Optimal Transmission Strategies for Dynamic Spectrum Access in Cognitive Radio Networks

Senhua Huang, Xin Liu, and Zhi Ding

Abstract—Cognitive radio offers a promising technology to mitigate spectrum shortage in wireless communications. It enables secondary users (SUs) to opportunistically access low-occupancy primary spectral bands as long as their negative effect on the primary user (PU) access is constrained. This PU protection requirement is particularly challenging for multiple SUs over a wide geographical area. In this paper, we study the fundamental performance limit on the throughput of cognitive radio networks under the PU packet collision constraint. With perfect sensing, we develop an optimum spectrum access strategy under generic PU traffic patterns. Without perfect sensing, we quantify the impact of missed detection and false alarm, and propose a modified threshold-based spectrum access strategy that achieves close-to-optimal performance. Moreover, we develop and evaluate a distributed access scheme that enables multiple SUs to collectively protect the PU while adapting to behavioral changes in PU usage patterns. Our results provide useful insight on the trade-off between the protection of the primary user and the throughput performance of cognitive radios.

Index Terms—Wireless Communication, Cognitive Radio, Dynamic Spectrum Access, Optimization

1 Introduction

Cognitive radio (CR) technology can potentially alleviate spectrum shortage in wireless communications by allowing secondary users (SUs) to opportunistically utilize spectral white spaces of primary users (PUs). Because primary (legacy) users have access priority, secondary cognitive radio networks are required to exert minimal effect on PUs if and when PUs become active. For example, in the DARPA XG project [1], one of the three major test criteria in the field test is "to cause no harm" [2]. The protection of PUs is vital to the future of cognitive radio system because no PU operators would be inclined to accommodate secondary cognitive networks without such assurance. Thus, the spectrum access strategy of the SU should aim to maximize the performance of SUs while operating under the strict protection requirement of PUs.

The protection of PUs is more challenging when there are multiple (decentralized) SUs accessing a common PU channel. Because these SUs may be widely distributed in a geographic location, one SU may not be able to sense all other SUs' transmissions. On one hand, this enables spatial reuse among SUs, which improves secondary network capacity. On the other hand, the protection of the PU becomes harder because individual SU's intrusion/interference can accumulate to a degree that is unacceptable to the PU. To address this issue, one may consider a centralized SU controller to coordinate all SUs. However, practical deployment may not always be able to set up a centralized controller. Additionally, unless an out-of-band channel is used, coordination among SUs will incur much overhead and delay, even if such centralized cooperation is possible. Thus, one of our chief objectives is to achieve the collective protection of the PU without a centralized controller while still enabling spatial reuse.

In this work, we apply PU packet collision probability as the metric for PU protection. SUs access the PU channel using a sensing-based scheme. Multiple SUs can consider access to the idle primary band independently or collaboratively. Regardless, in order not to degrade PU performance, the total collision probability must be restricted to satisfy the protection requirement set by the PU (operator). Under such a constraint, we investigate the fundamental limit on the throughput performance of opportunistic spectrum access by SUs.

Our contributions are:

- Development of an optimal spectrum access policy under the assumption of perfect sensing by the SUs.
- Derivation of (tight) lower and upper bounds on the SU throughput.
- Analysis of the impact of imperfect sensing on the SU throughput performance and the development of a modified spectrum access policy.
- Development of a distributed access scheme that enables multiple SUs to operate under the PU protection requirement and to adapt to changes in the PU traffic characteristics.

In summary, our results illustrate and quantify the effect of (a) the PU traffic characteristics, (b) the packet collision constraint, and (c) SU sensing capability, on the performance of multiple SUs.

The following notations are used in this paper. Upper

This work is supported in part by the National Science Foundation Grant CNS-0520126 and Grant CCF-0515058

S. Huang is with the Department of Electrical and Computer Engineering, the University of California, Davis, CA 95616, USA (e-mail: senhua@ece.ucdavis.edu).
X. Liu is with the Department of Computer Science, the University of California, Davis, CA 95616, USA (e-mail: liu@cs.ucdavis.edu).
Z. Ding is with the Department of Electrical and Computer Engineering, the University of California, Davis, CA 95616, USA (e-mail: zding@ucdavis.edu).

case letters are used to denote random variables. $\mathbb{E}_X[\cdot]$ denotes the expectation operation with regard to random variable X, $\Pr[\cdot]$ denotes the probability of random events, and $\mathbb{I}[\cdot]$ is an indicator function. For a random variable X, the corresponding lower case letter x is used to denote its mean.

2 Related Work

Cognitive radio has attracted much research attention. Both distributed and centralized schemes have been proposed to facilitate the spectrum sharing between SUs and PUs. For example, in [3], [4], centralized mechanisms are proposed to implement spectrum leasing and real time spectrum auctions of unused PU bands in which spectrum brokers and/or auctioneers match the demand and supply of "white space" between providers and consumers. Our works differ from these preceding works in the following aspects: (a) we address the asymmetrical relationship in access priority and protection requirement of the PU and SUs when designing SU spectrum access schemes; (b) we quantify explicitly the impact of the PU traffic pattern, protection requirement, and SU detection performance on the SU spectrum opportunity exploitation; and (c) characterize the interplay between PU protection and the SU performance.

The closest related works are those in [5], [6], [7], [8], [9], [10], [11], [12], [13]. For example, the work in [5], [6] studies the problem of opportunistic SU spectrum access over multiple PU bands under the partially observable Markovian decision process framework, and proposes cognitive MAC protocols that optimize the SU performance while limiting the interference to the PU. However, it is assumed that the PU idle/busy time is slotted with unchanged Markov transition between the PU idle and busy states, which implies a geometric PU busy/idle time distribution. Under such assumptions, the collisions perceived by the PU are solely due to the missed detections at the spectrum sensor. However, other causes of collisions exist, as explained in Section 3. In addition, we assume no synchronizations between SUs and PUs; nor do we require any restriction on the PU idle/busy time distributions.

In [7], [8], the authors showed that by choosing a suitable sensing time, the SU can achieve an optimal tradeoff between the false alarm probability and throughput under a constraint on the missed detection probability. With the assumption that the PU idle/busy periods follow exponential distributions, in [13], the authors propose CMA (Cognitive Medium Access) for an SU to utilize the PU spectrum opportunities optimally under the protection constraint from the PU; we show in [11] that the maximum usable time proportion for the SU cannot exceed the product of the collision probability constraint and the idle percentage of the PU's traffic.

However, as shown by the measurement results on WLAN traffic experiments in [14], assuming exponential distribution on PU idle time may not be practical for all PU traffic. Therefore, general PU idle/busy time distributions are considered here in our work. With this generalization, we are able to build the connection between the PU traffic pattern and the SU access strategy, and quantify the impact of PU idle time distribution on the SU throughput performance under the collective protection requirement from the PU.

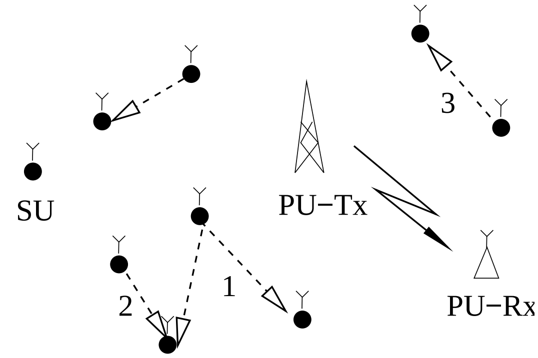


Fig. 1. Overlaying CR Network to Primary Network.

3 System Model

We consider the scenario in which one spectrum band is assigned to PUs while multiple SUs try to exploit spectrum opportunities vacated by PUs, as shown in Fig. 1. Although multiple bands may be open to the SUs, the results obtained here can be used directly for low-cost SUs that cannot frequently switch over channels.

Note that PUs and SUs may be of the same type, but of different service levels. One example is DARPA XG program in which both PUs and SUs are military users, but SUs have lower access priority, thereby requiring more advanced capabilities to achieve non-intrusive opportunistic spectrum access on the PU channels. We assume that an SU does not distinguish different PUs, and can only access the channel when no PU is active. Thus, SUs treat the collective of all PUs as one "aggregated" PU in designing their spectrum access schemes.

For convenience, our frequently used notations are listed in Table 1.

3.1 PU activity model

We assume that the PU activities follow an alternating IDLE-BUSY pattern, of which the idle time can be exploited by the SUs. The PU's channel access is not affected by SUs' behavior. In other words, whenever the PU has traffic to send, it will transmit on the channel without sensing. The sojourn time that the PU stays idle/busy depends on the PU traffic pattern, and varies randomly. We first assume that the PU traffic activities are ergodic and stationary for the concerned time-scale of SU access in the analysis, and allow the PU to change its traffic statistics in Section 6.

