CS 222 Fall 2007,

Homework 1, due October 8. Put in the homework box in Kemper hall (Television students - find out from your contacts how to submit homeworks)

Problem 1: This is a "discovery" problem not related to any material we have yet discussed in class. It arose in the literature of co-speciation analysis - but you don't need to understand anything about the application.

We are given a rooted binary tree T with n leaves labeled (as part of the input) with the integers 1 through n. Assume we have drawn the root at the top, so that the tree hangs down. Each interior node has exactly two children. A *planar layout* of T is defined by choosing, at each interior node, which of the two children are placed on the left, and which are placed on the right. Therefore in a planar layout, no edges of T cross. Given a specific planar layout of T, the order of the leaf labels (reading left to right) specifies a permutation of the integers 1 through n. We let $\mathcal{R}(\mathcal{T})$ be the set of permutations that are specified by all the possible planar layouts of T.

Question 1a: How large is $\mathcal{R}(\mathcal{T})$? Justify your answer.

We say that two leaves labeled i and j respectively, are "out of order", or "cross", in a permutation $\pi \in \mathcal{R}(\mathcal{T})$ if $i < j$ and i comes after j (somewhere) in π. One way to visualize this is to write the integers 1 through n in their natural order below the leaves of T, and draw a straight line from each leaf of T (labeled k say) to integer k in that list. Then leaves i and j cross if and only if the lines from leaves i and j cross.

Question 1b: Is the number of crossing leaves (or lines) affected by exactly where we write the integers 1 through n, as long as they are written in order and below the leaves? What if we write the integers above the root or to the side of the tree? Consider vertical and horizontal layouts of the list of integers. Explain your answers.

Question 1c: Given T and its leaf labels, we want to select a planar layout of T to minimize the number of crossing leaves. That is, we want to find a permutation $\pi \in \mathcal{R}(\mathcal{T})$ to minimize the number of crossing leaves. Create an algorithm to solve this problem and analyze its worst-case running time as a function of n. You should shoot for a time bound of $O(n^2)$ but you may be able to find a faster method. Explain and justify the idea behind the algoirthm, its correctness and its running time. You may provide code if it helps you think, but it will be ignored. Hence, you must *explain* the idea of the algorithm and the details of the algorithm in clear written text along with (maybe) some very simple pseudocode, rather than relying on code.

Problem 2: Read p. 1-12 on the stable matching problem. You may not need to read all the proof details, but it might help. Then answer the following problem.

Given the preference lists, let M and M' be two stable matchings (assuming there is more than one stable matching). For each man m, let $F(m, M, M')$ be m's favorite between his mate in M and in M'. Let $W(m, M, M')$ be the other woman. Prove that if each man is assigned to $F(m, M, M')$ then the result is a stable matching. Be sure to prove that it is in fact a matching, i.e. each man is assigned a different woman. Prove also that it is a stable matching. What about if each man is assigned to $W(m, M, M')$?

Problem 3. Do Exercise 1 on page 246 of the text. Be sure to explain your algorithm and its analysis. The analysis is facilitated by setting up a recurrence relation.