On the Characteristics of Spectrum-Agile Communication Networks

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***Abstract*— Preliminary studies as well as general observations indicate the presence of a significant amount of "white space" in radio spectrum, varying on frequency, time, and geographic locations. Thus, it is likely that spectrum access, instead of true spectrum scarcity, is the limiting factor of potential growth of wireless services. Enabled by regulatory changes and radio technologies advances, opportunistic usage of the white space has the potential to significantly mitigate the spectrum scarcity. In this paper, we study the characteristics of opportunistic spectrum availability and its exploration. We present two new metrics to capture unique characteristics in networks with spectrum agility. We also study the spatial and temporal properties of opportunistic spectrum availability. Last, the existing resource sharing problem formulation is modified to incorporate the effect of opportunistic spectrum availability.**

I. INTRODUCTION

Spectrum is among the most heavily regulated and expensive nature resource around the world. However, although almost all spectrum suitable for wireless communications have been allocated, preliminary studies and general observations indicate that much of the radio spectrum is not in use for a significant amount of time, and at large numbers of locations. For instance, experiments conducted by Shared Spectrum Company indicates 62% percent of "white space" (unused space) below 3GHz band even in the most crowded area near downtown Washington, DC, where both governmental and commercial spectrum usage are intensive [10]. In the experiment, a band is counted as white space if it is wider than 1MHz and remains unoccupied for 10 minutes or longer. Furthermore, spectrum usages vary dramatically in time, geographic locations, and frequency. A lot of precious spectrum (below 5GHz), perfect for wireless communications that worth billions of dollars, sit there silently. The large portion of white space indicates that opportunistic or dynamic spectrum usage may significantly mitigate the spectrum scarcity.

In this paper, we focus on the *opportunistic spectrum utilization* by users other than the primary licensed ones on a non-interfering or leasing basis. Such usage is being enabled by regulatory policy initiatives and radio technology advances. First, both the Federal Communications Commission (FCC) and the federal government have made important initiatives towards more flexible and dynamic spectrum usage, e.g., [4], [3], [2], [5], [1]. Furthermore, opportunistic spectrum sharing is enabled by software-defined radio or cognitive radio technologies, where such technology advances provide the capability for a spectrum-agile radio device to sense and operate on a wide range of frequencies using appropriate communication mechanisms, and thus enable dynamic and more intense spectrum reuse in space, time, and frequency dimensions.

We focus on the study of the secondary users who observe channels available dynamically and explore them opportunistically. We refer secondary users as spectrum users who are not owners of the spectrum and operate based on agreements/etiquettes imposed by the primary users of the spectrum. Because spectrum availability of a secondary user is determined by primary users, it is called opportunistic spectrum availability. We study the characteristics of opportunistic spectrum availability and its potential benefits. (Note that the secondary users may have their own licensed/allocated bandwidth where they are primary users, which is not the concern of this paper.) The objective of the paper is to better understand systems with opportunistic spectrum availability. We have the following contributions.

- We introduce two new metrics, effective non-opportunistic bandwidth and space-bandwidth product, to capture the inherent properties of white space and the benefits of opportunistic spectrum utilization.
- Spectrum availability observed by a secondary user is time-varying and location-dependent. We analyze such spatial and temporal properties.
- We modify the existing resource sharing problem formulation to incorporate opportunistic spectrum availabilities.

The paper is organized as follows: we first discuss the system model in Section II. We then study two new metrics in Section III. The temporal and spatial correlation of channel availability for secondary users are analyzed in Section IV. Last, we propose modifications to the existing resource allocation problem formulation so that opportunistic spectrum availability is captured. (This part is omitted in the extended abstract due to space limitation.) We conclude the paper in Section V.

II. SYSTEM MODEL

We consider two types of users. Primary users are the rightful owners and have strict priority on spectrum access. Secondary users are spectrum-agile devices that can sense the environment and adapt to appropriate frequency, power, and transmission schemes. They can opportunistically access unused spectrum vacated by idle primaries. Primary users are conventional legacy users whose hardware and protocols should not be required to retrofit secondary user access needs. Therefore, channel availability of secondary users is inherently determined by the activities and properties of primary users. We focus on the characteristics of such opportunistic channel availability. We assume that when a primary user is using the band, secondary users are not allowed to co-exist in vicinity. We assume that information on channel availabilities is given to secondary users. Although it is a great challenge to obtain such information, it is not the focus of the paper and there are various schemes proposed.