The idle/busy evolution of the PU is assumed to follow a semi-Markov renewal process. Here, we denote

TABLE 1
Notations

Symbol	Definition
L_I (L_B)	the sojourn time of PU idle (busy) state
l_I (l_B)	average PU idle (busy) time
α	percentage of PU idle time
$f(\cdot)$ ($F(\cdot)$)	PDF (CDF) of PU idle time
N_p (N_c)	number of (collided) PU packets in a PU busy time
n_p (n_c)	mean of N_p (N_c)
l_p (l_s)	the packet length of PU (SU)
p_c	collision probability perceived by the PU
p_c^A (p_c^B)	type-A (B) collision probability
η	collision probability constraint
N_c^A (N_c^B)	number of type-A (B) collisions in a PU busy time
n_c^A (n_c^B)	mean of N_c^A (N_c^B)
Γ_s	SU throughput performance
G_s	SU successful transmission time
q	SU transmission probability
τ_s (f_s)	sensing time (sampling frequency)
P_m (P_f)	probability of missed detection (false alarm)
σ_p^2 (σ_n^2)	power of PU signal (noise)
t	amount of time elapsed since latest PU idle state
k (m)	Index of PU idle-busy periods (SUs)
β, μ, κ, σ	parameters of PU idle time distribution

the sojourn time of the PU idle state as L_I, its probability density function (PDF) as $f(\cdot)$, its cumulative distribution function (CDF) as $F(\cdot)$, and its mean as l_I. We denote the sojourn time of the PU busy state as L_B with mean l_B. The idle percentage of the primary channel is then $\alpha = l_I/(l_I + l_B)$, which is the upper-bound on the percentage of time that the SU can transmit on the PU channel. For brevity, we assume that there are N_p PU slots (packets) in a PU busy period, where N_p is a random variable with mean n_p, and the length of each slot is denoted as l_p. Thus, $l_B = n_p l_p$. Note that here we loosely use the term "packet" to denote the time granularity of the PU's traffic.

Denote the collision probability "perceived" by the PU in a long-run as p_c. Relying on the stationarity and ergodicity of the PU's traffic pattern, we have

$$p_c = \lim_{K \to \infty} \frac{\sum_{k=0}^{K} N_c(k)}{\sum_{k=0}^{K} N_p(k)}, \tag{1}$$

where $N_c(k)$ and $N_p(k)$ denote the total number of collided packets and transmitted packets of the PU in the kth busy-idle cycle, respectively.

The PU (or the regulator) imposes the following constraint on the collective access activities from all SUs:

$$p_c \leq \eta. \tag{2}$$

For PUs that do not use a packet-based transmission scheme, they may be more inclined to limit the amount of overlapping time between them and SUs. In this case, minor alteration is required to transform the packet collision probability (2) to the overlapping time constraint.

3.2 Access behaviors of SUs

We assume that the SUs' transmission is slotted, with slot length fixed as l_s, and $l_s \ll l_I$, and $l_s \ll l_B$. In other words, the SU access unit (slot) is much shorter than the average PU idle/busy time. On one hand, a slowly varying PU activity pattern is beneficial for SU access. On the other hand, a small value of l_s provides more freedom for designing the SU access strategy since the SU can assess its spectrum opportunity and do so in finer granularity. For example, the whole duration of a cellphone call is on the order of seconds, while the typical length of WLAN packet is on the order of milliseconds. Furthermore, we assume that $l_s \leq l_p$. Note that we do not require the SUs to synchronize with the PU. We also assume that SUs always have packets to transmit, which is consistent with most related works that focus on the maximum usage of spectrum opportunities.

Each SU is equipped with an access controller, whose role is to make decisions for the SU to interact with the PU. The access controller follows the listen-before-talk principle. As illustrated in Fig. 2, the SU senses the channel to detect the PU state, and only initiates transmission when the PU is detected to be idle.

The competition or/and cooperation among multiple SUs are handled using the concept of "virtual channel," which is a logical channel constructed over the spectrum holes of the PU channel. Existing well-designed MAC protocols can be used readily by multiple SUs in vicinity to share the virtual channel, which greatly simplifies the design of cognitive radio devices. Consider the example illustrated in Fig. 1. When the PU is idle, the access controllers at SUs 1, 2, and 3 detect the opportunity, declare the virtual channel. Since SU 1 and SU 2 may interfere with each other and thus they can share the virtual channel using protocols such as CSMA/CA. On the other hand, they do not interfere with SU 3, which enables the spatial reuse.

We consider both perfect and imperfect sensing cases with respect to the SU spectrum sensor. Perfect sensing means that there is no sensing error and the sensing time is negligible; while imperfect sensing considers both sensing error and sensing time. There are two types of collisions with the PU in opportunistic spectrum access, as illustrated in Fig. 2.

- A type-A collision happens when the SU has not finished its transmission before the PU returns to the channel. This is because many wireless radios cannot receive and transmit using the same spectrum at the same time. Even if sensing at the SU is perfect at the beginning of the transmission, type-A collisions exist unless the SU can predict when the PU returns to transmit. Due to the potential type-A collisions, the SU may not transmit even when the PU is currently idle.

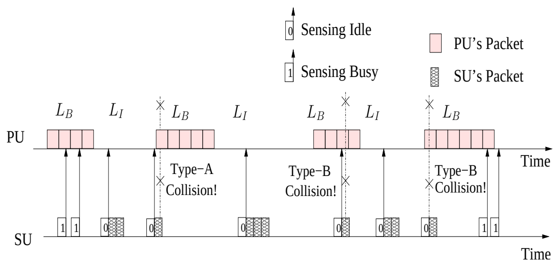


Fig. 2. Illustration of Dynamic Spectrum Access Schemes.

- A type-B collision happens when the SU mistakenly declares a busy channel as idle and transmits its packet, thereby causing additional collisions to the PU. This type of collision is a result of imperfect detection of the spectrum sensor at the SU. Note that type-B collision also includes the case when the PU transmission starts in the middle of sensing but the spectrum sensor may not be able to detect the presence of the PU.

We assume that an SU can detect its collision with the PU. Though not the main focus of the paper, several mechanisms can be used. For example, the SU can perform sensing after each packet transmission. Since we have $l_s \leq l_p$, if the SU senses the presence of PU signal after the SU's transmission, a collision with the PU is detected. An alternative is to check the acknowledgment (ACK) from the secondary and/or primary receiver. Normally, the absence of ACK indicates a likely collision. The SU can perform additional sensing to detect its collision with the PU by exploiting features of the PU signal transmission. Omitting lengthy discussion on this matter, we assume that the SU can detect the collision perfectly after it transmits a packet. Note that this is a commonly used assumption in many existing works (e.g. [6]). Imperfect collision detection is considered in [15].

The performance metric of the SU is **the percentage time** during which the SU can transmit without colliding with the PU. This is referred to as **throughput** in this paper. Note that this definition of throughput also includes the transmission time of all control signals among SUs if any. Similar definitions are found in related works (e.g., [7], [8], [13]). For a given SU access strategy, the successful SU access time in each PU idle-busy period is i.i.d. (independent and identically distributed), and can be represented by a random variable G_s. Thus we have the throughput of each SU as:

$$\Gamma_s = \frac{\mathbb{E}[G_s]}{l_I + l_B}. \tag{3}$$

Obviously, Γ_s is upper bounded by α, the percentage of PU idle time. Our objective is to study the fundamental limit on the achievable SU throughput performance and to develop access schemes for SUs to maximize the throughput, Γ_s, while satisfying the constraint $p_c \leq \eta$.

4 Optimal Access for General Primary User Idle Time Distribution

In this section, we derive the optimality of a threshold-based spectrum access policy for a single SU assuming perfect sensing, and present bounds on the achievable SU throughput performance with regard to the PU idle time distribution.

In certain applications, perfect sensing could be a reasonable approximate scenario. For instance, the results in this section can apply to scenarios in which the SUs are deployed well inside the service area of the PU transmitter. Indeed, the SU transmitter may even be co-located with the PU transmitter. Thus, a reliable detection of the PU transmission can be achieved with a very short sensing time in the presence of strong PU signals. An experimental example of such a scenario can be found in [14].

4.1 Definition of spectrum access policy

In general, the optimal policy depends on the current and the historic channel activities of the PU. Therefore, we express the spectrum access policy as a function of $h \in [0, T]$ and Λ, where h is the current (absolute) time, $T \to \infty$ is the time horizon of the SU access, and Λ is the observation history of the PU's activities until now. Since the PU can be either busy or idle, we have $\Lambda(h) = \{\tau | \Phi(\tau) = \text{Busy}, \tau \leq h\}$, where $\Phi(\tau)$ is the sensing outcome at time τ. The SU spectrum access policy, denoted by π, determines whether to transmit a packet at a particular time instance. Denote the transmission probability of the SU at h by $q(h, \Lambda(h))$. Obviously, we have $0 \leq q(h, \Lambda(h)) \leq 1$, and in particular, $q(h, \Lambda(h)) = 0$ when $\Phi(h) = \text{Busy}$.

Let $\tau_{max}(h)$ be the time instant of the latest PU busy period until h, i.e.,

$$\tau_{max}(h) = \max_{\tau \leq h} \{\tau : \Phi(\tau) = \text{Busy}\}. \tag{4}$$

We define a time variable t that reflects the time elapsed since $\tau_{max}(h)$; i.e.,

$$t = h - \tau_{max}(h). \tag{5}$$

In particular, $t = 0$ indicates the beginning of the latest idle period. Let $t_0, t_1, \cdots, t_k, \cdots$ denote the sequence of time instants when the SU transmits a packet since the beginning of the latest PU idle time, where $t_0 \geq 0$. In the following, we consider policies of the form $\pi = [q(t_0), q(t_1), \cdots]$ instead of $\pi = [q(h, \Lambda(h)) : h \in [0, T]]$, i.e., we consider the stationary policies that only depend on t instead of h and $\Lambda(h)$. In addition, since the SU can detect the beginning of the PU idle time at least within the range of certain sensing periods, the value of t can be obtained from the value of h. Whenever it is not confusing, we use $\Phi(t)$ instead of $\Phi(h) = \Phi(\tau_{max}(h) + t)$ to represent the sensing result.

