# Optimal Bandwidth Selection in Multi-Channel Cognitive Radio Networks: How Much is Too Much?

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***Abstract*— Cognitive radio improves spectrum efficiency by allowing secondary users (SU) to dynamically exploit the idle spectrum owned by primary users (PU). This paper studies optimal bandwidth allocation for SUs. Consider the following tradeoff: a SU increases its instantaneous throughput by accessing larger bands, but channel switching overhead (due to the dynamics of PU activities) and contention among multiple SUs create higher liability for larger bandwidths. So how much is too much? In this paper, we find the optimal bandwidth allocation in both the single SU and multiple SU cases, accounting for the effects of channel switching overhead. Our result is validated through both numerical simulation and real channel activity traces. We also study the impact of various factors on SU performance, namely, PU channel idle time and probability, PU channel correlation, and SU sensing and access schemes. The work sheds light on the design of spectrum sharing protocols in cognitive radio networks.**

## I. Introduction

Recently, research in cognitive radio networks has attracted a lot of attention, with many large organizations, companies, and universities at the forefront. DARPA's NeXt Generation project [1], the new IEEE standard, 802.22, Wireless Regional Area Networks (WRAN) [2], and the White Spaces Coalition (WSC), which includes both Microsoft and Google, are some of the high profile projects and organizations working in this area.

Cognitive radio (CR) can capture or "sense" temporal and spatial variations in the radio environment, allowing it to find unoccupied portions of spectrum in real-time. To exploit these holes in spectrum activity, CR is also capable of dynamically adjusting operating frequency, bandwidth, and other physical layer parameters [3][4]. The major challenge is to efficiently utilize the spectrum opportunities due to PU activity while protecting the performance of PUs.

Consider a set of licensed channels that are made available to SUs. This is often referred to as spectrum pooling [5]. At a given time, SUs can use a subset of these pooled channels if they are not occupied by the PUs. A SU may need to decide the number of PU channels to access in the available spectrum pool. Although intuitively it seems that a SU would want to exploit more available spectrum for higher throughput, several factors may affect the optimal bandwidth selection of a SU. First, using a larger number of PU channels implies more frequent switching because in a wider band, a PU is likely to reclaim a part of the band more quickly, and the SU has to switch to a different set of channels. Channel switching infers overhead, which may prevent a SU from selecting a large number of PU channels even if they are available. Second, if there are multiple SUs, SUs need to share the available PU bands among them. Motivated by such tradeoffs, we study the following issue in this paper: how much bandwidth is appropriate for a SU to optimize the overall throughput? We consider both the cases of one SU and multiple SUs, and take into account switching overhead due to the dynamics of PU activity. Our results reveal the relationship between PU behavior and SU performance. Our main contributions are as follows.

- In the single SU scenario, we find the optimal bandwidth to maximize the SU throughput while considering switching overhead under both saturated and limited traffic scenarios. We consider both the cases where the SU is required to use a set of consecutive channels or not. We then study PU channel correlation and show its impact on SU performance.
- In the multi-SU scenario, we derive closed-form approximations for the optimal SU throughput in both the discrete and consecutive channel cases. In the context of the consecutive channel case, we define the channel reconfiguration problem and present an efficient scheme for channel reconfiguration.
- In addition to extensive simulations, we also test our scheme using a set of real channel trace data, which exhibits both heterogeneous PU behavior and some dependence among PU activities in adjacent channels. Our scheme is shown to be robust.

The rest of the paper is organized as follows. We describe the PU and SU models as well as our channel measurement configurations in Section II. In Section III, we study the optimal bandwidth selection problem in saturated traffic case, limited traffic case, and correlated channels scenario in the single SU case. We then propose the optimal channel configuration in multi-SU scenario in Section IV. After reviewing related work in Section V, we conclude the paper in Section VI.

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TABLE I: Notations of Main Variables

| | |
|---|---|
| $B_p$ | Bandwidth of a channel for PUs (MHz) |
| $M$ | Total number of channels |
| $P$ | Probability of a channel being idle |
| $N$ | Number of channels a SU uses, bounded by $N_{max}$ |
| $N^*$ | Optimal number channels of a SU |
| $\phi(N)$ | Transmit rate of a SU, a function of $N$ (Mbps) |
| $T^h(N)$ | Channel holding time of a SU (s) |
| $C$ | General term on overhead of a SU (s) |
| $R_s(N)$ | Average throughput of a SU, a function of $N$ (Mbps) |
| $D$ | Traffic amount of a SU (Mb) |
| $K$ | Number of SUs which interferes with each other |
| $K_{avg}(N)$ | Average number of SUs which can access spectrum simultaneously |
| $Q$ | Channel reconfiguration scheme |
| DISC | Use discrete channels |
| CONC | Use consecutive channels |

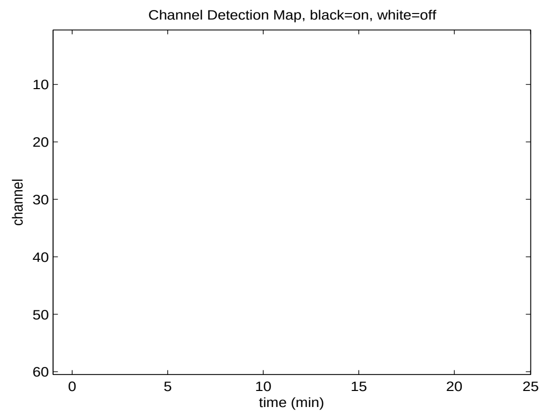



Fig. 1: Channel activity map from trace data of 60 real channels.

