A Framework for Maximum Capacity in Multi-channel Multi-radio Wireless Networks

(Invited Paper)

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***Abstract*— Wireless networks with multi-channel multi-radio availability are attracting more and more attention from the research community because of its performance improvement and relatively low cost and complexity. Most work in the literature consider the performance of multi-channel multi-radio wireless networks with predefined numbers of channels and radios, and develop algorithms for channel allocation and transmission scheduling. The capacity limit on multi-channel multi-radio wireless networks was seldom addressed before. In this work, we study two problems: In a specific topology, 1) what is the maximum capacity we can get given the number of channels and radios? and 2) what is the impact of the number of radios on the system performance? To answer the above questions, we propose a general framework to find the maximum capacity given a multi-channel multi-radio wireless network. Its result also provides an indication of the "goodness" of a topology. We then use the framework to study the impact of radio constraints.**

I. Introduction

Despite significant advances in physical layer technologies, today's wireless LAN still cannot offer the same level of sustained bandwidth as their wired counterpart. The current 802.11 protocols use only a single channel at any given time, although multiple non-overlapping channels are available. In particular, the IEEE 802.11 b/g standard and IEEE 802.11a standard provide 3 and 12 orthogonal frequency channels, respectively, which can be used simultaneously within a neighborhood. If we can make use of all available channels, intuitively, the total throughput can be increased multiplicatively at the cost of additional radio equipments. Because the price of radio equipments has decreased dramatically due to technology advances and mass production, the idea of multi-channel wireless networking is quite promising.

To exploit the advantage of multi-channel availability, one or more radios can be equipped at each node. For example, we can use one radio for each available channel. Then the channel utilization problem for N channels can be decoupled into N independent subproblems, where each node use one radio to share one channel with all the other nodes as in the traditional scenario. However, due to practical considerations such as cost, space, power, and heat, we may also want to limit the number of radios equipped on each node, which is a current trend in the research on MAC layer design of multi-channel multi-radio wireless networks. For example, in [1], [4], [5], [6], the number of radios used is fixed to two, or even one. Then certain heuristic channel assignment algorithm is used on a per packet basis to coordinate the transmissions among nodes. Optimal throughput in multi-channel multi-radios networks has also been studied [8], [9], [10]. Both [8] and [9] use linear programming (LP) and integer linear programming (ILP) to find the maximum throughput as well as the corresponding routes of the network. They assume the source-destination pairs, the number of channels and radios are given. Kyasanur *et. al.* study the impact of the ratio between number of radios and channels on the system performance in the asymptotic case [10].

In this paper, we study the following two questions: 1) what is the maximum capacity that can be achieved for a given number of channels and radios; 2) how many radios are necessary to fully utilize a certain number of channels? We propose a framework to find the maximum capacity in a given topology with a certain number of channels, and optionally, the number of radios on each node. We first use the approach in [7] to generate the resource contention graph based on the topology graph. We extend the resource contention graph into a max-flow graph. Then we formulate it as an ILP problem by adding the constraints derived from the resource contention graph. The framework enables us to study the impact of radio constraints on the system performance.

Different from [8], [9], our work does not assume any traffic patterns. We consider links among nodes and study the maximum possible spectrum reuse among links given the channel and radio constraints. The optimal capacity we study depends on the topology itself, rather than a specific traffic pattern. The maximum capacity obtained in our framework will upper bound the maximum throughput of any given traffic pattern. In addition, because our work does not involve routing, the complexity to find a numerical solution is much less significant. Furthermore, we also consider the impact of number of channels and radios. In comparison with [10], which focuses on the asymptotic bound, our framework study the relationship between channels and radios on any given topology.

We have the following contributions: First, we propose a framework to find the maximum capacity in a given topology. The framework can be easily modified to achieve other objective such as fairness in channel allocation. In addition, without inputting the radio constraint, the framework shows a