Since the sensing is perfect, we have only type-A collision with the PU, which happens at the beginning of the PU's busy period. Because $l_s \leq l_p$, at most one PU packet collision occurs in each PU busy period. If $\eta \geq \frac{1}{n_p}$, the SU access strategy is trivial: the SU simply transmits until a collision happens. Therefore, we consider the case in which $\eta < \frac{1}{n_p}$. For a given access policy π, the number of collided PU packets in each PU busy-idle cycle is i.i.d. (represented by a random variable N_c with mean n_c), and by the law of large numbers, we have:

$$p_c(\pi) = \frac{n_c}{n_p}. \tag{6}$$

Let $Z(k)$ be the event that the PU returns in $[t_k,\, t_k + l_s)$, we have the expected number of collided packets in a PU idle-busy period as:

$$\begin{aligned} n_c &= \sum_{k=0}^{\infty} 1 \Pr[Z(k), \text{SU transmits at } t_k] \\ &= \sum_{k=0}^{\infty} q(t_k) \Pr[Z(k)]. \end{aligned} \tag{7}$$

where $\Pr[Z(k)] = \Pr[t_k \leq L_I < t_k + l_s]$.

The throughput of the SU for a given access strategy π, which is the percentage of time that the SU successfully utilizes the spectrum resource, can be written as

$$\Gamma_s(\pi) = \frac{\sum_{k=1}^{\infty} (\sum_{l=0}^{k-1} l_s q(t_l)) \Pr[Z(k)]}{l_B + l_I}, \tag{8}$$

where $\sum_{l=0}^{k-1} l_s q(t_l)$ is the expected total successful transmission time when the PU returns during $[t_k,\, t_k + l_s)$.

We have the following result.

Proposition 1: Under the PU packet collision probability constraint $p_c(\pi) \leq \eta$, the SU throughput $\Gamma_s(\pi)$ increases as l_s decreases.

The result is intuitive. Whenever we split the SU packet into two halves, the first half of the packet has a higher success rate while the total collision perceived by the PU remains unchanged. Therefore, $\Gamma_s(\pi)$ increases as l_s decreases. We omit the proof here.

Letting $l_s \to 0$, we can convert the summation into the Riemann integral:

$$p_c(\pi) = \frac{1}{n_p} \int_0^{\infty} f(\tau) q(\tau)\, d\tau. \tag{9}$$

The maximum SU throughput for given access strategy π can then be expressed as:

$$\Gamma_s(\pi) = \frac{G_s(\pi)}{l_I + l_B} = \frac{\int_0^{\infty} f(t) \int_0^t q(\tau)\, d\tau\, dt}{l_I + l_B}. \tag{10}$$

Here $G_s(\pi)$ is used to explicitly reflect the dependence of the throughput performance on the access strategy π.

Therefore, we can maximize the SU throughput by solving the following optimization problem:

$$\begin{aligned} \text{maximize}_{\pi} \quad & \Gamma_s(\pi) \\ \text{subject to} \quad & p_c(\pi) \leq \eta, \end{aligned} \tag{11}$$

where the expressions of $\Gamma_s(\pi)$ and $p_c(\pi)$ are given in (10) and (9), respectively. Since we have a large number of time instances $t \in [0, \infty)$ at which the optimal spectrum access decision $q(t)$ must be made, it is computationally prohibitive to exhaustively search over the policy space. However, we can exploit the structure of the problem (in both the objective function and the constraint) to obtain an optimal policy.

4.2 Optimal spectrum access policy

First, we define a time-related decision metric as follows:

$$g(t) = \frac{1 - F(t)}{f(t)}. \tag{12}$$

Since $\frac{f(t)}{1-F(t)}$ is the conditional probability density that the PU will return at time instance t given that it has been idle until t, a larger value of $g(t)$ indicates a smaller collision possibility when the SU transmits a packet at t. We will show that this metric captures the connection between the packet collision probability constraint and the throughput performance. We have the following theorem on the optimal SU policy.

Theorem 1: The following SU spectrum access policy solves the constrained optimization problem in (11) among all stationary policies, i.e., it maximizes the SU throughput under the PU packet collision probability constraint $p_c \leq \eta$:

$$q^*(t) = \begin{cases} 1, & \text{if } g(t) > \gamma^*,\ \Phi(t) = \text{Idle} \\ p^*, & \text{if } g(t) = \gamma^*,\ \Phi(t) = \text{Idle} \\ 0, & \text{otherwise}, \end{cases} \tag{13}$$

where the value of γ^* is determined as:

$$\gamma^* = \inf \left\{ \gamma : \int_{\tau: g(\tau) > \gamma} f(\tau)\, d\tau \leq n_p \eta \right\}. \tag{14}$$

If $\int_{\tau: g(\tau) > \gamma^*} f(\tau)\, d\tau = n_p \eta$, $p^* = 0$; otherwise,

$$p^* = \frac{n_p \eta - \int_{\tau: g(\tau) > \gamma^*} f(\tau)\, d\tau}{\int_{\tau: g(\tau) = \gamma^*} f(\tau)\, d\tau}. \tag{15}$$

Proof: Consider any policy π, we have:

$$\begin{aligned} G_s(\pi) &= \int_0^{\infty} q(\tau) \int_{\tau}^{\infty} f(t)\, dt\, d\tau \\ &= \int_0^{\infty} q(\tau)(1 - F(\tau))\, d\tau \\ &= \int_0^{\infty} q(\tau) f(\tau) \frac{(1 - F(\tau))}{f(\tau)}\, d\tau. \end{aligned} \tag{16}$$

Divide the whole integral interval in the above expression into three disjoint sets as below:

$$\begin{aligned} S_1 &= \{\tau : g(\tau) > \gamma^*\}, \\ S_2 &= \{\tau : g(\tau) < \gamma^*\}, \\ S_3 &= \{\tau : g(\tau) = \gamma^*\}. \end{aligned} \tag{17}$$

In addition, since $\forall \tau, 0 \leq q(\tau) \leq 1$, for $q^*(\tau)$ defined in (13), we have the following relationship

$$\begin{aligned} q^*(\tau) - q(\tau) \geq 0, \quad & \forall \tau \in S_1, \\ q^*(\tau) - q(\tau) \leq 0, \quad & \forall \tau \in S_2. \end{aligned} \tag{18}$$

Then, we have $\forall \tau \in S_1 \bigcup S_2$,

$$[q^*(\tau) - q(\tau)]g(\tau) \geq [q^*(\tau) - q(\tau)]\gamma^*. \tag{19}$$

Using the above relations, we have:

$$\begin{aligned} & G_s(\pi^*) - G_s(\pi) \\ =& \int_0^\infty \big[q^*(\tau) - q(\tau)\big] f(\tau)g(\tau)d\tau \\ =& \int_{\tau:S_1} \big[q^*(\tau) - q(\tau)\big] f(\tau)g(\tau)d\tau \\ &+ \int_{\tau:S_2} \big[q^*(\tau) - q(\tau)\big] f(\tau)g(\tau)d\tau \\ &+ \gamma^* \int_{\tau:S_3} \big[q^*(\tau) - q(\tau)\big] f(\tau)d\tau \\ \geq& \gamma^* \left\{ \int_0^\infty q^*(\tau)f(\tau)d\tau - \int_0^\infty q(\tau)f(\tau)d\tau \right\} \\ =& \gamma^*(n_p\eta - n_p\eta) = 0. \end{aligned} \tag{20}$$

Because $\Gamma_s(\pi) = \frac{G_s(\pi)}{l_I + l_B}$, we have $\Gamma_s(\pi^*) \geq \Gamma_s(\pi)$.

Note that, a smaller value of γ^* corresponds to a larger set of S_1, and thus, a more aggressive SU transmission. The collision probability perceived by the PU when the SU adopts π^* can be expressed as:

$$p_c(\pi^*) = \frac{1}{n_p}\Big[\int_{\tau:S_1} f(\tau)\, d\tau + p^* \int_{\tau:S_3} f(\tau)\, d\tau\Big]. \tag{21}$$

For the optimal policy, we have $p_c = \eta$. Substituting the value of γ^* obtained from (14) into (21), the value of p^* is either 0 or determined by (15). □

Note that randomized transmission is required when $g(t) = \gamma^*$, $p^* \neq 0$. In other words, the SU should transmit with probability $0 < p^* < 1$. When $\forall t$, $0 < q(t) < 1$, we call the spectrum access policy "random access".

We have shown that the threshold-based policy (13) is optimal, which is intuitive in that the SU should transmit when the conditional collision probability is low. For practical systems, the SU can detect previous PU transmissions with high accuracy at the PHY layer. When the PU switches from busy to idle, the SU initializes an internal timer $t = 0$, computes the value of $g(t)$, and then compares $g(t)$ with the threshold γ^*. Finally, the SU decides whether to transmit based on the comparison result.

Next, we consider a few examples of PU idle time distributions to illustrate how to obtain the values of γ^* and p^* in the optimal spectrum access policy.