We use footprint to abstract the space occupancy of a primary user in a channel. Secondary users outside the footprint of primary users are allowed to transmit on the channel given the maximum transmission power constraint. This is motivated by the idea that each communication consumes space, e.g., [7] and many others. It is also justified by the co-existence study of TV stations and WLAN devices. It has been shown that one or more WLAN devices can operate outside a certain range from the TV station without violating the D/U (desired/undesired signal strength) requirement of legitimate TV receivers that are within the service contour of the TV station [8], [9]. Similar analysis can be applied to other types of primary users. Therefore, the footprint of a primary user is modelled by a disk of radius r. The channel used by an active primary user cannot be utilized by secondary users in its footprint. On the hand, if it is outside the footprints of all primary users of a channel, the secondary user can operate on the channel. Intuitively, footprint can be considered as a model for the interference/communication range of a user. Transmission power and its effect are incorporated in the size of a footprint. For instance, in the case of TV/WLAN sharing, the (maximum) transmission power of a WLAN device affects both the footprint of itself and that of a TV station [8], [9].

Footprints of different users may or may not overlap depending on application scenarios. For instance, two TV stations using the same channel cannot have overlapping service contours. Otherwise, TV receivers in the overlapping area may not receive desired TV signal quality. On the other hand, in many application scenarios, especially mobility and ubiquitous coverage are required, overlapping is a necessity. For instance, the coverage areas of different base stations have to overlap to support user mobility and good coverage. Furthermore, multiple WLANs can have overlapping footprints and share a channel using CSMA/CA. On the other hand, they can also avoid overlapping by choosing different channels providing the number of available channels is large enough.

We use the following model for channel availability that is observed by secondary users at a snapshot. We first abstract the networks of secondary users into a graph, where vertexes represent secondary users. If two wireless users are within the communication/interference range of each other, then the two nodes are connected by an edge. If two vertexes are connected by an edge in the graph, we assume that these two nodes cannot use the same spectrum simultaneously. This abstraction is widely used in the literature, such as [11]. In addition, we associate with each vertex a set, which represents the available channels of the user. As mentioned earlier, if a secondary user is outside the footprint of all (active) primary users of a channel, then the channel is available at the secondary user.

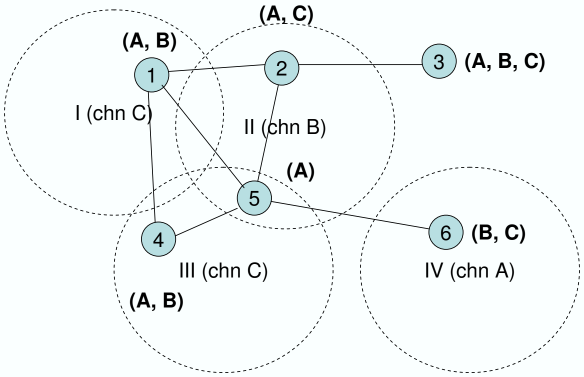

Fig. 1. A snapshot of channel availability at secondary users.

We show an example in Figure 1. The six vertexes 1-6 represent six different secondary users. There are three frequency bands, namely A, B, and C, which are communication channels that are opportunistically available to the secondary users. We assume that all channels have the same bandwidth, which can be generalized easily. In addition, four (active) primary users I-IV are present, using bands C, B, C, A, respectively. The footprint of a primary user is represented by the dashed circle centered at it. For instance, Node 5 is within the footprints of primary users II and III, who use channels B and C, respectively. Therefore, channels B and C are not available at Node 5 while channel A is. Because of location differences, each node may have access to a different set of channels. In our figure, the available channels are (A,B) at vertex 1, (A,C) at vertex 2, etc. The resource allocation problem is how they should share these channels. Note that Figure 1 shows a snapshot of the network. At different time instance, due to the traffic variation of primary users, channel availabilities at secondary users vary.

The paper focuses on characteristics of spectrum agile networks. In other words, we study the inherent properties instead of the design/control of the system. To elaborate, primary users are legacy users and we cannot control their locations, traffic patterns, communications, and other parameters. Furthermore, secondary users, e.g., WLAN devices, have their own properties such as maximum transmission power, location, and density. We consider these properties given as well.

III. METRICS FOR OPPORTUNISTIC SPECTRUM UTILIZATION

The basic concept of opportunistic spectrum utilization is to utilize spectrum unutilized at time, frequency, and space. Because such spectrum availability is dynamic and different from traditional spectrum availability defined by command-and-control manner, we first introduce two new metrics for its characterization.