## II. Models and Assumptions

### A. Primary User Model

We consider PUs as legacy devices that access a block of spectrum through static channelization. We assume that there is a set of $M$ consecutive PU channels, each with the same bandwidth denoted by $B_P$. This is a common system description in many licensed spectrum bands. For example, in the U.S., a TV channel has a bandwidth of 6 MHz and there are more than 100 TV channels.

At any moment, each channel can be in either a busy or idle state, which refers respectively to times when a PU occupies the channel or does not. We also refer to these as the ON/OFF states. We assume the channel states are statistically independent unless stated otherwise. We let $P$ denote the probability that a channel is idle, and assume that each channel has the same idle probability. We can write $P = E(T^I)/(E(T^I) + E(T^B))$, where $T^I$ and $T^B$ denote the idle and busy time respectively. In this paper, there is no assumption on the arrival patterns of PUs unless specifically stated. Main notations for this paper are listed in Table I.

### B. Secondary Users' Model

A SU is an opportunistic user equipped with a cognitive radio which can dynamically change its operating frequency and bandwidth. For simplicity, we assume that a cognitive radio transmits on a positive integer number of licensed PU channels. This assumption is reasonable. For example, in the Microsoft KNOWS prototype, the minimum bandwidth of a cognitive radio is 5MHz and the prototype operates on bandwidths that are multiples of 5MHz [6].

In this paper, let $N$ denote the number of channels a SU operates on, where $N = 1, \ldots, N_{max}$, with $N_{max}$ being the upper limit of channels that a SU can operate on. The objective is to find the optimal value of $N$ such that per SU throughput is optimized. In this paper, we use bandwidth and number of channels interchangeably. We note that the $N$ channels that a SU operates on can be consecutive or discrete, based on its physical layer capability. OFDM is a viable technology to satisfy either of these requirements. The difference between the discrete and consecutive channel requirement is very important, as will be shown in both Section III and Section IV. Going forward, we use CONC to denote a SU requires consecutive channels and DISC to denote when a SU uses discrete channels.

To protect PU communications, a SU can only operate on the channels when they are idle, and must evacuate channels immediately when PUs return. In this paper, we assume that a SU must switch as soon as any one of the channels is reclaimed by a PU. The remaining unoccupied channels are returned to the spectrum pool and become candidates for SU transmission.

We use Shannon capacity to model the available rate of a SU. To elaborate, when a SU uses $N$ channels, its achievable rate $\phi(N)$ satisfies

$$\phi(N) = NB_p \log(1 + \frac{P_t}{n_0 N B_p}), \quad (1)$$

where $P_t$ denotes the transmit power of a SU. We assume that each SU has the same fixed transmit power throughout the paper. In addition, $n_0$ denotes noise variance, which we assume is uniform throughout the spectrum in which the SU network operates. Our analytical models can be directly extended to other rate models.

The overhead that a SU incurs before it can access the spectrum is denoted by $C$. The main contributors to overhead are the channel evacuation, link setup, and channel sensing times. First we define the cost $C_0$ as the sum of the channel evacuation and link setup times. $C_0$ is modeled as a random variable that is i.i.d. for all SUs. In practice, the channel evacuation and link setup times are based on the system's specific hardware and operating environment. In XG field tests [7], the evacuation time is up to 0.45s and link setup time is 0.165s. The full overhead $C$ consists of different components depending on the network model being investigated, which will be detailed in later sections. Generally, $C$ is a random variable and also a monotonically increasing function of $N$, because it is more difficult to get a large number of channels.

### C. Real Channel Trace Collection

We also collected real trace data to test the robustness of our models. Spectral measurements were taken in the 850-870MHz band. The spectral data was collected in 0.01s snapshots, with a DFT frequency resolution of 8.333kHz. The measurements were taken over a duration of 100 minutes.

Within the measured band, 60 channels were selected. The channels exhibit high power PU transmissions and low noise levels when idle, resulting in easily detectable on/off PU activity traces. PU activity is determined by the application of a simple energy threshold method, with low-level processing to eliminate false alarms caused by noise. We note that each of the channels only has a bandwidth of 25kHz; these channels are mainly of interest because they exhibit a large number of on/off cycles, and statistical heterogeneity. Some of the selected channels are adjacent in the spectrum, and experience adjacent channel interference (ACI) which leads to some correlation among the channels.

Fig. 1 visualizes the real trace data used in our model. We use an energy threshold to determine whether each channel is busy, denoted by a black tick, or idle, denoted by a white tick, for any 0.01s time interval, and each channel in our data trace exhibits a number of busy/idle cycles. Observing Fig. 1, we can intuitively see that when a larger number of channels is accessed by a SU, a PU is more likely to interrupt the transmission sooner than if the SU uses a smaller number of channels. However, a SU's instantaneous transmission rate will also be higher on a larger number of channels. These observations reveal a key tradeoff between the rate of SU transmission, and the transmission time.