When the PU has a slotted-structure and transits between idle and busy states according to a static Markov process (e.g., as assumed in [5], [6]), the PU idle time follows a geometric distribution. Specifically, $f(t) = \sum_{k=1}^\infty p_k \delta(t - kl_I)$, where $\delta(t)$ is the Dirac delta function, and p_k can be easily calculated using the state transition probabilities. We have $\forall k$:

$$g(t) = \begin{cases} \infty, & t \in ((k-1)l_I, kl_I), \\ 0, & t = kl_I. \end{cases} \tag{22}$$

We have $\gamma^* = 0$ and $p^* = n_p\eta$ in the optimal transmission policy, i.e., it is optimal to transmit during the slot interval with probability 1 when the PU is sensed to be idle, and transmits with probability $n_p\eta$ at the slot boundaries. The corresponding SU throughput limit with slotted PU idle time is α, which is the entire white space. Note that in this simple case collision free transmission is possible.

When the PU idle time is exponentially distributed, we have $g(t) = l_I$, which is a constant. In this case, we have $\gamma^* = l_I$, and $p^* = n_p\eta$. Note that the randomization is important here — SU transmits with probability p^* when it senses the channel being idle. The corresponding maximum throughput of the SU is $\Gamma_s = \alpha n_p \eta$, which serves as an upper bound on the SU throughput performance achieved by the random access schemes in [11] and CMA scheme proposed in [13] for single PU channel (both works assume a fixed non-zero SU packet length). Measurement results in [16] show that the inter-arrival time of voice users (i.e., idle time) in CDMA systems follows exponential distribution. In such systems, a random access policy is advantageous for SUs in terms of achieving the optimal tradeoff between protecting the PU and improving spectrum utilization efficiency.

For a class of PU idle time distributions, we derive an optimal transmission policy with a simpler form as shown below:

Corollary 1: For a given PU idle time distribution $f(\cdot)$, when $g(\cdot)$ is monotonically decreasing and continuous, the following spectrum access policy is optimal under the collision probability constraint $p_c \leq \eta$:

$$q^*(t) = \begin{cases} 1, & \text{if } t \leq T^*,\ \Phi(t) = \text{Idle} \\ 0, & \text{otherwise,} \end{cases} \tag{23}$$

where T^* can be obtained from the collision probability constraint $\int_0^{T^*} f(t)\, dt = n_p\eta$. The optimal throughput is:

$$\Gamma_s = \frac{\int_0^{T^*} tf(t)\, dt + T^* \int_{T^*}^\infty f(t)\, dt}{l_I + l_B}. \tag{24}$$

Proof: Since $g(t)$ is monotonically decreasing, relying on Theorem 1, there exists a T^*, such that for any $0 \leq t \leq T^*$, $g(t) \geq \gamma^*$, and $g(T^*) = \gamma^*$. Thus, the Corollary follows. □

Similarly, for monotonically increasing and continuous $g(\cdot)$, the optimal access strategy requires the SU to transmit with probability 1 when t is larger than some threshold T^*.

We use two simple examples to illustrate the optimal time threshold-based transmission policy for the SUs. When the PU idle time is uniformly distributed on the interval $[0, 2l_I]$ (therefore the mean idle time is l_I), $g(t) = 2l_I - t$, $t \in [0, 2l_I)$ is strictly decreasing, and relying on Corollary 1 we have:

$$T^* = 2l_I n_p \eta. \tag{25}$$

The corresponding maximum SU throughput is

$$\Gamma_s = 2\alpha n_p \eta - \alpha n_p^2 \eta^2. \tag{26}$$

When the idle time of the PU follows a two-parameter Weibull distribution with parameters β and μ, i.e.,

$$f(t) = \frac{\beta}{\mu}(\frac{t}{\mu})^{\beta-1}e^{-(\frac{t}{\mu})^{\beta}}, \tag{27}$$

where $\beta > 0$ is the shape parameter, and $\mu > 0$ is the scale parameter. When $\beta = 1$, the Weibull distribution degenerates into the exponential distribution. Weibull, therefore, is more general than exponential, and may have broader application for modeling the PU traffic pattern. When $\beta > 1$, $g(t)$ is a decreasing function of t, so relying on Corollary 1 we have:

$$T^* = \mu[-\ln(1 - n_p\eta)]^{\frac{1}{\beta}}. \tag{28}$$

We stress here that the proposed threshold-based access strategy can be applied to an arbitrary sojourn time distribution, including those without closed-form expression of PDF or CDF.

4.3 Impact of PU idle time distribution

As observed from previous sections, the PU idle time distribution has significant impact on the fundamental limit of the SU throughput performance. Therefore, we compare the maximum SU throughput under different PU idle time distributions in the following corollary.

Corollary 2: The optimal SU throughput performance under the PU packet collision probability constraint, η, is in $[\alpha n_p\eta, \alpha]$. The upper bound is achieved if $\int_0^{\infty} f(t)\mathbb{I}\{g(t) > 0\} \leq n_p\eta$; the lower bound is achieved if the PU idle time follows the exponential distribution.

Proof: When the PU idle time distribution satisfies $\int_0^{\infty} f(\tau)\mathbb{I}\{g(\tau) > 0\}\, d\tau \leq n_p\eta$, with $\gamma^* = 0$, $p^* = 0$ (i.e., the SU does not transmit when $g(t) > 0$), the SU achieves the upper bound. Examples of this kind include fixed and slotted PU idle time, as discussed in Section 4.2.

Next, we prove the lower bound constructively. Consider a random access policy $\hat{\pi} = [\hat{q}(t) : t = 0, 1, \cdots]$ as:

$$\hat{q}(t) = \begin{cases} n_p\eta, & \text{if } \Phi(t) = \text{Idle} \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

Obviously, $\hat{\pi}$ is sub-optimal. We have:

$$p_c = \frac{1}{n_p}\int_0^{\infty} \hat{q}(t)f(t)\, dt = \eta, \tag{30}$$

and

$$G_s(\hat{\pi}) = n_p\eta\int_0^{\infty} (1 - F(t))\, dt = n_p\eta l_I. \tag{31}$$

It follows that $G_s(\pi^*) \geq G_s(\hat{\pi}) = n_p\eta l_I$. Therefore, $\Gamma_s \geq \Gamma_s(\hat{\pi}) = n_p\eta\alpha$, which is the maximum SU throughput when the PU idle time is exponentially distributed. □

The random access policy in (29) has interesting properties. In the absence of idle time distribution information, the scheme in (29) achieves a capacity of $n_p\eta\alpha$ regardless of the actual distribution but is optimal for the exponential distribution. In addition, it does not require the SU to perform persistent sensing to discern the beginning of the PU idle time.

In Fig. 3, we compare the maximum SU throughput performance under different PU idle time distributions for collision probability constraint $\eta \in [0, 1]$. In the calculation, we set l_I = 2ms, l_B = 2ms, n_p =1. We can observe that the exponential PU idle time distribution results in the worst SU throughput performance. Weibull distribution with $\beta = 2$ may be better or worse than the uniform PU idle time distribution over $[0, 2l_I]$. Generalized Pareto (GP) distribution with scale parameter σ = 1.87ms has slightly better SU throughput performance than the exponential PU idle time distribution. Since random access policy achieves the same SU throughput performance as the exponential distribution for all PU idle time distributions, results in Fig. 3 show the advantage of the threshold-based policy over random access policy.

In summary, we have the following observations:

- The knowledge of PU traffic behavior is important. Knowing the distribution may significantly improve the SU's throughput. For example, the maximum throughput obtained in (26) under uniform distribution is much larger than $n_p\eta\alpha$. Therefore, in order to fully utilize the spectrum opportunities in the primary band, the SU should acquire knowledge of the PU behavior. Here, we focus on the time domain behavior, but spatial correlation knowledge of the PU signal can also be exploited (e.g. [17]).
- When the distribution information is absent, an SU can exploit the random policy in (29) as a lower bound of its throughput. Not knowing the distribution is equivalent to the memoryless exponential distribution. On the other hand, access control design based on the exponential idle time distribution is robust in the sense that the SU throughput performance varies little when the PU idle time distribution changes (e.g., [13]).
- When the PU idle time distribution is not available or is time varying (as validated by the measurement data in [16]), the SU may learn the PU traffic pattern in real time to take advantage of the time correlation in the PU traffic pattern. We propose such an adaptive scheme in Section 6.

4.4 Numerical results

Next we compare the performance achieved by the proposed threshold-based policy and the CMA scheme proposed in [13]. In the simulation, the CDF of the PU idle time is given by [14] as:

$$F(t) = p_uF_u(t) + (1 - p_u)F_{gp}(t), \tag{32}$$

where p_u denotes the probability that the PU idle time follows a uniform distribution, $F_u(t)$ is the CDF of a uniform distribution on the interval [0, 0.7ms], and

$$F_{gp}(t) = 1 - (1 + \kappa\frac{t}{\sigma})^{-1/\kappa} \tag{33}$$

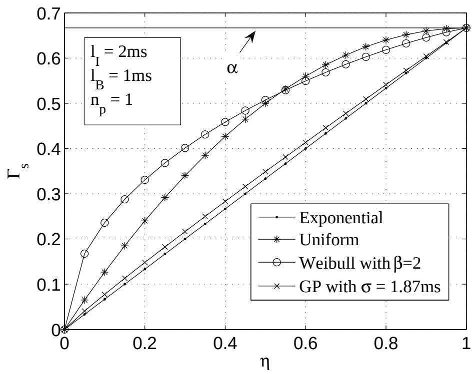


Fig. 3. Comparison of SU Throughput Performance.