A. *Effective Non-opportunistic Bandwidth*

Consider the following question: suppose that there are 62% of white space under 3GHz. If we can fully utilize such white space, is it equivalent to gaining an additional spectrum band of $0.62 \times 3 = 1.86$GHz? The answer is that it depends. To address this question, we introduce the notion of effective non-opportunistic bandwidth (ENOB). It is defined as the equivalent non-opportunistic bandwidth required to achieve the same performance as in the case of opportunistic spectrum availability. A non-opportunistic channel is referred to a channel that is always available to all (secondary) users in consideration, which is how spectrum is allocated in the traditional command-and-control manner. This metric is designed to study the impact of opportunistic spectrum availability.

We elaborate the idea with a naive example. Consider a simple network with only two nodes. They cannot use the same channel simultaneously due to interference. Consider a channel with bandwidth W that is opportunistically available at these two nodes. The channel is available at each node with probability p independently. Suppose that a user obtains one unit of throughput per unit of spectrum. Then the total throughput gained by the two users is:

$$W(p^2 \times 1 + 2p(1-p) \times 1 + (1-p)^2 \times 0) = Wp(2-p).$$

The first term represents the case where the channel is available to both nodes and only one of them can use it due to interference. The second term is for the case where only one of the users observes the spectrum availability and uses it. The last term is the case where the spectrum is available to neither of the two users. To achieve the same total throughput, $B_e = Wp(2-p)$ unit of non-opportunistic bandwidth is required. To elaborate, when a spectrum of B_e is available to both users in the traditional way (i.e., the spectrum is always available to both users), the throughput is $Wp(2-p)$ because only one of the users can use it at any given time. Thus, we claim that $B_e = Wp(2-p)$ as ENOB in this simple example. Suppose $W = 3G$Hz and $p = 62\%$. In this example, we see the impact of 62% white space under 3G is equivalent to $Wp(2-p) = 2.76$GHz of spectrum in the traditional way. The correlation on spectrum availabilities at secondary users also plays an important role. For instance, if two secondary users are very close and observe the same spectrum opportunity, then $B_e = Wp$ instead of $Wp(2-p)$ as in the independent case in this example.

Note that spectrum is not being "created" by secondary users. Instead, they simply explore the spectrum holes generated by primary users. Inherent characteristics of primary users, such as communication range, transmission power, traffic pattern, node density, and topology, determine the spectrum opportunities of secondary users. The purpose of ENOB is to quantify the potential of such spectrum opportunities for a given set of secondary users.

The intuition is similar to that of the effective bandwidth used to capture the statistic multiplexing gain. However, what is being capture here is the degree of spatial reuse and statistical multiplexing between primary and secondary users. In general, because of the characteristics of primaries mentioned above, users observe different channel availabilities and yield utilization gain, which is quantified by ENOB.

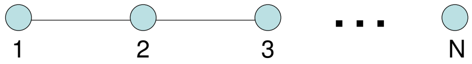

Fig. 2. A Chain Topology

1) ENOB of a Chain Topology: We use a chain topology to further illustrate the idea of ENOB. We consider a chain topology of length N. We assume that only the two nearest nodes interfere with each other and thus cannot use the same spectrum simultaneously, as shown in Figure 2. We calculate its ENOB as follows. Consider a spectrum band of width W. Let p_0 be the probability that a node observes the channel available. Let p_c be the probability that node A observes the channel available given that its neighbor B observes the channel available. Note that p_c is indeed a function of distance between A and B, as discussed in Section IV-A. Here we assume all nodes on the chain are evenly spaced and thus omit the distance in the notation. Let q_c be the probability that A observes the channel available given that B does not, where

$$q_c = p_0 \frac{1-p_c}{1-p_0}.$$

As shown in the Appendix (in the full paper), the ENOB of the chain topology is

$$B_e(N) = W \frac{U(N)}{\lfloor (N+1)/2 \rfloor}$$

where

$$\begin{aligned} U(N) \quad = \quad & \frac{2(1-p_0)q_c}{1-p_c^2}\left(\lfloor \frac{N}{2} \rfloor - \frac{1-p_c^{2\lfloor \frac{N}{2} \rfloor}}{1-p_c^2} p_c^2\right) \\ & + p_0 \frac{1-p_c^{2\lfloor \frac{N+1}{2} \rfloor}}{1-p_c^2} - (1-p_0) q_c \frac{1-p_c^N}{1-p_c^2} \mathbf{1}_{\{N \text{ is even}\}} \end{aligned}$$