## III. One Secondary User Case

In this section, we focus on the optimal bandwidth selection problem for a single SU. We assume a SU has a large pool of PU channels to choose from. This model can be applied to a network with a small number of SUs, where interactions among SUs' is insignificant due to a large pool of channel availability, while PU activity may have a large effect on SU bandwidth strategy and performance. We also assume that in the small network scenario, the channel sensing burden falls on the individual SUs, and $C$ therefore includes channel sensing time as part of the general overhead of a SU communication. The focus here is to study the tradeoff between higher bandwidth and more frequent switching. We consider two scenarios, the saturated traffic scenario and limited traffic scenario.

### *A. Saturated Traffic Case*

Consider a SU with saturated traffic. The objective is to find the optimal bandwidth, i.e., the optimal value of $N$ that maximizes the SU throughput. Using a larger bandwidth leads to higher instantaneous rate, but also larger sensing overhead. In addition, operating on a larger number of channels will cause more frequent collisions with PUs and therefore more frequent channel evacuations and less channel holding time. All these factors imply that there is a tradeoff between transmit rate and overhead.

The procedure for a SU communication is as follows. The SU conducts channel sensing to find a set of $N$ available channels and sets up the communication link with the receiver. The SU then begins transmission. When one of the channels is reclaimed by PUs, the SU stops transmission and evacuates all of the channels immediately[1], and the second run of channel sensing and link setup begins. Let $t_i^h(N)$, $i = 1, \dots, n$, denote the $i$th cycle's channel holding time. Note here $t_i^h(N)$ is a realization of the random channel holding time $T^h(N)$, which is a function of $N$. Let $c_i$, $i = 1, \dots, n$, denote the $i$th cycle's overhead, which is a realization of the random overhead $C$. We will discuss $T^h(N)$ and $C$ later.

The SU's throughput is calculated by dividing total throughput by total time required for transmission, i.e.,

$$R_s(N) = \lim_{n\to\infty} \frac{\phi(N)\sum_{i=1}^n t_i^h(N)}{\sum_{i=1}^n c_i + \sum_{i=1}^n t_i^h(N)} = \frac{\phi(N)E(T^h(N))}{E(C)+E(T^h(N))} \tag{2}$$

The optimal channel selection is the number of channels that maximizes the overall throughput. In other words, the objective function is formulated as

$$N^* = \text{argmax}_N R_s(N). \tag{3}$$

To find $N^*$, we need the parameter $E(C)$, and $E(T^h(N))$. Practically, these parameters can be obtained through a self-learning approach. The SU can record the instances of overhead and channel holding time that it experiences, or query the average channel holding time from other SUs or a database infrastructure. In our model, we assume that these parameters have already been obtained by the SU. Note that the closed-form representation of $N^*$ is hard to derive from (2), but can be searched from 1 to $N_{max}$.

Now let us examine the properties of channel holding time $T^h(N)$. $T^h(N)$ is the minimum residual idle time of all $N$ channels. That is $T^h(N) = \min(T_1^r, T_2^r, \dots, T_N^r)$, where $T_i^r$, $(i = 1, \dots, N)$ is the residual idle time of channel $i$. Let $F_{T_i^I}(x)$ denote the CDF of the idle time of channel $i$, denoted by $T_i^I$. Then the unconditional PDF of $T_i^r$ is $f_{T_i^r}(x) = (1 - F_{T_i^I}(x))/E(T_i^I)$. If each channel's residual idle time is i.i.d, the PDF of $T^h(N)$ is $f_{T^h}(x) = N(1 - F_{T_i^r}(x))^{N-1} f_{T_i^r}(x)$. For exponentially distributed idle time of each channel, we have $E(T^h(N)) = E(T^I)/N$. For uniformly distributed channel idle time, we have $E(T^h(N)) = 2E(T^I)/(2N+1)$. There are iterative algorithms to calculate the average minimum residual time given a general distribution of $T_i^r$ [8]. Generally speaking, $E(T^h(N))$ is a decreasing function of $N$. The larger the number of channels a SU uses, the less the channel holding time.

Next, we consider the overhead $C$. The overhead $C$ consists of the channel sensing time and $C_0$, which is the sum of channel evacuation time and link setup time. Since channel sensing time is a function of $N$, we then write the total overhead cost as $C = C(N)$.

Let $t_s$ denote the basic unit of channel sensing time, which is a constant. Let $C_s(N)$ denote the number of sensing time units required for a SU to get a set of idle channels, which is a function of $N$. We make the following conservative assumption: after a PU arrives and a SU evacuates its channels, this SU must conduct sensing as if it has no knowledge of

[1]This is needed to coordinate the communication with the receiver.

any of the channel states [2]. This means that even if only one channel in the SU's channel selection has become occupied, the SU must re-scan each of those remaining channels if it wishes to use them.