TABLE 2
Simulation Parameters

	$\mathbf{l_I}$	$\mathbf{l_B}$	α	$\mathbf{p_u}$	κ	σ
Set-1	2.90ms	1.03ms	0.7379	30.1%	0.0151	3.95
Set-2	1.39ms	1.03ms	0.5605	47.7%	0.0169	2.35

with the values of the shape parameter κ and scale parameter σ taken from [14] for PUs that have typical WLAN traffic. We list the PU parameters used here as in Table 2. The value of n_p is set to be 1. Note that the optimality of the CMA scheme under exponential PU idle time distribution is obtained without consideration of the sensing overhead and errors [13]. Here, we use the same assumptions.

For single channel, the CMA scheme essentially allows the SU to transmit with probability

$$q_{CMA} = \min\{\frac{n_p \eta l_s}{l_I(1 - e^{-l_s/l_I})}, 1\}, \tag{34}$$

when the SU senses the PU as idle.

The threshold-based policy proposed in Theorem 1 is tailored to accommodate the situation in which $l_s \not\to 0$. We divide the time after the beginning of the PU idle time into sequential slots, each with length l_s. For slot k, the decision metric $g(k)$ is calculated as:

$$g(k) = \min\{g(t), t \in [(k-1)l_s, kl_s)\}. \tag{35}$$

For each k, we have the value

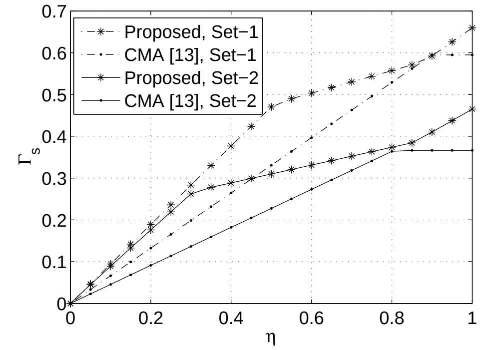
$$\Pr[Z(k)] = \Pr[(k-1)l_s \le L_I < kl_s] = F(kl_s) - F((k-1)l_s).$$

To obtain the values of the threshold γ^* and p^*, we first sort the array of $g(k)$ in an ascending order. Then, we find the smallest value of γ such that

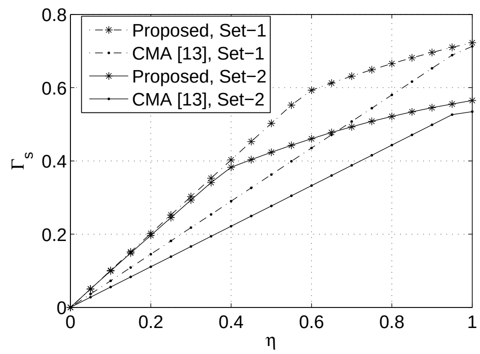
$$\sum_k \mathbb{I}[g(k) \ge \gamma] \Pr[Z(k)] \le \eta. \tag{36}$$

Denote this value as $\bar{\gamma}$. If the equality in (36) holds, then $\gamma^* = \bar{\gamma}$, and $p^* = 0$. Otherwise,

$$\gamma^* = \max\{g(k) : g(k) < \bar{\gamma}\}. \tag{37}$$



(a) SU Packet Length: $l_s = 625\,\mu$s.



(b) SU Packet Length: $l_s = 100\,\mu$s.

Fig. 4. Comparison with the CMA Scheme.

The value of p^* in this case can be obtained as:

$$p^* = \frac{n_p \eta - \sum_k \mathbb{I}[g(k) > \gamma^*] \Pr[Z(k)]}{\sum_k \mathbb{I}[g(k) = \gamma^*] \Pr[Z(k)]}. \tag{38}$$

The simulation results for both sets of PU traffic parameters are shown in Fig. 4(a) and Fig. 4(b) for the SU slot length $l_s = 625\mu$s (the slot size of Bluetooth) and $l_s = 100\mu$s, respectively. In other words, the SU slot length is roughly 50% and 10% of the average PU idle time. We observe that the proposed threshold-based policy always obtains better SU throughput than the CMA algorithm proposed in [13]. In addition, with a smaller value of l_s, the SU throughput performance is a smoother function of η due to a smaller quantization effect, and is better than that of a larger value of l_s. This verifies the statement in Proposition 1. However, with a fixed control overhead, the SU may choose a proper slot length to achieve the most efficient payload delivery.

In summary, to quantify the spectrum opportunities, important factors are the percentage of the idle time (α), the PU idle time distribution $f(\cdot)$, the availability of such information, and the PU protection requirement (η). When there are multiple spectrum bands available for SUs to choose from, SUs should consider all these factors. For example, in a secondary spectrum leasing market, the SU throughput performance limit obtained here can serve as a reference for spectrum pricing.

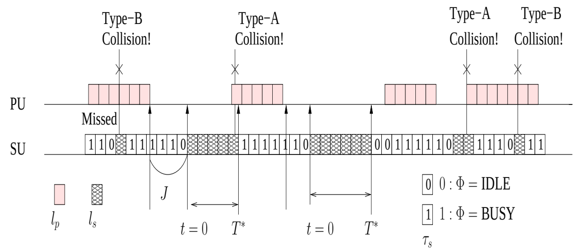


Fig. 5. Illustration of Imperfect Sensing.

5 Cognitive Access under Imperfect Sensing

In this section, we consider the impact of imperfect sensing. In this case, as illustrated in Fig. 5, two types of collisions caused by different SU actions exist. As in the previous section, we assume that the SU has the knowledge of the PU idle time distribution $f(\cdot)$. The sensing time is denoted as τ_s. In general, the greater the τ_s, the more reliable the spectrum sensing, and thus the better the protection on the PU. However, a longer sensing time also means less transmission time for the SU. Therefore, there exists a trade-off between detection performance and SU throughput performance.

5.1 Model and analysis

We model the SU sensing performance by a (P_f, P_m) pair, where P_f is the probability of false alarm (recognizing the idle PU as busy), and P_m is the probability of missed detection (recognizing the busy PU as idle). As mentioned in Section 3, missed detections cause type-B collisions. On the other hand, false alarm leads to waste of spectrum opportunities. The results here apply to any generic sensing method, including cooperative sensing schemes, as long as the sensing performance can be described by the (P_f, P_m) pair. We assume that for each sensing action, the events of having a false alarm or missed detection error are independent across time.

Let $N_c^A(k)$ and $N_c^B(k)$ denote the numbers of type-A and type-B collisions in the kth PU idle-busy cycle, respectively. By assuming stationary access policy and independent sensing outcomes, both $N_c^A(k)$ and $N_c^B(k)$ are i.i.d. random processes, and thus the collision probability perceived by the PU can be expressed as:

$$p_c = \frac{\mathbb{E}[N_c^A] + \mathbb{E}[N_c^B]}{\mathbb{E}[N_p]} = \frac{n_c^A}{n_p} + \frac{n_c^B}{n_p}. \tag{39}$$

In what follows, we use $p_c^A = n_c^A/n_p$ and $p_p^B = n_c^B/n_p$ to denote the type-A and type-B collision probabilities, respectively.

We first discuss the impact of missed detection on the collision perceived by the PU. As illustrated in Fig. 5, the SU only initiates packet transmission after it senses the channel being idle, and keeps sensing otherwise. Though the optimal spectrum access policy under imperfect sensing may require the SU to transmit even when it detects the PU being busy, it is reasonable for the SU to be conservative in order to protect the PU. Type-B packet collisions happen after each missed detection, except in rare cases when the PU has a state transition from the busy to idle state at the precise moment the SU starts transmitting (known as the edge effect).

For the PU, the following case causes the most type-B collisions: the SU transmits a packet immediately after mistakenly detecting the busy PU as idle. Let X denote the residual time of the PU's packet when the SU starts transmitting. Since there is no synchronization between the SU and the PU, X is uniformly distributed over $[0, l_p]$. Define Z_c as the number of collided PU packets due to a missed detection. Ignoring the edge effect, we have:

$$Z_c = \begin{cases} 1, & \text{if } X > l_s, \\ 2, & \text{if } X \le l_s. \end{cases} \tag{40}$$

For $l_p \ge l_s$, the expectation of Z_c is given by

$$z_c = \mathbb{E}_X[Z_c] = \frac{l_p + l_s}{l_p}. \tag{41}$$

Note that, for continuous PU busy time, z_c depends on the distribution of L_B, which can be readily calculated.

Next, we calculate the average number of missed detections for a given P_m. Denote the number of missed detections during a PU busy period with length $N_p l_p$ as N_m. We have the expected value of N_m given N_p as:

$$\mathbb{E}[N_m|N_p] = \sum_{n=1}^{\lceil \frac{N_p l_p}{\tau_s + l_s} \rceil} n P_m^n (1-P_m)^{\lceil \frac{N_p l_p - n l_s}{\tau_s} \rceil - n} \binom{\lceil \frac{N_p l_p - n l_s}{\tau_s} \rceil}{n},$$

where $\lceil\cdot\rceil$ represents the operation of rounding up a real number to the closest integer.