Note that if $p_c = 1$, then

$$B_e(N) = Wp_0.$$

Figure 3 illustrates ENOB for different values of p_c, where p_0 is 0.1 and 0.7, respectively. In the figures, the x-axis is the value of p_c, where $p_c \in [p_0, 1]$. The y-axis is the ENOB normalized over Wp_0, where is Wp_0 is the ENOB when $p_c = 1$. It is clear that the larger the value of p_c, i.e., the higher the correlation between neighboring nodes, the lower the value of ENOB. In addition, the normalized ENOB is higher for smaller values of p_0. The trend is similar for different values of N.

The value of ENOB depends on many factors, such as the probability of the spectrum availability, the correlation among different users, and the topology of the network. In general, the higher the correlation of spectrum availability among users, the lower the value of ENOB. The intuition is that a system with high correlation behaves similarly to a system with the traditional spectrum availability, and thus has low statistic multiplexing gain. On the other hand, the more complex the

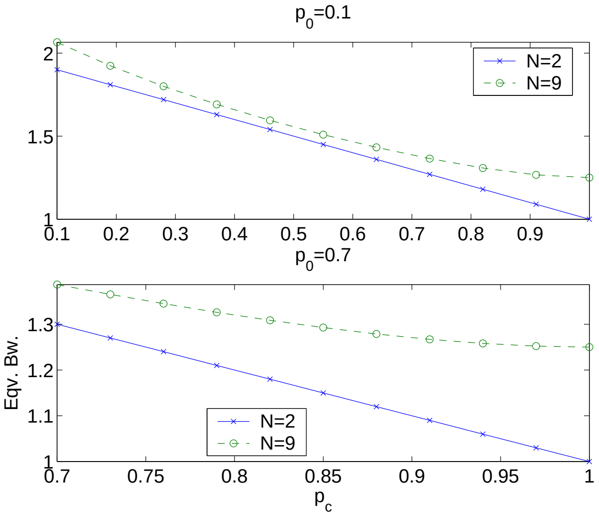

Fig. 3. Normalized equivalent bandwidth of a chain topology.

topology, the higher the value of ENOB. For instance, consider a clique of size N. Assume that each node observes channel available with probability p independently. Then the effective bandwidth of the channel is $B_e = W\left(1-(1-p)^N\right)$. For very small p, $B_e \approx NWp$, which shows a gain of roughly N-fold. The intuition is that the multiplexing gain is higher for a dense network, especially when p is small.

In summary, the metric, ENOB, is introduced to capture the impact of the opportunistic spectrum availability. In general, ENBO is difficult to calculate because the network topologies are more complicated and the availability of channels at different nodes are correlated in a complex manner. Thus, we may need to resort to numerical evaluations.

B. Space-bandwidth Utilization

Consider a wireless device. It is clear that a wireless device consumes a certain amount of spectrum for its communication, which is an irreproducible natural resource. Equivalently important is that a certain amount of space is also occupied by the communication [7]. Wireless devices, especially primary users and secondary users, are different in communication power and therefore occupy different amount of space for communications. For instance, the service range of a TV station is about 100km. In comparison, the transmission range of an unlicensed wireless LAN (WLAN) device is about 100m. Therefore, 6MHz of bandwidth (the bandwidth of an analog TV channel) occupied by a TV station has a different communication result than the same bandwidth of a WLAN device. Thus, to quantify such differences in utilization, we use a simple metric, space-bandwidth product, which is defined as the product of the space and bandwidth utilization of a user in communication. This definition is analogous to the transport capacity defined in [7] to study capacity scaling laws in large wireless networks. Another analogy is the man-mile metric used by airline industry. We note that open spectrum enables spectrum reuse by secondary users in both space and frequency dimensions. Thus, this metric is important to understand the effect of opportunistic spectrum utilization.

In this section, we use footprint to abstract the space utilization of both primary and secondary users. We model footprint of a wireless device as a disk of radius r. The radii of primary and secondary users can be dramatically different. For instance, it has been proposed that unlicensed users can transmit in the unused TV band [6]. Another potential is for unlicensed wireless device to share radar bands. In both applications, a primary user has a dramatically larger transmission power and thus larger footprint than that of a secondary user.