First, let us consider a sensing scheme in which the SU scans the channels one by one (OBO sensing) [3]. For OBO sensing, $t_s$ is the amount of time required to sense one channel, and $C_s(N)$ is different for the DISC and CONC case. For the former case, $C_s(N)$ is the sum of $N$ geometrically distributed variables with parameter $P$. So for DISC with OBO sensing the average channel sensing time is $Nt_s/P$. For the latter case, $C_s(N)$ is the number of channels that have been scanned until $N$ consecutive idle channels have been found. We present the results on $E(C_s(N))$ in the CONC case as follows:

$$E(C_s(N)) = \begin{cases} \left(\frac{1}{P^{N-1}} - 1\right)\left(\frac{1}{P(1-P)} - \frac{(N-1)P^N}{P-P^N}\right) + \frac{1}{P} + N, \\ \quad N > 1; \\ \frac{1}{P}, \ N = 1. \end{cases} \quad (4)$$

If the SU senses $N$ channels simultaneously, we refer to this as parallel sensing. In this case, $t_s$ is the time required to perform sensing on one block of $N$ channels. For this sensing scheme, $C_s(N)$ is the same for both DISC and CONC cases. Since the probability of $N$ channels being idle is $P^N$, $C_s(N)$ is a geometrically distributed variable with parameter $P^N$. Therefore the average channel sensing time is $t_s/P^N$. Note that $t_s$ is assumed to be the same for OBO sensing and parallel sensing, which can be easily generalized.

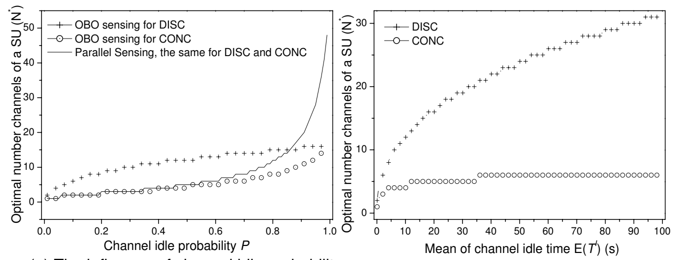
In Fig. 2, we numerically study how PUs' activity, i.e. channel idle time and idle probability, influence bandwidth selection of the SU, under different sensing and channel access behaviors. We assume channel idle times are i.i.d and exponentially distributed.

Fig. 2(a) shows how optimal bandwidth selection changes under different sensing models as a function of the channel idle probability. We see that as $P$ increases, optimal bandwidth increases because channel sensing overhead is lower for both CONC and DISC cases, regardless of the sensing scheme. We also observe that if SUs use OBO sensing, $N^*$ is larger for DISC than CONC. This is because CONC requires larger overhead to find a set of $N$ channels compared to DISC. This implies that it is less favorable to use a large bandwidth for CONC. In addition, parallel sensing causes a larger overhead for DISC than OBO sensing when $P$ is below 0.9. However, when $P$ is above 0.9, $N^*$ increases quickly. In this case, most channels are idle, and thus sensing multiple channels simultaneously results in low overhead.

Given these results, we conclude that when $P$ is large, it is wise to use parallel sensing for both DISC and CONC cases. Otherwise, it is better to use OBO for the DISC case, and there is no significant difference between OBO and parallel sensing for CONC. Note that without the conservative assumption we made earlier, in the case of DISC, the SU can always use OBO

[2]This conservative assumption also has practical meaning. The event triggering the SU to evacuate could come from the upper layer where the SU has no information on which specific channel is reclaimed.

[3]We assume that the earlier scanned channels are still available after the whole channel sensing period. This assumption is reasonable when $t_s$ and $C_s(N)$ are small.




(a) The influence of channel idle probability $P$, E($T^I$)=10s

(b) The influence of mean of channel idle time E($T^I$), $P$=0.5

Fig. 2: The impact of PUs' activity on the SU's optimal bandwidth. $P_t = 0.5$mw, $n_0 = -56.5$dB/MHz (They are used throughout the paper). $t_s = 0.01$s, $C_0 = 0.05$s, $N_{max} = 100$.

since it only needs to search for a single channel. For CONC, however, the conservative assumption makes little difference because the $N-1$ channels are usually useless in the case where a SU needs $N$ consecutive channels.

Fig. 2(b) shows how channel idle time influences the bandwidth of a SU. We consider OBO sensing for DISC and CONC. Not surprisingly, larger channel idle time leads to larger bandwidth, since a larger channel idle time will result in a longer channel holding time for SUs. More surprising is the difference in sensitivity to channel idle time between DISC and CONC. This difference can also be attributed to the fact that CONC has a larger sensing overhead, which requires enough increase in channel holding time to offset. In summary, DISC achieves higher performance, but needs to be more sensitive to the changes in the average idle time of PU channels. In contrast, CONC has lower throughput, but is also less sensitive to changes in PU behavior.