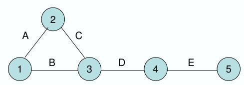

Fig. 1. The topology graph

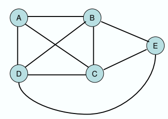

Fig. 2. The interference graph

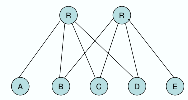

Fig. 3. The resource contention graph

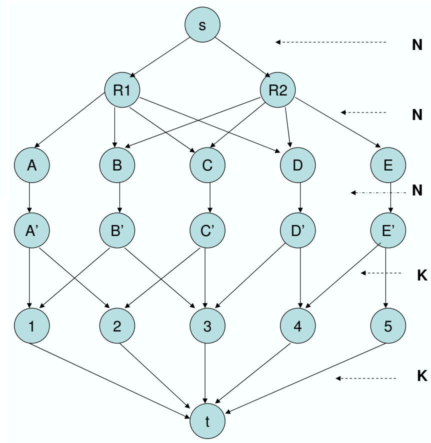

Fig. 4. The proposed framework

heterogenous radio allocation to maximize capacity. Second, the impact of the number of radios on the system throughput is studied using the proposed framework. We find that when the number of radios is small compared with the number of channels, radio is the limiting factor of the system performance. Within this range, the capacity increases multiplicatively with the number of radios. If the number of radios keeps increasing, at a certain turning point, channel becomes the constraint to limit the capacity. The capacity improvement is limited or even stopped beyond this turning point. This provides us a guideline on how many radios is appropriate to fully utilize the available channels in a specific topology.

The rest of the paper is organized as follows. Section II describes the proposed framework in detail. Section III evaluates the relationship between channels and radios in specific topologies using the proposed framework. Section IV discusses some related issues and followed by the conclusions.

II. The Framework for Resource Allocation

A. A MAC Layer Framework for Single Channel [7]

Our proposed framework is inspired by [7], where a unified MAC layer framework is proposed to achieve proportional fairness among MAC layer users. The proposed framework can be used on various physical technology such as omnidirectional antennas, switched beam antennas and MIMO. It formulates the channel access problem as a four-phase process. The inputs are the topology graph and the links to be scheduled (Figure 1). It first generates the flow interference graph (Figure 2) bases on the traffic flows. The vertex in the interference graph represents a flow in the topology graph. Two vertices are connected if the corresponding flows interfere with each other in the topology graph. Then it generates the resource contention graph (Figure 3) based on the interference graph. The resource contention graph captures the various contention regions in the network topology by identifying all the maximum cliques in the interference graph. It is essentially a bipartite graph with one set of vertices representing the resource on contention regions, the other set of vertices representing the links. There is an edge between a resource vertex and a link vertex if this link belongs to the contention region represented by the resource vertex. In the above example, there are two maximal cliques that can be identified from Figure 2, namely $ABCD$ and $BCDE$, which are represented by two resource vertices R1 and R2 in the resource contention graph. After the resource contention graph is formed, the interference constraint can be easily obtained: A link is allocated a time slot if and only if it is allocated resources in all the contention regions it belongs to.

B. The Proposed Framework

We extend the framework described in Section II-A to incorporate multi-radio and multi-channel constraints. We consider all possible links (i.e., there is a link between two nodes if they are within the transmission range of each other) among nodes and find the maximum possible aggregated throughput. This is the achievable throughput in a scenario where all links have infinite amount of traffic and can fully utilize all resource (channel and radio) allocated. In other words, our framework indicates the maximum possible spectrum usage (and spatial reuse) for a give topology under channel and radio constraints. It also provides an upper bound on throughput for any specific traffic pattern with optimal routing decisions.

In the following presentation, let us assume the network is homogeneous. That is, each node observes the same channel availability, and equips with the same number of radios. It can be easily generalized to a heterogeneous network, as will be discussed in Section IV. Let N be the number of channels available and K be the number of radios available.

We next explain the new framework using the example topology in Figure 1. Figure 4 is the resulting modified resource constraint graph. In the figure, the resource vertices ($R1$ and $R2$) connects to the link vertices ($A-E$) as in the original resource contention graph. But now the edges are directional, starting from the resource vertices to the link vertices. We also change the resource contention graph by increasing the number of resources from one to N since there are N available channels. It is done by increasing the capacity of the links from the resource vertices (i.e., R1 and R2). In addition, we have a dummy source s pointing to the two resource vertices. Each link vertex X (A-E) has one edge pointing to an image link vertex $X^{'}$ (A'-E'). All the edges mentioned above have