The space-bandwidth product has a clear physical meaning in broadcast applications. For instance, a TV station with coverage area of 100 square miles has the same space-bandwidth product as 10 smaller TV station, each with 10 square miles of coverage. Assume receivers are uniformly distributed in the area. Then they serve the same number of users. In other words, the space-bandwidth product is proportional to the number of users served. In the case of peer-to-peer communication, space-bandwidth product is an indication of coverage. It is also an indirect implication of the tradeoff between the cost of an AP (and its installation) and the data rate for each user.

A spectrum-agile system exploits holes in both spatial and frequency dimensions. However, there could be spectrum intervals that cannot be (easily) utilized. For instance, if a secondary user requires a spectrum band of 5MHz, then an isolated free TV band of 6MHz may not be fully utilized. We note that OFMD technology may be able to utilize scattered spectra. However, stringent requirements on band filters may limit its practical implementation. Space-bandwidth product quantify such holes in utilization in both spectrum and space dimensions. We next illustrate its usage in a special case that focuses on the spatial dimension.

1) An Illustration Example: Consider one spectrum band that both primary and secondary users can fully utilize. Assume primary and secondary users have dramatically different footprint sizes. Such difference gives us another type of spatial gain. An analogy is to put big stones in a jar. After the jar is filled with big stones, one can still fill a large amount of sand into the jar by taking advantage of the space among big stones. In this analogy, it is unfair to compare a big stone with a grind of sand unless the space/volume is taken into account. Similarly, we use space-bandwidth product to illustrate the difference in footprint size.

We first consider the case where footprints are not allowed to overlap. We assume that primary users are stationary. They decide their locations in the following manner. Consider a large area with no users. The first primary user randomly selects its location. A following primary user also chooses its location randomly, but the choice is limited to the area where its footprint does not overlap with those of the previous users. In the end, if no additional primary users can be assigned, we consider the system full and can calculate the maximum space utilization of the primary users. The process mimics the location determination of primaries, say TV stations. The location of a TV station is in general determined by population density and geographic areas. For instance, it is more likely to locate a TV station in a big city than in a deserted area just to fully utilize the space. Subsequent TV stations have to respect the footprints of prior allocations. After the primary users pick their locations, secondary users subsequently locate themselves without overlapping footprints with previous users, both primary and secondary. An example of the scenario is that an AP avoids channels used by primary users and other WLANs operating in vicinity when it selects a channel.

We consider an area large enough so that boundary effects are ignored to simplify analysis. We consider the full capacity case. In other words, primary users randomly pack in as much as possible followed by secondary users. It is interesting to see that the footprints of primary users can only occupy about μ portion of the space. Numerical evaluation indicates that $\mu \approx 0.45$. If the radii of secondary users are much smaller than that of a primary user, secondary users can occupy μ portion of the **unoccupied** space of primaries. In other words, secondary users can occupy $\mu * (1-\mu) \approx 0.25$ portion of the total space. This is a total space-bandwidth utilization of 70%. Consider a heterogenous network with TV stations, WLAN devices, pico sensors, then the overall utilization is 0.83 instead of 0.45 with only primary users.

We have the following observations. First, this type of packing results in no cost at primary users. In other words, with or without secondary users, primary users achieve the same spatial utilization. Furthermore, the secondary users packed in space holes are able to use the same channel regardless whether primary users are active or not. Secondly, any one type of users can only achieve a utilization of μ, even if it is pico sensors that occupy the entire space. In other words, the gain comes from the **heterogeneity** in space consumption of different types of devices. Note that the sequence is also important — larger users need to be allocated first to achieve better utilization. In a spectrum-agile communication networks, it is likely that secondary users are smaller than primaries. Therefore, spectrum-agile networks can achieve better utilization of space, which can be captured by the space-bandwidth product.

If the dynamics in spectrum utilization of primary users is also taken into account, then the gain is even higher. Assume each primary user is active with probability p_0 on average. Consider the case where there are enough secondary users that demand spectrum. Then the space-bandwidth utilization of a primary-user-only system is only $p_0\mu$ on average while one tier of secondary users can improve the utilization to $p_0\mu + \mu(1-p_0\mu)$.

In the above, we consider the case where the overlap of footprints is not allowed. On the other hand, if such overlap is allowed, then the utilization depends on the density of primary and secondary users. Let λ_p and λ_s, and R_p and R_s be the density and radius of primary and secondary users, respectively. The utilization of primary users is $1 - \exp(-\lambda_p \pi R_p^2)$. One tier of secondary users improves the utilization by $\exp(-\lambda_p \pi R_p^2) \times (1 - \exp(-\lambda_s \pi R_s^2))$.