### *B. Limited Traffic Case*

We now assume that a SU has a fixed amount of traffic. The objective of maximizing the SU's throughput is then equivalent to minimizing the total amount of time taken to send this fixed amount of traffic, which we refer to as the finishing time. Many factors affect the finishing time. Due to PU activity, the SU may need to interrupt its transmission, evacuate the channels and configure to another set of channels before sending out the data. To account for the disruption of SU transmission caused by PU activity, we model the finishing time to include the switching overhead and channel holding time. The total finishing time can be represented by

$$T(N) = \sum_{i=1}^{\rho} C_i(N) + t_x, \quad (5)$$

where $\rho$ is the number of times that the SU has to switch channels due to PU activity, $C_i(N)$ is a random variable to denote the overhead of the $i$th switching, and $t_x$ is the data transmission time. Letting $D$ denote the fixed traffic amount, $t_x = D/\phi(N)$ if the SU operates on $N$ channels. The objective is then to minimize $E(T(N)) = E(\sum_{i=1}^{\rho} C_i(N)) + t_x$. Let $T_i^h(N)$, $i = 1, \ldots, \rho$, denote the channel holding time after the $i$th channel switching instance, we have

$$\rho = \text{argmin}_i (T_1^h(N) + T_2^h(N) + ... + T_i^h(N) \geq t_x). \quad (6)$$

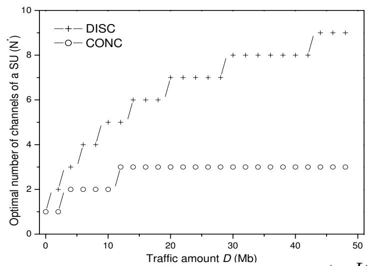



Fig. 3: Traffic-aware optimal bandwidth, $E(T^I) = 10$s, $C_0 = 0.05$s, $t_s = 0.01$s, $P = 0.5$.

We note that the random number $\rho$ is a stopping time for the random process $\{T_i^h(N), i > 0\}$ because the event $\{\rho = i\}$ does not depend on the random process $\{T_j^h(N), j > i\}$. Then if $T_i^h(N)$, $i \geq 1$, are i.i.d, following *Wald's* equation [9], we have

$$E(\sum_{i=1}^{\rho} T_i^h(N)) = E(\rho)E(T_i^h(N)). \tag{7}$$

Because $E(\sum_{i=1}^{\rho-1} T_i^h(N)) < t_x \leq E(\sum_{i=1}^{\rho} T_i^h(N))$, by (7), we have $t_x/E(T_i^h(N)) \leq E(\rho) < (t_x + E(T_\rho^h(N)))/E(T_i^h(N))$. We note that $T_\rho^h(N)$ has different distribution with $T_i^h(N)$, $i < \rho$, since $\rho$ is the stopping time. We can use $1 + t_x/E(T_i^h(N))$ to approximate $E(\rho)$. Generally, the approximation is accurate when $t_x$ is large. We have validated the accuracy of $E(\rho) = 1 + t_x/E(T_i^h(N))$ under several common used distributions. $T_i^h(N)$ and $C_i(N)$ have the same distribution as $T^h(N)$ and $C(N)$ of Section (III-A) respectively.

In Fig. (3) we numerically study the properties of traffic-aware optimal bandwidth selection for the DISC and CONC case. We assume each channel's idle time is i.i.d and exponentially distributed. We observe that a larger amount of data leads to larger $N^*$ for both cases. This figure implies that as a SU is transmitting, it should decrease the number of channels it uses since the traffic amount is decreasing. The decrease should be more frequent when the it has a low overhead to access the spectrum and in case that the amount of traffic becomes smaller. We also see that $N^*$ is much more sensitive to the change of traffic amount in the DISC case than in the CONC case, although CONC is also fairly sensitive when the total traffic is small.

### C. The Impact of Channel Correlation

In our spectrum measurement experiments, we often observe that a channel is in the busy state when there is high power transmission in a neighboring channel. This implies that some of the activity observed in this channel is due to interference from its neighboring channel, which we refer to as adjacent channel interference (ACI). This introduces channel correlation into our data. Therefore, we are motivated to study the channel correlation problem and its impact on SU performance. Although there can be many causes of channel correlation, such as a PU which operates in several channels at once, we choose mainly to focus on channel correlation resulting from ACI. We first model the so-called 1-ACI case where a PU interferes only with its nearest adjacent channels. Then we extend the results to a more general $\kappa$-ACI case.

Consider a channel $i$. Some PUs may use a low transmit power which causes no notable interference to any surrounding channels. But other PUs may have a high transmit power and will cause interference to its adjacent channels $i-1$ and $i+1$. For simplicity, we assume the arrival processes of the two kinds of PU traffic are modeled as two Poisson processes, denoted by $P_i^0(t, \lambda_i^0)$ and $P_i^1(t, \lambda_i^1)$ respectively.

In [10], the fatal shock model is introduced to model multi-variant exponential distribution. The model has the following physical meanings in our system. When a PU accesses a channel, this event acts as a "fatal shock" that terminates the idle period of the channel. Therefore, both the occurrence of low power PUs and high power PUs in channel $i$ are "fatal shocks" to channel $i$. Furthermore, the occurrence of a high power user at channel $i$ is also a "fatal shock" to both channels $i-1$ and $i+1$. The "fatal shocks" of a channel make the SU using that channel stop transmission.