It can be easily shown that averaging on N_m gives the following upper and lower bounds:

$$P_m \frac{N_p l_p}{\tau_s + l_s} \le \mathbb{E}[N_m|N_p] \le P_m \frac{N_p l_p}{\tau_s}. \tag{42}$$

Since we have

$$n_c^B = \mathbb{E}_{N_p}[\mathbb{E}[N_m|N_p]] z_c, \tag{43}$$

the type-B packet collision probability is bounded by

$$\frac{P_m l_p z_c}{\tau_s + l_s} \le p_c^B \le \frac{P_m l_p z_c}{\tau_s}. \tag{44}$$

Using the lower bound in (44), we observe that it is impossible for SUs with $P_m > \frac{\eta(\tau_s + l_s)}{l_p + l_s}$ to achieve the required PU protection. Therefore, the operating region of the spectrum sensor is constrained. In order for the SUs to satisfy the collision probability constraint safely, we require that

$$P_m \le \frac{\eta(l_p + l_s)}{\tau_s}. \tag{45}$$

While determining a feasible value of P_m using (45) is simple, a good approximation on p_c^B for a given P_m can also be found. For a given PU busy length $N_p l_p$, an event of missed detection occupies $\tau_s + l_s$ seconds while a correct PU detection requires τ_s seconds, and thus the total number of two SU sensing outcomes (missed and correct detection) can be estimated as $\lceil \frac{N_p l_p}{(1-P_m)\tau_s + P_m(\tau_s+l_s)} \rceil$. Then the average number of missed detections during a PU busy period can be approximated as:

$$\mathbb{E}[N_m|N_p] \simeq P_m \lceil \frac{N_p l_p}{(1-P_m)\tau_s + P_m(\tau_s + l_s)} \rceil. \tag{46}$$

Now we can estimate the value of n_c^B via (43), and obtain the following approximation on p_c^B:

$$\begin{aligned} p_c^B &\simeq \frac{P_m \mathbb{E}[\lceil \frac{N_p l_p}{(1-P_m)\tau_s + P_m(\tau_s+l_s)} \rceil] z_c}{n_p} \\ &\simeq \frac{P_m(l_p + l_s)}{\tau_s + P_m l_s}, \end{aligned} \tag{47}$$

which is much easier to calculate than using the exact value of $\mathbb{E}[N_m|N_p]$.

Next, we focus on the impact of false alarm. The allowance for type-A collisions is

$$p_c^A \leq \hat{\eta} = \eta - p_c^B. \tag{48}$$

With the value of P_m satisfying condition (45), we have $\hat{\eta} > 0$, and the following discussion is meaningful.

Due to false alarm errors, the SU does not know the beginning of the idle period accurately. Let J (as shown in Fig. 5) be the number of consecutive sensing actions that the SU takes to detect the idle state after the PU becomes idle. It is easy to see that J follows a geometric distribution:

$$\Pr[J = j] = P_f^{j-1}(1 - P_f), j = 1, 2, \cdots$$

Since $l_s \leq l_p$, a type-A collision only leads to one packet loss for the PU. However, since the SU does not know whether it made a false alarm, its access policy should not depend on the realization of J. Instead, the SU access strategy determines the transmission probability $q(k)$ for a sequence of time slots $k = 1, 2, \cdots$ after it detects the channel being idle. Given that the SU transmits at time slot k, a type-A collision happens when the PU returns during time interval $[(k-1)l_s + j\tau_s, kl_s + j\tau_s]$. We define this event as $Z(k, j)$ with $j = 1, 2, \cdots$, with

$$\Pr[Z(k, j)] = F(kl_s + j\tau_s) - F((k-1)l_s + j\tau_s). \tag{49}$$

For a given $J = j$, the average type-A collision probability during an idle-busy period can be expressed as:

$$\frac{\sum_{k=0}^{\infty} q(t_k) \Pr[Z(k, j)]}{n_p}. \tag{50}$$

Then, taking the expectation over J, we have the following expression for p_c^A:

$$\begin{aligned} p_c^A &= \frac{1}{n_p} \mathbb{E}_J \left\{ \sum_{k=0}^{\infty} q(k) \Pr[Z(k, J)] \right\} \\ &= \frac{1 - P_f}{n_p} \sum_{j=1}^{\infty} P_f^{j-1} \sum_{k=0}^{\infty} q(t_k) \Pr[Z(k, j)]. \end{aligned} \tag{51}$$

Similarly, we have the expression for the throughput performance as below:

$$\Gamma_s = \frac{\mathbb{E}_J[\sum_{k=0}^{\infty} l_s q(kl_s)(1 - F(kl_s + J\tau_s))]}{l_I + l_B}. \tag{52}$$

For a given pair of (P_f, P_m) (the sensing time should be long enough to guarantee that $\hat{\eta} > 0$), the optimal spectrum access strategy for the SU can be formulated as a similar optimization problem as in (11) with constraint $p_c^A \leq \hat{\eta}$ replacing $p_c \leq \eta$. However, the joint optimization on the spectrum detector and access controller is complicated due to the dependency of Γ_s on P_f and $\hat{\eta}$, which is then determined by P_m. Here, we adopt a heuristic approach based on the result in Theorem 1. Specifically, we have a modified threshold-based spectrum access policy for the SU.

First, we define the following decision metric for a given (P_f, P_m) pair:

$$\tilde{g}(t) = \frac{\mathbb{E}_J[(1 - F(t + J\tau_s))]}{\mathbb{E}_J[f(t + J\tau_s)]}.$$

For slot k ($k = 1, 2, \cdots$), similar to (35), we calculate the decision metric as:

$$\tilde{g}(k) = \min\{\tilde{g}(t) : t \in [(k-1)l_s, kl_s)\}. \tag{53}$$

Then, we have the modified threshold-based spectrum access policy as:

$$q(k) = \begin{cases} 1, & \text{if } \tilde{g}(k) > \tilde{\gamma}; \\ \tilde{p}, & \text{if } \tilde{g}(k) = \tilde{\gamma}; \\ 0, & \text{otherwise.} \end{cases} \tag{54}$$

The calculation of threshold $\tilde{\gamma}$ and $\tilde{p}$ follows a similar approach to the calculation of γ^* and p^* in Sec. 4.4, with $\tilde{g}(k)$ replacing $g(k)$ in (36), (37), and (38), $\hat{\eta}$ replacing η in (36), $\mathbb{E}_J[\Pr[Z(k, J)]]$ replacing $\Pr[Z(k)]$ in (36).

5.2 Spectrum access with energy detector

Next, we use energy detector as an example to illustrate how to jointly determine the spectrum sensor operating point and the SU access strategy.

As in [18], the detection statistic of the SU spectrum sensor can be approximated by Gaussian distribution. Denote the sampling frequency as f_s, and the power of the AWGN and PU signals as σ_n^2 and σ_p^2, respectively. Then, we have

$$\begin{aligned} P_f &= \mathbb{Q}\left(\frac{\zeta - \tau_s f_s \sigma_n^2}{\sqrt{2\tau_s f_s \sigma_n^4}}\right), \\ P_m &= 1 - \mathbb{Q}\left(\frac{\zeta - \tau_s f_s(\sigma_n^2 + \sigma_p^2)}{\sqrt{2\tau_s f_s(\sigma_n^2 + \sigma_p^2)^2}}\right), \end{aligned} \tag{55}$$

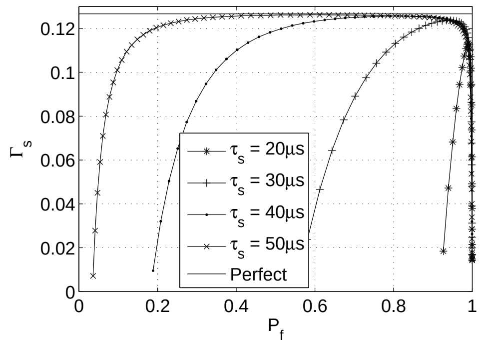
where $\mathbb{Q}(\cdot)$ is the well-known Q function, and ζ is the detection threshold. The ROC (receiver-operating characteristic) curve of the energy detector is jointly determined by τ_s, f_s, and SNR (Signal-to-Noise-Ratio).

By adjusting the detection threshold ζ, we can obtain different values for the (P_f, P_m) pair.

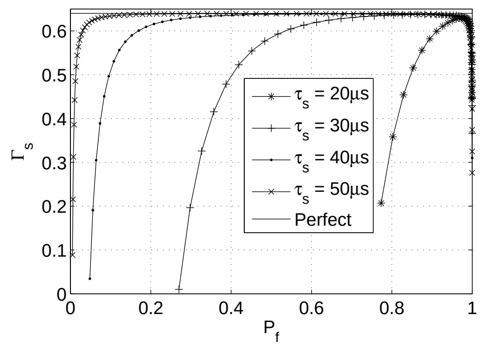
Last, we present numerical results on the SU performance with imperfect sensing. The PU idle time is assumed to be uniformly distributed in $[0, 2l_I]$ and the number of packets in a busy period is set to 100. We use the following parameters in the simulations: $l_p = l_s = 1$ ms, $l_I = 200$ ms, $l_B = 100$ ms, $f_s = 10$ MHz, and SNR $= \sigma_p^2/\sigma_n^2 =$ -3 dB. Different values of sensing time $\tau_s = 20, 30, 40, 50\,\mu$s are used.

Fig. 6(a) and Fig. 6(b) show the throughput performance of the proposed modified threshold-based spectrum access scheme when $\eta = 10^{-3}$, and $\eta = 8 \times 10^{-3}$, respectively. For comparison, we also plot the throughput performance assuming perfect sensing. Recall that P_m has to be small enough to satisfy $\hat{\eta} > 0$. It is easy to note that, for a smaller packet collision probability constraint, the range of the feasible operating points is smaller. We observe that there is an optimal operating point of the spectrum sensor to achieve the maximum throughput. This point represents the optimal trade-off between P_m and P_f. For a given P_f, the longer the sensing time, the larger the throughput (within a reasonable range). This implies in practical systems, a longer sensing time is desirable, since the access of the SU has to be conservative enough to protect the PU.

