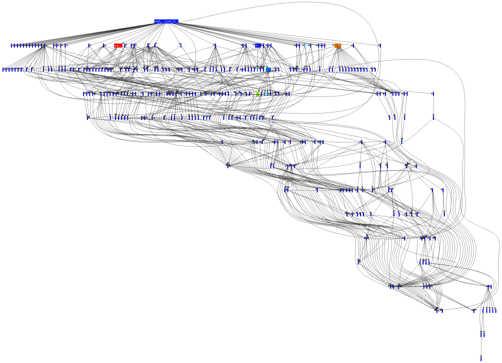

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- TS algorithm
  1. run DFS(G) to compute finishing times $f[v]$ for all $v \in V$
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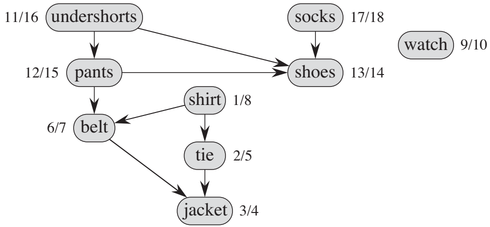
- Running time: $\Theta(|V| + |E|)$

Topological sort

Example: "Getting-dressed-graph" and DFS



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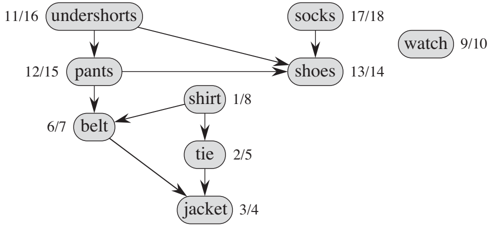
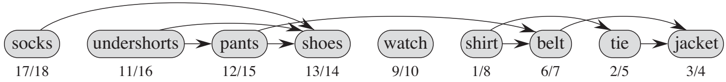


Topologically sorted



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Theorem (correctness of the algorithm):
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Proof: *Just need to show that if* $(u, v) \in E$*, then* $f[v] < f[u]$.
When we explore edge (u, v), u is gray, what's the color of v?

- Is v gray too?
  no, because then v would be ancestor of u, edge (u, v) is a back edge, a contradiction of a DAG.
- Is v white?
  yes, then v is descendant of u, by DFS, $d[u] < d[v] < f[v] < f[u]$
- Is v black?
  yes, then v is already finished. Since we're exploring (u, v), we have not yet finished u, therefore $f[v] < f[u]$