Following the *fatal shock* model, we model the channel idle time correlation. We omit correlated channel idle probability, which can be similarly derived. Consider a SU that operates on $N$ consecutive channels, indexed by $i = j, \ldots, j+N-1$. For each channel $i$, the idle period still follows exponential distribution since for each channel the PUs arrive as a Poisson process. It follows that the residual idle time of a channel $i$, $T_i^r$, also follows exponential distribution. We calculate the minimum residual idle time of $N$ correlated channels, which is the channel holding time for a SU, denoted by $T_c^h(N)$. We have

$$\begin{aligned} &\Pr(T_c^h(N) > t) = \Pr(T_j^r > t, T_{j+1}^r > t, \ldots, T_{j+N-1}^r > t) \\ &= \Pr(P_{j-1}^1(t) = 0, P_i^0(t) = P_i^1(t) = 0, P_{j+N}^1(t) = 0) \\ &= \exp[-(\lambda_{j-1}^1 + \sum_{i=j}^{j+N-1} (\lambda_i^0 + \lambda_i^1) + \lambda_{j+N}^1)t], \\ &i = j, ..., j+N-1. \end{aligned} \tag{8}$$

Therefore, $T_c^h(N)$ is exponentially distributed with parameter $\lambda_{j-1}^1 + \sum_{i=j}^{j+N-1} (\lambda_i^0 + \lambda_i^1) + \lambda_{j+N}^1$.

To highlight the impact of channel correlation, let us examine the channel independence case. If a SU observe each channel independently, for channel $i$, both the idle time and the residual idle time are still exponentially distributed with parameter $\lambda_{i-1}^1 + \lambda_i^0 + \lambda_i^1 + \lambda_{i+1}^1$. After observing the same $N$ channels, the independence assumption leads to a minimum residual idle time that is exponentially distributed with parameter $\sum_{i=j}^{j+N-1}(\lambda_{i-1}^1 + \lambda_i^0 + \lambda_i^1 + \lambda_{i+1}^1)$. Consider homogenous channels, where $\lambda^0 = \lambda_i^0$ and $\lambda^1 = \lambda_i^1$ $\forall i$. By the independence assumption,

$$E(T^h(N)) = \frac{1}{N(\lambda^0 + 3\lambda^1)}. \tag{9}$$

By comparison, for correlated channel, we have

$$E(T_c^h(N)) = \frac{1}{N\lambda^0 + (N+2)\lambda^1} > E(T^h(N)). \tag{10}$$

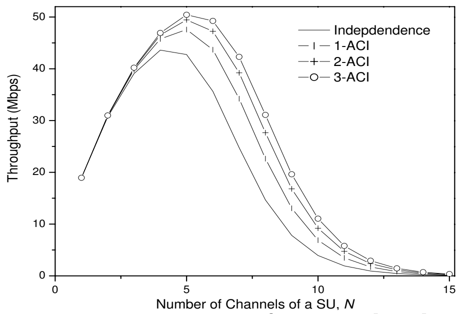



Fig. 4: The impact on a SU's performance by channel correlation resulting from adjacent channel interference. $E(T^I) = 10$s, $C_0 = 0.05$s, $t_s = 0.01$s, $P = 0.5$. Consider the CONC case and OBO sensing.

We can extend the study to the $\kappa$-ACI case. For a channel $i$, we may have a set of PUs that have different transmit power levels, and therefore interfere with different ranges of adjacent channels. Let $\lambda_i^\alpha$ denote the rate of PUs that interfere with $\alpha$ adjacent channels from $i$, $0 \leq \alpha \leq \kappa$. If a channel does not have such a PU, the rate is 0. It is not difficult to show that the minimum residual idle time for $N$ channels (indexed by $i = j, ..., j+N-1$) is exponentially distributed with parameter $\sum_{i=j}^{j+N-1} \sum_{\alpha=0}^{\kappa} \lambda_i^\alpha + \sum_{n=1}^{\kappa} \sum_{\alpha=n}^{\kappa} (\lambda_{j-n}^\alpha + \lambda_{j+N+n-1}^\alpha)$. On the other hand, under the assumption of channel independence, it is exponentially distributed with parameter $\sum_{i=j}^{j+N-1} \sum_{n=0}^{\kappa} \sum_{\alpha=n}^{\kappa} \lambda_{i\pm n}^\alpha$. Similar with 1-ACI case, if each channel is homogenous, the parameters for the correlation and independence are $N \sum_{\alpha=0}^{\kappa} \lambda^\alpha + 2(\sum_{\alpha=1}^{\kappa} \alpha\lambda^\alpha)$ and $N(\sum_{\alpha=0}^{\kappa} \lambda^\alpha + 2\sum_{\alpha=1}^{\kappa} \alpha\lambda^\alpha)$ respectively.

Applying the models to (2), we study how channel correlation influences the performance of a SU in Fig. 4. In the simulation, we assume that the arrival rates for PUs with different power levels are the same. The figure clearly shows that if channels are correlated, there is an obvious increase in throughout compared to that under independence assumption. In fact, we can see a direct relationship between $\kappa$ and throughput, i.e., higher correlation leads to higher throughput.

