ECS 20 Final Review Exercises

1. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving

   (a) a containment proof. (That is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

   (b) a Venn diagram illustration.

2. Let the relation R on the set $\{1, 2, 3, 4, 5\}$ defined by the rule $(x, y) \in R$ if $x = y - 1$.

   (a) List the elements of R

   (b) List the elements of R^{-1}

   (c) List the domains of R and R^{-1}, respectively.

   (d) List the ranges of R and R^{-1}, respectively.

   (e) Is the relation reflexive, symmetric, antisymmetric, and/or transitive?

3. Let the relation R defined on the set of positive integers as

   $$(x, y) \in R \quad \text{if} \quad 3 \quad \text{divides} \quad x - y.$$

   Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive, and/or a partial order.

4. Suppose $f(n) = f(n/3) + 2n$, $f(1) = 1$. Find $f(27)$.

5. Consider the function $f(n) = 2\lfloor n/2 \rfloor$ from $\mathbf{Z}$ to $\mathbf{Z}$. Is this function one-to-one? Is this function onto? justify your answer.

6. (a) Prove or disprove: If $a \equiv b \pmod 5$, where a and b are integers, then $a^2 \equiv b^2 \pmod 5$.

   (b) Prove or disprove: If $a^2 \equiv b^2 \pmod 5$, where a and b are integers, then $a \equiv b \pmod 5$.

7. Prove that $6 | (3^k + 4^k + 5)$ for $k \geq 1$.

8. Use mathematical induction to show that $f_n^2 = f_{n-1} f_{n+1} + (-1)^n$ for $n \geq 2$, where $\{f_n\}$ is the Fibonacci sequence with $f_0 = f_1 = 1$.

9. Suppose that $|A| = 5$ and $|B| = 10$.

   (a) Find the number of functions $f\colon A \to B$.

   (b) Find the number of one-to-one functions $f\colon A \to B$.

   (c) Find the number of onto functions $f\colon A \to B$.

10. Suppose you have 30 books (15 novels, 10 history books, and 5 math books). Assume that all 30 books are different. In how many ways can you:

    (a) put the 30 books in a row on a shelf.

    (b) get a bunch of 4 books to a friend.

    (c) get a bunch of 3 history and 7 novels to give to a friend.

    (d) put the 30 books in a row on a shelf if the novels are on the left, the math books in the middle, and the history books are on the right.

11. What is the minimum number of students required in a class to be sure that at least six will received the same grade, if there are five possible grades A, B, C, D, and F?

12. Solve the given recurrence relations for the initial conditions given

    (a) $a_n = 7a_{n-1} - 10a_{n-2}$, $a_0 = 5$, $a_1 = 16$.

    (b) $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = a_1 = 1$.

13. Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

    (a) Show that $a_n = -2^{n+1}$ is a solution of this recurrence relation.

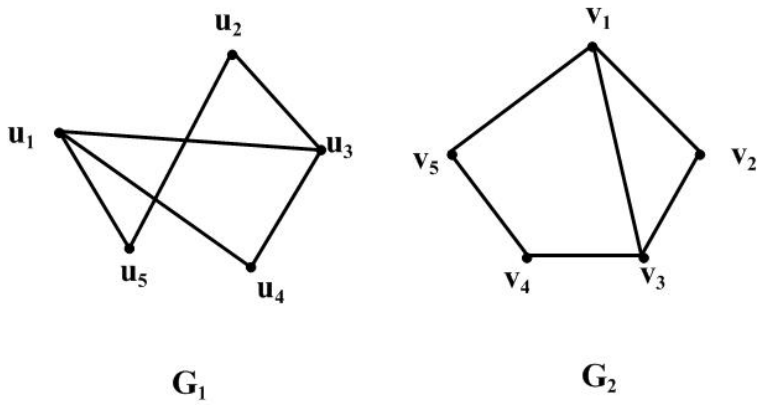
    (b) Find all solutions of this recurrence relation.

    (c) Find the solution with $a_0 = 1$.

14. Discrete probability – Part I of Homework Problem Set 8

15. Graphs and Tree – Part II of Homework Problem Set 8

16. Extra: Determine whether the graphs G_1 and G_2 shown below are isomorphic, justify your answer.



G_1 G_2