In summary, the main purpose of space-bandwidth product is to quantify the gain of opportunistic spectrum utilization, in both the space and frequency dimensions. The space-bandwidth product is an indicator of resource consumed and thus can be directly linked economic values of spectra. We note that there are limitations in the definition of space-bandwidth product. For instance, the definition does not directly apply to spreading spectrum communication systems. Another limitation is that the product is not a direct indication of throughput or transport capacity in peer-to-peer communication systems.

IV. SPATIAL AND TEMPORAL PROPERTIES

The channel availability observed by a secondary user is time-varying and location-dependent. In this section, we quantify such spatial and temporal properties.

A. Spatial Correlation

It is intuitively clear that when two secondary users are close to each other, they are more likely to observe the same channel availability. When they are far away, the observations are more likely to be independent. We quantify this spatial correlation next.

Consider a channel. Primary users of the channel locate in the area following a Poisson distribution with density λ. This is a typical assumption used to study random wireless networks, e.g., in [7]. The footprint of each primary user is a disk with radius R_p. In this model, overlapping of footprints is allowed. The location of a secondary user is independent of that of primaries. Then a secondary user, A, observes the channel available when there is no active primary users within radius R_p. Denote $P(A)$ as the probability that A observes the spectrum availability. We have

$$P(A) = e^{-\lambda \pi R_p^2}.$$

Consider another secondary user, B, at distance d. We are interested in the probability that B also observes the availability given that A observes spectrum availability, denoted as $P(B|A)$. We have

$$P(B|A) = \begin{cases} \exp(-\lambda(\pi R_p^2 - a_0)), & 0 \le d \le 2R_p \\ P(B), & d \ge 2R_p, \end{cases}$$

$$a_0 = 2R_p^2 \cos^{-1}\left(\frac{d}{2R_p}\right) - d\sqrt{R_p^2 - (d/2)^2}.$$

When $d \ge 2R_p$, we have $P(B|A) = P(B)$; i.e., two nodes are so far away that they observe independent channel availabilities.

We use a relative radio between $P(B|A)$ and $P(B)$ as an indication of the spatial correlation. If B observes availability independent of A, then the ratio is 1. The larger the value, the higher the spatial correlation. Figure 4 illustrate this correlation. In the figure, the x-axis is the normalized distance, i.e., d/R_p. The y-axis is the radio between $P(B|A)$ and $P(B)$ to show the impact of A's availability at B. In the figure, $R_p = 1$, and the value of λ is indicated on the corresponding curve. It is clear from the figure that the closer the two secondary nodes, the more likely that they observe the same channel availability; i.e., the higher the spatial correlation. For larger values of λ, the higher the normalized correlation. In fact, when the value of λ is large, both $P(B|A)$ and $P(B)$

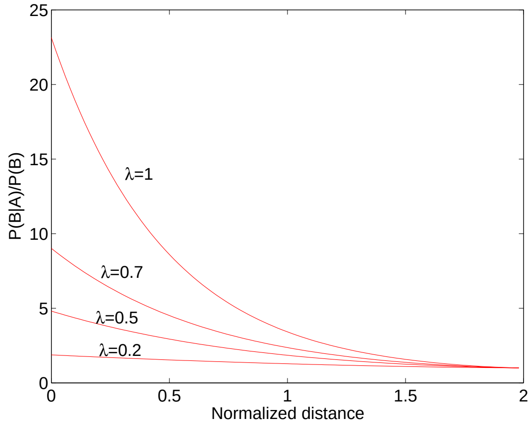

Fig. 4. Spatial Correlation between secondary users

are small; i.e., the lower the probability of a secondary user observing an available channel. However, the impact of A's availability is relatively large; i.e., the curve is steeper for a larger value of λ.

In summary, distance has a significant impact on the spatial correlation on channel availability between secondary users. We are extending the study to the case where the footprint of primary users do not overlap and the case where the footprint of a primary user is irregular. Numerical results indicate that similar trends exist.

B. Temporal Correlation

The temporal properties of secondary users include the distributions of channel availability and unavailability, and their prediction. Clearly, such temporal characteristics depend on the properties of primary users. Consider a particular channel. We note that a secondary user can access a channel if it is not within the footprints of any active primary users of the channel. Based on the discussion in Section III-B, we know that there are secondary users that are outside the footprints of any primary users simply by exploring the space holes. For such users, this spectrum band is always available. However, there are also secondary users that are within the footprints of one or more primary users. They can access the channel only when the corresponding primaries are not active. We study these users next.