In addition, for the benefit of the SU, the transmission given a missed detection contributes nothing to its throughput, but "consumes" the collision probability allowance. So it is much more important to ensure a small missed detection probability than false alarm probability. In other words, the capability to protect the PU (measured by a small missed detection probability) can benefit the SU's throughput performance. We also observe that when τ_s is large enough, the performance of the optimal operating point is very close to that of perfect sensing. The larger the value of τ_s, the larger the region where the performance is close to optimal.



(a) Packet Collision Constraint: $\eta = 10^{-3}$



(b) Packet Collision Constraint: $\eta = 8 \times 10^{-3}$

Fig. 6. SU Throughput with Imperfect Sensing.

6 Adaptive Opportunistic Spectrum Access

As shown in Fig. 1, the protection of PUs is more challenging when there are multiple SUs accessing a common PU channel. Because SUs are widely spread in the area, one SU may not be able to sense all other SUs' transmissions or their collisions with the PU. While this enables spatial reuse among SUs, the protection of the PU is more difficult because the SUs' disruption accumulates. Consider the case where two SUs cause random collisions with the PU. Assume that the two SUs are not aware of each other. If each SU causes 1% collision randomly (e.g., using the random access policy (29) in Section 4.3), the cumulative collision perceived by the PU will be 1.99%. When there are more SUs, the impact of such cumulative intrusion can be devastating.

Another challenge is to acquire knowledge of the PU idle time distribution. While there are schemes to estimate distribution functions [19], [20], [21], they usually require a large number of observations to achieve certain accuracy. This is especially challenging when the PU traffic pattern and the idle time distribution is time varying (e.g., [16]).

In this section, we address the above challenges by proposing a distributed and adaptive spectrum access scheme for multiple SUs. The proposed scheme explicitly explore the structure of the optimal scheme presented in Sec. 4, in particular the time-threshold-based policy as described by Corollary 1. When function $g(\cdot)$ is monotonically decreasing (or increasing), the proposed adaptive strategy approaches the optimal performance for each SU while satisfying the collective protection requirement from the PU. Each SU executes the proposed adaptive algorithm and makes transmission decisions independently.

Here, we assume that all SUs have knowledge about the average number of PU packets in a busy period, which is much easier to obtain than the distribution knowledge. We also assume that SUs are far away from each other, and thus no collision among SUs is considered. Note that collision among nearby SUs can be controlled by any existing MAC protocol given synchronization on the virtual channel. In the sequel, we use superscript $m = 1, 2, \cdots$ to index SUs. We first consider perfect sensing case, and then extend the algorithm to the imperfect sensing case.

6.1 Perfect sensing case

Because the statistics of the PU traffic pattern may change over time, and different SUs may join the network at different moments, the interruption to the PU is dynamically changing. This requires that each SU intelligently adjust its transmission strategy based on its observation of the PU behavior and its estimate on the collision probability perceived by the PU. With the time-threshold-based access structure derived in Sec. 4, each SU only needs to iteratively update one access parameter, i.e., the threshold T^*.

Each SU records $N_c^m(k)$, defined as the number of collisions caused by it during the kth PU busy-idle period. With perfect sensing, SUs can detect the beginning of the PU idle time accurately. Each SU transmits its packets after sensing the idle channel, and stops when the internal timer t reaches the threshold $T^m(i)$ (the ith estimate of T^*) or it collides with the PU. Each updating interval for the time threshold $T^m(i)$ consists of W idle-busy periods of the PU. The SU estimates the collision probability in the ith interval as

$$\tilde{\eta}^m(i) = \frac{\sum_{k=(i-1)W+1}^{kW} N_c^m(k)}{W n_p}, \tag{56}$$

and updates the threshold T^m as follows:

$$T^m(i+1) = T^m(i) + \xi(i) T^m(0) \frac{\eta - \tilde{\eta}^m(i)}{\eta}, \tag{57}$$

where $T^m(0)$ is the initial value, and $\xi(i)$ is the step-size. Note that updating methods similar to (57) are often seen in the field of adaptive signal processing, control, and stochastic approximation (e.g., [22], [23]). Here, with the insights obtained from Corollary 1, we are able to adopt existing techniques to solve the practical challenges of satisfying the accumulated interruption constraint without the knowledge PU idle time distribution and achieve good SU throughput performance.

A natural choice of $T^m(0)$ is the value of T^* for exponential PU idle time distribution, i.e., $T^m(0) = -l_I \ln(1 - n_p\eta)/l_s$. When $\tilde{\eta}^m > \eta$, the SU reduces its transmission time; otherwise, the SU increases its transmission time. The step-size determines the trade-off between the convergence speed and the variance of the updated values. For example, one can show that $T^m(i)$ converges to T^* when $\xi(i) = 1/i$, using standard stochastic approximation techniques.

As an illustration, we use numerical simulations where the idle time initially follows the Weibull distribution with $\beta = 2$ and changes to uniform distribution later without informing SUs. Simulation parameters are set as: $l_I = 200\,\text{ms}$, $n_p = 100$, $l_s = l_p = 1\,\text{ms}$, $\eta = 0.1\%$, $W = 50$, and $\mu(i) = 0.4$. Each SU has the knowledge of n_p and l_I (both are set to be constants), but not $f(\cdot)$. Initially, there is only one SU in the network. Another SU joins the network after 50000 idle-busy periods (about 7500 seconds) and runs the proposed algorithm independently without observing the collisions caused by the first SU to the PU. The two SUs cannot sense each other. After 150000 idle-busy periods, the PU's idle time distribution changes from Weibull to uniform distribution.

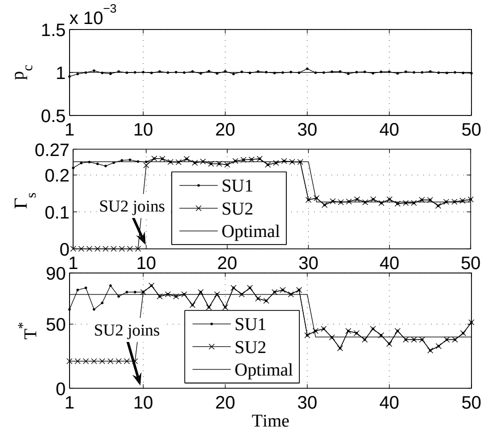


Fig. 7. Convergence of the Proposed Adaptive Algorithm (Unit of x-axis: 5000 idle-busy periods).

The performance of the proposed algorithm is shown in Fig. 7, where the value of p_c and Γ_s is evaluated every 5000 idle-busy periods, or 100 updating periods. For comparison purposes, we plot the optimal throughput and the optimal value of T^* in the figure. From Fig. 7, we can observe that the proposed algorithm has several desirable properties. First, it satisfies the protection requirement of the PU when there are multiple SUs opportunistically accessing the spectrum. Second, it converges very quickly to the optimal value of T^*, and obtains close-to optimal throughput performance derived for $l_s \to 0$. Note that each SU independently achieves similar throughput Γ_s rather than sharing the total available time, thus achieving desirable spatial reuse. Third, it adapts to the dynamics of the PU's traffic pattern rapidly. Additionally, it is a distributed algorithm with no central controller. Each SU performs its algorithm independently.

The proposed adaptive algorithm works well for a wide class of idle time distributions with monotonically decreasing $g(\cdot)$. When this condition is not satisfied, the T^* policy from Corollary 1 and the adaptive scheme may not be optimal. However, the adaptive scheme can still guarantee the collaborative protection of the PU in a distributed manner while enabling spatial reuse. In the future, we plan to extend the adaptive scheme to general idle time distributions.

6.2 Imperfect sensing case

In the case of imperfect sensing, the SU can neither track accurately the status of the PU nor the exact beginning of the idle time. We address the impact of the imperfect sensing by modifying the above adaptive scheme.

First, each SU estimates the collision probability caused by missed detection, i.e., p_c^B, using (47) or the upper bound (44). Then, it calculates the margin for type-A collisions as $\hat{\eta} = \eta - p_c^B$. Here, we assume that an SU

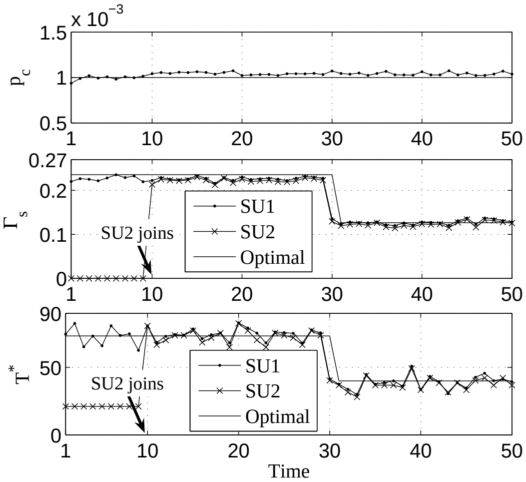


Fig. 8. Performance of the Proposed Adaptive Algorithm with Imperfect Sensing (Unit of x-axis: 5000 idle-busy periods).

does not know the presence of other SUs, and has no knowledge about other SUs' missed detection probabilities or the information about other SUs' collisions with the PU. SUs update their time threshold independently using (57) where η is replaced by $\hat{\eta}$.