However, this does not mean that PUs with higher transmit powers are better for SUs. As stated before, the comparison made here is based on fixed mean channel idle times. If we have a set of channels which are independent, and each channel has a fixed mean idle time, this will result in more switching than if the channels are positively correlated with the same fixed mean idle times. Less switching will then lead to higher channel residual times and thus greater performance. It is clear from these simulations that identifying channel correlation has an important impact on SU performance. We also note that a set of consecutive channels are usually more correlated than the discrete channels, which implies that a SU can get a longer channel holding time in the CONC case than the DISC case and therefore the larger overhead of CONC is offset to some degree.

## IV. Multiple Secondary User Case

In this section, we investigate the optimal bandwidth configuration problem when there are multiple SUs and limited channels. First, we present the model for a multiple SU network and derive the throughput equation for each SU. We then consider the optimal bandwidth problem for DISC case and CONC case. For the former case, the optimal bandwidth solution is easy to derive based on the model presented. For the latter case, obtaining the optimal bandwidth configuration consists of two steps. First, we derive the optimal bandwidth solution of each SU based on the per SU throughput equation derived in the system model. Second, we define the *channel reconfiguration problem*, in which SUs are relocated to other channels to allow more SUs to be accommodated in the spectrum. We present an efficient scheme for this problem.

### A. Optimal Bandwidth of Each SU in the Multi-SU Case

In our model, we consider a network of $K$ SUs operating on $M$ licensed channels. All SUs are within each other's interference range and must be scheduled on disjoint channels when they communicate concurrently. Our goal here is to derive $N^*$, the optimal bandwidth that maximizes the average throughput of a SU.

We assume there is a central infrastructure that gathers channel state information. This infrastructure provides channel availability information to the SUs in real-time as channels become available. Because SUs do not need to conduct channel sensing themselves, $C$ consists of link setup and channel evacuation times, i.e., $C = C_0$, which is different from the single SU case. This generally results in a much smaller $C$ than in the single SU case, and more closely matches real world models.

The model also assumes that each SU has the same behavior, and therefore equal channel access opportunity. Specifically, we assume that each SU has the same mean of overhead, $E(C)$, and that the pool of available channels is the same for all SUs. Based on these assumptions, we have the following throughput equation for each SU as a function of the number of channels $N$ that each SU uses.

$$R_s(N) = \frac{K_{avg}(N)\phi(N)}{K}. \tag{11}$$

Here, $K_{avg}(N)$ denotes the average number of users that a system can support, which clearly depends on $N$, the number of channels a SU uses. Although the value of $N^*$ is difficult to derive analytically, we can numerically find $N^*$ that maximizes average SU throughput. To do this, we must first derive the expression for $K_{avg}(N)$, which is different in the DISC and CONC cases. Thus, the main goal of the following sections becomes the derivation $K_{avg}(N)$ in both cases, to be used in the numerical search for $N^*$. We first consider the DISC case, which leads to a much simpler derivation for $K_{avg}(N)$.

*1) DISC case:* Let us first consider a case where there is no overhead for a SU to access the spectrum, i.e. $E(C) = 0$. When each SU uses $N$ discrete channels, the number of SUs that the spectrum can support is upper-bounded by one of two values, either the total number of SUs $K$, or the maximum number of SUs that $M$ channels can support given $N$, $\lfloor \frac{M}{N} \rfloor$. We denote this upper-bound by $K_{max}$, where $K_{max} = \min(K, \lfloor \frac{M}{N} \rfloor)$. We also note that the system can support at most $i$ SUs when the number of available channels

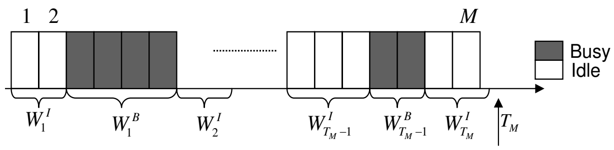



Fig. 5: The alternative idle and busy channel blocks and the stopping time $T_M$ due to channel boundary $M$.

is less than $(i+1)N$ but greater than or equal to $iN$. Therefore the average number of SUs in the spectrum can be written as:

$$K_{avg}(N) = \sum_{i=0}^{K_{max}-1} \sum_{j=i*N}^{(i+1)N-1} i * P_M(j) + \sum_{j=K_{max}N}^{M} K_{max} P_M(j) \tag{12}$$

where $P_M(j) = \binom{M}{j} P^j (1-P)^{M-j}$, which is the probability that $j$ channels are idle.

To expand this model to include non-zero overhead $C$, we can consider the overhead as a part of the channel busy period, and proceed as if $E(C) = 0$. The idle probability of each channel is changed to $\hat{P} = \frac{E(T^I)-E(C)}{E(T^I)+E(T^B)}$. $E(C)$ should be smaller than $E(T^I)$ in practice for the channels to be useful for dynamic access. We will also apply $\hat{P}$ as defined here to the CONC case. Note this conversion will potentially undermine the channel independence assumption since each SU will use $N$ channels, since it will add some constant $C$ to the busy periods of all $N$ channels. However, as stated in our model, the time overhead is insignificant compared to the length of idle and busy periods. We run simulations on real channel trace to test the robustness of our model.