Temporal properties of secondaries depend on the activity pattern of primaries. For instance, if the activities of primary users are periodic, such as TV stations, then the channel availability of a secondary user within footprints is also periodic and is perfectly predictable. Another special case we illustrate next is when the arrival processes of primary users are independent Poisson processes. Note that we do not assume the service time distribution or busy period of a primary user. It has been shown that the service time distribution is well modelled as exponential for voice traffic, but not for data traffic. However, the idle period of primary users is the channel available period of secondary users. Because the arrival process of a primary user can be modelled as Poisson, the idle period of primary users is exponential. Thus, the available period of a secondary user is also exponential. Let $1/\mu^I$ be the mean of the idle period of a primary user. If footprints of primary users do not overlap, then the available period of a secondary user within a footprint is exponentially distributed with mean $1/\mu^I$. If footprints of primary users can overlap and primary users locate with density λ_p, then the available period of a secondary user is exponentially distributed with mean $1/\mu^a$ where

$$\mu^a = \begin{cases} 0 & \text{w. p.} \quad 1 - e^{-\lambda_p \pi R_p^2} \\ k\mu_I & \text{w. p.} \quad e^{-\lambda_p \pi R_p^2} \frac{(\lambda_p \pi R_p^2)^k}{k!} \end{cases} .$$

In the above equation, the first one represents the case where the user is outside all footprints of primaries and thus always observes the channel available. The second one is the case the user is within the footprint of k primary users. A challenging issue is to understand the distribution of un-available period for secondary users. Because the busy period of a primary user is in general not exponential, the distribution analysis is the same as the busy period analysis of an M/G/n queue.

V. CONCLUSION

In this paper, we characterize opportunistic spectrum utilization in spectrum-agile communication networks. The characteristics and potentials of spectrum holes in both frequency and space are studied. Spatial and temporal properties are quantified. Such characteristics are considered as inherent. The paper contributes to better understandings of the properties of spectrum agility.

VI. APPENDIX

A. Calculation of Spatial Correlation

We calculate $P(B|A)$ given the distance between B and A is d using Figure 5. The event that A observes the channel available implies that there is no active primary users in the left circle. Therefore, the overlapping area has no active primaries and B also observes the channel available if and only if the rest of the circle around B has no primary users. Let the size of the overlapping area be a_0. We have

$$P(B|A) = \exp\left(-\lambda_p(\pi R_p^2 - a_0)\right) .$$

We calculate a_0 next. We have

$$\alpha = \cos^{-1}\left(\frac{R}{d/2}\right) .$$

The size of the triangle, b_0, is $R\sin(\alpha) \times d/2$. Therefore,

$$\begin{aligned} a_0 &= \pi R^2 \times \frac{2\alpha}{2\pi} - 2b_0 \\ &= 2R_p^2 \cos^{-1}\left(\frac{d}{2R_p}\right) - d\sqrt{R_p^2 - (d/2)^2}. \end{aligned}$$

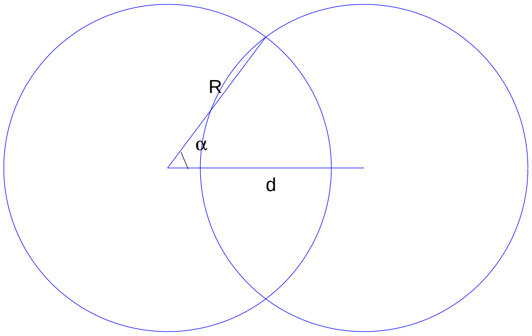

Fig. 5. Two nodes with distance d.

B. Calculation of ENOB for the chain Topology

Let X_i be the event that node i observes the channel available, Y_i be the event that node i uses the channel, and $\bar{x}$ be that event x does not happen. Let $p_c = P(X_a|X_b)$, where $p_c \in [p_0, 1]$. Because $p_0 = P(X_a) = P(X_a|X_b)P(X_b) + P(X_a|\bar{X}_b)P(\bar{X}_b)$, we have

$$q_c = P(X_a|\bar{X}_b) = p_0\frac{1-p_c}{1-p_0}.$$

Let p_i^u be the probability that node i uses the channel in consideration. We find p_i^u iteratively. We assign channel from left to right. A node is assigned the channel if the channel is available and the neighbor on the left is not assigned the channel. This scheme can be shown to yield the maximum utilization. We have

$$\begin{aligned} p_1^u &= p_0, && (1)\\ p_2^u &= P(Y_2) = P(\bar{Y}_1)P(X_2|\bar{Y}_1) && (2)\\ &= (1-p_1^u)P(X_2|\bar{X}_1) && (3)\\ &= (1-p_0)q_c. && (4) \end{aligned}$$