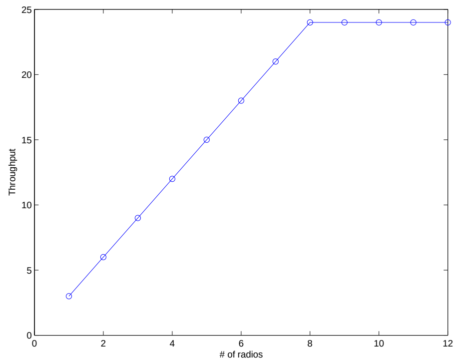

Fig. 5. Performance on a seven-node chain

capacity N. Below the image link vertices, we add vertices (1-5) corresponding to the nodes $(1-5)$ in the topology graph. Each image link vertex connects to two end node vertices. This is introduced for radio constraints. Each link, if allocated x channels, requires x radios from each of the end node of the link. For example, in the original topology graph, two end nodes of link A is node 1 and node 2. In our framework, image link vertex $A^{'}$ has two edges pointing to vertex 1 and 2 respectively. At last, each node vertex has one edge pointing to the dummy sink node t. All the new edges introduced above have capacity K, which is the number of equipped radios.

The edges with capacity of N reflect the channel constraint in our scenario. Similarly, the edges with capacity of K reflect the radio constraint. The intuition is as follows. If a transmission link is allocated a channel, each of its two end nodes must also allocate one radio to utilize the allocated channel. The construction of the proposed framework can well incorporate such dependency. The objective to maximize the channel utilization is equivalent to maximizing the flow from source to the sink. Although the framework looks similar to the max-flow problem, there is a major difference. In the max-flow problem, except for the source and sink vertices, each vertex must satisfy the flow conservation constraint, i.e., the summation of the incoming flows equals to the summation of the outgoing flows. In our framework, besides the s, t, there are two set of vertices on which the flow conservation constraint is not satisfied. The first set is the link vertices $(A-E)$. As mentioned above, a link is allocated a channel if and only if it is allocated resources in all the contention regions it belongs to. So if any link vertex is allocated a channel, it should get a unit of flow from each resource vertex which has an edge pointing to it. To translate it into our framework, the link vertices should have the constraint that any incoming flow and any outgoing flow should be equal. For instance, the flow constraint of vertex B is $F_{R1B} = F_{R2B} = F_{BB'}$. The second set is image link vertices $(A^{'}-E^{'})$. If a link is allocated a channel, its two end nodes should also allocate one radio to use the channel. So in the framework, the image link vertices should have similar flow constraint as the link vertices. For example, the flow constraint of vertex $D^{'}$ is $F_{DD'} = F_{D'3} = F_{D'4}$. Except for the two sets of vertices above, all the other vertices satisfy the flow conservation constraint.

Although it is similar to max-flow problem, we cannot solve it using existing max-flow algorithms because some vertices do not satisfy the flow conservation constraints. So we solve it by modifying the LP formulation of the max-flow problem. Specifically, we change the flow constraint on the two special sets of nodes as described above. Note that, the objective of the LP formulation for max-flow problem is to maximize the flow directly coming from the source node s, which is not what we want to maximize in our formulation. This is because link vertices do not satisfy flow conservation constraint, the flow out of source s should be larger than or equal to the actual number of channels used by the link vertices. So the objective of our framework is to maximize the total flow coming from the link vertices to image link vertices.

Our proposed framework is formally described as follows. The first step is to generate the resource contention graph based on the procedures described in Section II-A given a topology graph. The only difference is that we do not consider actual flows. So the interference graph is based on all the links in the topology graph.

We next extend the resource contention graph by adding a set of image link vertices, a set of node vertices, a source vertex and a sink vertex. Connect the link vertex with the corresponding image link vertex. Connect the image link vertices with node vertices according to the topology and flow graph. Then connect the source vertex to the resource vertices, and connect the node vertices to the sink vertex. The edge capacity for the first three levels are N, for the last two levels are K.

The ILP problem is formulated in the following. Let L be the set of physical links; i.e., $L = \{A, B, C, D, E\}$ in the example. Let L' be the set of image links; i.e., $L' = \{A', B', C', D', E'\}$ in our example. Let $B(x)$ be the set of neighbors of a vertex in Figure 4. Let f_{ij} be the flow between vertices i and j, where $f_{ij} \geq 0$. We note that $f_{ij} = 0$ if there is no directional edge from i to j in the graph.