If the SU transmits until $T^m(i)$ without colliding with the PU, the SU will confirm the return of the PU after detecting N_s consecutive busy slots, where N_s is a design parameter. This reduces the mistake that the SU falsely detects a busy-idle transition when the PU is still idle.

We present simulation results to validate the performance of the proposed algorithm. The simulation setup is the same as in the previous section. The missed detection probabilities of SU 1 and SU 2 are 1.3203×10^{-8} and 10^{-6}, respectively. The false alarm probabilities are 0.67 and 0.2, respectively. The sensing time is $\tau_s = 50\,\mu$s. The number of packets in a busy period of the PU follows a uniform distribution with mean $n_p = 100$. The design parameter N_s is heuristically set as $N_s = n_p\, l_s/(10\tau_s)$. Each updating period for the SU consists of $W = 50$ idle-busy periods.

The performance of the proposed algorithm is shown in Fig. 8. The convergence to T^* is fast. Each SU achieves close-to optimal throughput performance. The proposed algorithm can approximate the PU protection requirement well. It is slightly above the threshold after the second SU joins the network. This is due to the fact that the type-B collisions accumulate at the PU since each SU is oblivious to other SUs. To address this issue, one can tune the spectrum sensor to reduce the missed detection probability (we intentionally left the missed detection probability of the second SU high to illustrate this accumulative effect). In addition, SUs should be aware of the potential existence of other SUs and thus leave a margin on the collision probability constraint.

7 Conclusion

We studied the fundamental limit on the throughput of cognitive SU networks under the primary packet collision constraint. Under the perfect sensing assumption, we derived an optimal threshold-based SU transmission strategy for generic PU idle time distribution. We showed that the idle time distribution and the packet collision probability constraint of the PU have significant impact on the throughput performance and provided both a lower and an upper throughput bound. We analyzed the impact of imperfect sensing on the SU performance, and proposed a modified threshold-based spectrum access scheme that achieves close-to-optimal performance. Moreover, we proposed a distributed scheme that adapts to the PU traffic pattern changes and works well in the case of multiple secondary users, under both perfect sensing and imperfect sensing assumptions. Our results in this paper provide insights on the trade-off between PU protection and SU throughput performance.

References

[1] Shared spectrum company: DARPA XG program information. [Online]. Available: http://www.sharedspectrum.com/technology/darpaxg.html

[2] F. W. Seelig, "A description of the August 2006 XG demonstrations at fort A.P. Hill," *the 2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 1–12, 2007.

[3] M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, "DIMSUMnet: new directions in wireless networking using coordinated dynamic spectrum," *the 6th IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks (IEEE WoWMoM)*, pp. 78–85, June 2005.

[4] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A general framework for wireless spectrum auctions," *the 2nd IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, pp. 22–33, 2007.

[5] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal on Selected Areas in Communications (JSAC): Special Issue on Adaptive, Spectrum Agile and Cognitive Wireless Networks*, vol. 25, no. 3, pp. 589–600, 2007.

[6] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," *IEEE Transactions on Information Theory*, vol. 54, no. 5, pp. 2053–2071, 2008.

[7] P. Wang, L. Xiao, S. Zhou, and J. Wang, "Optimization of detection time for channel efficiency in cognitive radio systems," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007, pp. 111–115.

[8] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, April 2008.

[9] A. Ghasemi and E. S. Sousa, "Optimization of spectrum sensing for opportunistic spectrum access in cognitive radio networks," *the 4th Consumer Communications and Networking Conference (CCNC)*, pp. 1022–1026, Jan. 2007.

[10] Q. Zhao and K. Liu, "Detecting, tracking, and exploiting spectrum opportunities in unslotted primary systems," *in IEEE Radio and Wireless Symposium (RWS)*, 2008.

[11] S. Huang, X. Liu, and Z. Ding, "Opportunistic spectrum access in cognitive radio networks," *The 27th IEEE Conference on Computer Communications (INFOCOM)*, pp. 1427–1435, April 2008.

[12] P. Pawelczak, R. Prasad, and R. Hekmat, "Opportunistic spectrum multichannel OFDMA," *IEEE International Conference on Communications (ICC)*, pp. 5439–5444, June 2007.

[13] S. Geirhofer, L. Tong, and B. Sadler, "Cognitive medium access: Constraining interference based on experimental models," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 95–105, Jan. 2008.

[14] S. Geirhofer, L. Tong, and B. M. Sadler, "Dynamic spectrum access in WLAN channels: Empirical model and its stochastic analysis," in *Proc. First International Workshop on Technology and Policy in Accessing Spectrum (TAPAS)*, 2006.

[15] S. Huang, X. Liu, and Z. Ding, "Optimal sensing-transmission structure for dynamic spectrum access," *to appear at IEEE INFOCOM*, 2009.

[16] D. Willkomm, S. Machiraju, J. Bolot, and A. Wolisz, "Primary users in cellular networks: A large-scale measurement study," *3rd IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Oct. 2008.

[17] M. Wellens, J. Riihijarvi, M. Gordziel, and P. Mahonen, "Evaluation of cooperative spectrum sensing based on large scale measurements," *3rd IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, Oct. 2008.

[18] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Experimental study of spectrum sensing based on energy detection and network cooperation," in *Proc. First International Workshop on Technology and Policy in Accessing Spectrum (TAPAS)*. New York, NY, USA: ACM Press, 2006.

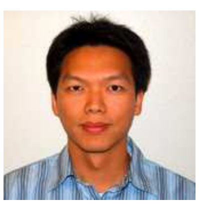
[19] H. Kim and S. K. G., "Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks," *IEEE Transactions on Mobile Computing*, vol. 7, no. 5, pp. 533–545, May 2008.

[20] A. Azzalini, "A note on the estimation of a distribution function and quantiles by a kernel method," *Biometrika*, vol. 68, no. 1, pp. 326–328, April 1981.

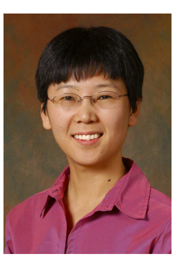
[21] S. Wang, "Nonparametric estimation of distribution functions," *Metrika*, vol. 38, no. 1, pp. 259–267, December 1991.

[22] P. R. Kumar and P. Varaiya, *Stochastic systems: estimation, identification and adaptive control*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1986.

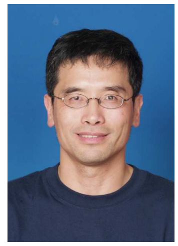
[23] H. Kushner and G. Yin, *Stochastic Approximation Algorithms and Applications*. New York: Springer-Verlag, 1997.



Senhua Huang Senhua Huang received his B.S. and Master of Engineering degrees in electrical engineering from the University of Science and Technology of China in 2003 and 2006. He is currently working toward his Ph.D. degree in the Department of Electrical and Computer Engineering at the University of California, Davis. His research interests are in the area of wireless communications and networks, with current emphasis on the dynamic spectrum access for cognitive radio networks.



Xin Liu Xin Liu received her Ph.D. degree in electrical engineering from Purdue University in 2002. She is currently an associate professor in the Computer Science Department at the University of California, Davis. Before joining UC Davis, she was a postdoctoral research associate in the Coordinated Science Laboratory at UIUC. Her research is on wireless communication networks, with a focus on resource allocation and dynamic spectrum management. She received the Best Paper of year award of the Computer Networks Journal in 2003 for her work on opportunistic scheduling. She received NSF CAREER award in 2005 for her research on "Smart-Radio-Technology-Enabled Opportunistic Spectrum Utilization." She received the Outstanding Engineering Junior Faculty Award from the College of Engineering, University of California, Davis in 2005.



Zhi Ding Zhi Ding is the Child Family Endowed Professor of Engineering and Entrepreneurship at the University of California, Davis. He received his Ph.D. degree in Electrical Engineering from Cornell University in 1990. From 1990 to 1998, he was a faculty member of Auburn University. From 1998-2000, he was a faculty member of Electrical and Computer Engineering at the University of Iowa. He joined the University of California, Davis in 2000 as a professor of Electrical and Computer Engineering. Currently, he is also a guest Changjiang Chair Professor of the Southeast University in Nanjing, China.

Prof. Ding is a Fellow of the IEEE and is named as 2004-6 Distinguished Lecturer by the Circuits and Systems Society. Professor Ding was associate editor for IEEE Transactions on Signal Processing from 1994-1997 and 2001-2004. He served as associate editor for IEEE Signal Processing Letters from 2002-2005. He was a member of the Editorial Board of the IEEE Signal Processing Magazine. He served as the Technical Program Chair for COMSOCs Globecom-2006. Prof. Ding was a member of IEEE Signal Processing Societys Technical Committee on Statistical Signal and Array Processing (1994-1998) and a member of Technical Committee on Signal Processing for Communications (1998-2003). He also served on the IEEE Signal Processing Society Technical Committee on Multi-Media Signal Processing (2002-2005).

Professor Ding has published over 200 refereed research papers. He also coauthored two books: Blind Equalization and Identification (Taylor and Francis, 2001) and Modern Digital and Analog Communication Systems (Oxford University Press, 2009). His major research interests are wireless communications, networking, and signal processing. He is a Distinguished Lecturer of the IEEE Communications Society for the term 2008-09.