Furthermore,

$$\begin{aligned} p_i^u &= P(\bar{Y}_{i-1})P(X_i|\bar{Y}_{i-1})\\ &= P(\bar{Y}_{i-1}\bar{X}_{i-1})P(X_i|\bar{Y}_{i-1}\bar{X}_{i-1})\\ &\quad +P(\bar{Y}_{i-1}X_{i-1})P(X_i|\bar{Y}_{i-1}X_{i-1})\\ &= P(\bar{X}_{i-1})P(X_i|\bar{X}_{i-1})\\ &\quad +P(Y_{i-2})P(X_{i-1}|X_{i-2})P(X_i|X_{i-1})\\ &= (1-p_0)q_c + p_{i-2}^u p_c^2\\ &= (1-p_0)q_c + (p_{i-4}^u p_c^2 + (1-p_0)q_c)p_c^2\\ &= (1-p_0)q_c + (1-p_0)q_c p_c^2 + p_{i-4}^u p_c^4. \end{aligned}$$

For even i, we have

$$\begin{aligned} p_i^u &= (1-p_0)q_c(1+p_c^2+p_c^4+\cdots+p_c^{i-2})\\ &= \sum_{k=0}^{i-2}(1-p_0)q_c p_c^{2k}\\ &= (1-p_0)q_c\frac{1-p_c^i}{1-p_c^2}. \end{aligned}$$

For odd i, we have

$$\begin{aligned} p_i^u &= (1-p_0)q_c(1+p_c^2+p_c^4+\cdots+p_c^{i-3}) + p_1^u p_c^{i-1}\\ &= \sum_{k=0}^{i-3}(1-p_0)q_c p_c^{2k} + p_0 p_c^{i-1}\\ &= (1-p_0)q_c\frac{1-p_c^{i-1}}{1-p_c^2} + p_0 p_c^{i-1}. \end{aligned}$$

Thus, we have

$$p_i^u = \begin{cases} (1-p_0)q_c\frac{1-p_c^i}{1-p_c^2} & i \text{ is even}\\ (1-p_0)q_c\frac{1-p_c^{i-1}}{1-p_c^2} + p_0 p_c^{i-1} & i \text{ is odd} \end{cases}$$

Let $U(N)$ be the utilization of a unit spectrum. We have

$$\begin{aligned} U(N) &= \sum_{i=1}^{N} p_i^u\\ &= \sum_{i=2,\text{even}}^{N} 2(1-p_0)q_c\frac{1-p_c^i}{1-p_c^2} + \sum_{i=1,\text{odd}}^{N} p_0 p_c^{i-1}\\ &\quad -(1-p_0)q_c\frac{1-p_c^N}{1-p_c^2}\mathbf{1}_{\{N \text{ is even}\}}\\ &= \sum_{j=1}^{\lfloor N/2\rfloor} 2(1-p_0)q_c\frac{1-p_c^{2j}}{1-p_c^2} + \sum_{j=0}^{\lfloor (N-1)/2\rfloor} p_0 p_c^{2j}\\ &\quad -(1-p_0)q_c\frac{1-p_c^N}{1-p_c^2}\mathbf{1}_{\{N \text{ is even}\}}\\ &= \frac{2(1-p_0)q_c}{1-p_c^2}\left(\lfloor\frac{N}{2}\rfloor - \frac{1-p_c^{2\lfloor\frac{N}{2}\rfloor}}{1-p_c^2}p_c^2\right)\\ &\quad +p_0\frac{1-p_c^{2\lfloor\frac{N+1}{2}\rfloor}}{1-p_c^2} - (1-p_0)q_c\frac{1-p_c^N}{1-p_c^2}\mathbf{1}_{\{N \text{ is even}\}} \end{aligned}$$

On the other hand, if a channel is always available to the secondary users, then the utilization per unit bandwidth is $\lfloor(N+1)/2\rfloor$. Thus, the equivalent bandwidth if

$$B_e(N) = W\frac{U(N)}{\lfloor(N+1)/2\rfloor}.$$

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