$$maxmize \sum_{l \in L} f_{ll'} \tag{1}$$

subject to

$$f_{il} = f_{lj}, \quad \forall i, j \in B(l), \quad \forall l \in L \cup L',$$
$$\sum_{i \in B(n)} f_{in} = \sum_{j \in B(n)} f_{nj} \quad \forall n \notin L \cup L' \cup \{s, t\},$$

where f_{ij}s are integers. The first constraint is resource consumption (in terms of radio and channel) for the link and image link set. For the link set, the constraint indicates that if a link (A-E) is allocated x channels, it has to take x unit of resource from the resource pools (R1 and R2) it connected to. For the image link set, the constraint indicates that if a link (or its image link) is allocate x channels, it has to consume x radios at the end nodes of the link. The second constraint is the flow conservation constraint for all other nodes. The above

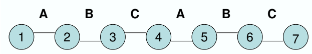

Fig. 6. Optimal transmissions when N = 12, K = 2

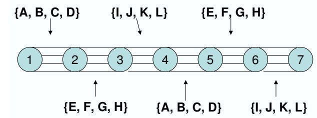

Fig. 7. Optimal transmissions when N = 12, K = 8

is an ILP formulation. We note that the number of variables in the problem is proportional to the number of flows. When the number of variables is not large, we can solve the ILP easily. Otherwise, we may need to relax the ILP to LP as in [8] to provide an upper bound.

III. Evaluations of the Framework

In this section, we use our framework to study the impact of the numbers of channels and radios on the maximum capacity. In the following, we fix the number of channels to be 12 (as in 802.11a, there are 12 orthogonal channels available). We vary the number of radios from 1 to 12. For each channel-radio setting, we calculate the maximum capacity using the proposed framework. We first study a simple chain topology to gain some insights. Then we study a random topology to further verify the observations we made in the chain topology.

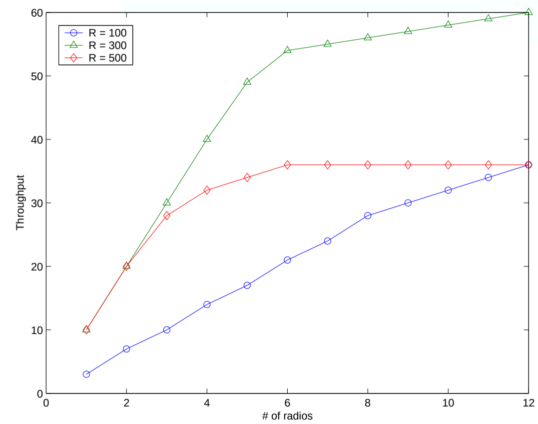

Fig. 8. Performance of random topology

A. A Chain Topology

The chain contains seven nodes and six links. The maximum capacity vs. the number of radios is plotted in Figure 5. We note that the capacity initially increases multiplicatively with the number of radios. When the number of radios is larger than 8, there is no further improvement. We explain the phenomenon as follows.

Let C_0 be the maximum capacity when $K = 1$ and $N = 12$. When K is small compared with N, the number of available radios limits the optimal capacity. One additional radio (at each node) can lead to the capacity improvement of C_0 because the number of channels is sufficiently large. We call this range the radio-constraint range. At a certain point, channels become the limiting factor of the capacity. Additional radios will lead to a smaller improvement or even no improvement because there is not enough residual channel to utilize. We call it the channel-constraint range. There is a turning point between these two ranges, which is topology-dependent (e.g., eight in the chain topology).

To elaborate, let us look at the channel allocation under different (N, K) pairs. Figure 6 shows the optimal transmission schedule given by the proposed framework when the number of radios is two. Link (1, 2) is active on channel A. Link (2, 3) has to use channel B to avoid interference with (1, 2). Link (3, 4) has to use channel C to avoid interference with link (1, 2) and (2, 3). The next link (4, 5) can reuse channel A, so on and so forth. The system capacity is 6. All the nodes except node 1, 7 use up its equipped radios. But only 3 out of 12 channels are used. Radio is the limiting factor in this scenario. Figure 7 shows the optimal transmission schedule when the number of radios is eight. Now all the nodes except 1 and 7 use up their radios, and all 12 channels are used. Note that such schedule is the best we can achieve with 12 channels. Beyond this point, channel availability limits the capacity and there is no further improvement.

B. A Random Topology

We uniformly and randomly place 20 nodes in a 1000 by 1000 square area. We assume that all nodes have the same transmission range. We assume two nodes are connected if and only if they are within the transmission range of each other. We vary the transmission range as 100, 300 and 500. The performance is shown in Figure 8. For a given (N, K) pair, the maximum capacity under $R = 300$ is the highest, followed by $R = 500$, and $R = 100$. When R is too small (e.g., $R = 100$), the connectivity of the graph is limited. We just do not have enough links to schedule the transmissions, which leads to a small value of maximum capacity. On the other hand, when R is too large, the interference range of a transmission is also large. Large interference range reduces spatial reuse and has a negative impact on the capacity. So there exists an optimal transmission range where the potential spatial reuse, and therefore the maximum capacity, is optimized.

In addition, it can be observed that a smaller transmission range leads to a larger turning point, and vice versa. An intuitive explanation is as follows. When the transmission range is larger, due to the larger interference in the network, more channels are needed for the same proportion of capacity improvement than the scenario with smaller transmission range. The channels will soon be used up and become the limiting factor, which accounts for the smaller turning point.

In general, our framework can be used to measure the potential spatial reuse of a given topology. Such a measurement can be used as an indication of the “goodness” of a topology. We can also use the framework to determine the desirable number of radios. For example, as in Figure 5, we may not want to equip more than eight radios on each node since there is no space for performance improvements.

IV. DISCUSSIONS

In this section, we further discuss some related issues to our framework. Some possible future work is also discussed here.

A. Channel Decoupling

As mentioned in Section II, the proposed framework provides an upper bound on throughput for any specific traffic pattern with optimal routing decisions. Our formulation and constraint tries to avoid interference when allocating channels on links. Although the output of the framework provides a channel allocation when it calculates the maximum capacity, such channel allocation may partition the network especially when the number of channel is the limiting factor. It may not be used for scenarios where multi-hop routing is needed for a specific traffic pattern. In such cases, the maximum capacity calculated in the framework provides an upper bound on the performance.

When $N = K = 1$, our framework is equivalent to find the maximum independent set of the topology [11]. When $N = K = \text{const}$ where $\text{const} > 1$, according to the channel decoupling property mentioned in Section I, the maximum capacity should be const times the capacity when $N = K = 1$. When $K < N$, it is not straightforward to get the maximum capacity using other schemes while our framework provides a simple way for it.

B. Heterogenous Network

In the above study, we assume a homogeneous network where the number of channels and the number of radios are the same for all users. Our framework can be easily modified to apply to a heterogenous network where each node equips different number of radios. Specifically, the edges with radio constraints (i.e., capacity equals to K) in the max-flow graph can have different capacities to reflect the non-uniform radio equipment. The non-uniform channel availability is not common in current wireless networks. But it could be possible in the next generation open-spectrum wireless networks [12]. We consider how to incorporate non-uniform channel availability in our framework as a future work.

In addition, the radio constraint (i.e., edges with the capacity K) can be relaxed if we want to distribute radios more effectively in a heterogenous network. For example, originally each node equips with one radio. Now we have a certain amount of additional radios to be distributed among nodes to improve the capacity. Instead of uniformly distributing them among nodes, is there any more efficient way to do it? We can do it by relaxing the radio constraint in our framework. Specifically, we change the capacity of the edges with radio constraint from K to ∞. Then after the maximum capacity is achieved, the flows in the edges from node vertices to the sink t reflect the relative congestion condition of different nodes if we want to fully utilize the available channels. This provides us a guideline on how to distribute additional radios among nodes, i.e., the nodes with higher flow value should be equipped with more radios. But the actual algorithm to do it may also need to consider a certain kind of fairness. We consider it as a future work.

V. CONCLUSIONS

In this work, we propose a framework to find the maximum capacity of a given topology with multiple channels and radios. The framework formulate it as an ILP problem. The first step is to generate the resource contention graph from the topology graph. The second step is to generate a max-flow-like graph using the resource contention graph. The traditional LP constraints of the max-flow problem is then modified using the interference constraints derived from the resource contention graph.

The solution of the ILP is the maximum possible spectrum usage (including spatial reuse) for a give topology under channel and radio constraints. For any specific traffic pattern, it provides an upper bound on throughput with optimal routing decisions.

We use the framework to analyze the impact of the number of radios on the system performance. We find that when the number of radios is small compared with the number of channels, radio is the limiting factor of the system performance. Within this range, the capacity increases multiplicatively with the number of radios. If the number of radios keeps increasing, at a certain turning point, channel becomes the constraint to limit the capacity. The capacity improvement is limited or even stopped beyond the turning point. This provides us a guideline on how many radios is appropriate to fully utilize the available channels in a specific topology.

VI. ACKNOWLEDGEMENT

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