*2) CONC case:* First, we discuss some properties of the PU activity in the $M$ channels. In Fig. 5 we see that at any time, the $M$ channels appear as a series of busy and idle channel blocks. Let $W_I^i$ and $W_B^i$ refer respectively to the width (in number of channels) of the $i$th idle channel block and $i$th busy channel block. Obviously, SUs can only transmit over the idle channel blocks, and if we know the number of channels contained in an idle channel block $i$, we can calculate the number of SUs that this block can support. If we also find the number of idle channel blocks that are occupied by SUs, we can obtain the average number of SUs. However, both $W_I^i$ and the number of idle channel blocks are random variables, and require consideration when we calculate $K_{avg}(N)$. As in the DISC case, the number of SUs in the spectrum is constrained by either the number of channels $M$ or the number of SUs $K$. First we consider the channel constraint.

Let us define a random variable $T_M$, as

$$T_M = \text{argmin}_i (W_1^I + W_1^B + ... + W_i^I + W_i^B \geq M). \tag{13}$$

$T_M$ is the minimum number of idle and busy channel blocks which taken together have a larger width than $M$. We note that $T_M$ is a stopping time for the random process $\{W_i^I + W_i^B, i \geq 1\}$, since the event $\{T_M = i\}$ only depends on $\{W_j^I + W_j^B, 1 \leq j \leq i\}$, and has no relation with $\{W_j^I + W_j^B, j > i\}$.

Now consider an idle channel block $i$ with width $W_i^I$. The block can support $\lfloor \frac{W_i^I}{N} \rfloor$ SUs. Since the width of idle channel blocks form a random process, $\{W_i^I, i \geq 1\}$, it follows that the number of SUs that can be supported by each idle channel block also forms a random process, defined as $\{\lfloor \frac{W_i^I}{N} \rfloor, i \geq 1\}$. $T_M$ is also a stopping time for $\{\lfloor \frac{W_i^I}{N} \rfloor, i \geq 1\}$. Let us first consider a case where there is an infinite number of SUs. Then the number of SUs that can be accommodated by the channels, denoted by $K_M$, follows:

$$\sum_{i=1}^{T_M-1} \lfloor \frac{W_i^I}{N} \rfloor \leq K_M \leq \sum_{i=1}^{T_M} \lfloor \frac{W_i^I}{N} \rfloor. \tag{14}$$

If there is a limited number of users $K$, the actual number of SUs occupying the channels is $\min(K_M, K)$, which is a random variable. The intuition is clear. When $M$ is large, or $\hat{P}$ is large, there is a large channel accommodating capacity, and the number of SUs in the spectrum is close to $K$. On the other hand, when $K$ is large, the channels are saturated with SUs due to the limited number of channels $M$. Now we can write

$$K_{avg}(N) = E(\min(K_M, K)). \tag{15}$$

The distribution for $K_M$ is difficult to derive in practice, so we make a reasonable approximation here. Let $K_{avg}(N) \approx \min(E(K_M), K)$. Note that $\min(E(K_M), K)$ is actually an upper bound of $K_{avg}(N)$.

Now let us calculate $E(K_M)$. According to (14), we have

$$\begin{aligned} E\left(\sum_{i=1}^{T_M-1} \lfloor \frac{W_i^I}{N} \rfloor\right) &\leq E(K_M) \leq E\left(\sum_{i=1}^{T_M} \lfloor \frac{W_i^I}{N} \rfloor\right) \\ \Rightarrow E(T_M)E(\lfloor \frac{W_i^I}{N} \rfloor) - E(\lfloor \frac{W_{T_M}^I}{N} \rfloor) &\leq E(K_M) \leq E(T_M)E(\lfloor \frac{W_i^I}{N} \rfloor). \end{aligned} \tag{16}$$

where $E\left(\sum_{i=1}^{T_M} \lfloor \frac{W_i^I}{N} \rfloor\right) = E(T_M)E(\lfloor \frac{W_i^I}{N} \rfloor)$ by *Wald's* equation. We can use $E(T_M)E(\lfloor \frac{W_i^I}{N} \rfloor)$ to approximate $E(K_M)$. We calculate $E(T_M)$ and $E(\lfloor \frac{W_i^I}{N} \rfloor)$ as follows.

From (13) and the fact that $T_M$ is the stopping time for $\{W_i^I + W_i^B, i \geq 1\}$, we can write

$$\begin{aligned} &\sum_{i=1}^{T_M-1} (W_i^I + W_i^B) < M \leq \sum_{i=1}^{T_M} (W_i^I + W_i^B) \\ \Rightarrow &E(T_M)E(W_i^I + W_i^B) - E(W_{T_M}^I + W_{T_M}^B) < M \leq \\ &E(T_M)E(W_i^I + W_i^B). \end{aligned} \tag{17}$$

We can use $M/E(W_i^I + W_i^B)$ to approximate $E(T_M)$. $W_i^I$ is a geometrically distributed random variable. We have $P(W_i^I = k) = \hat{P}^{k-1}(1-\hat{P}), k \geq 1$, and we can ignore the boundary effect. We also have $P(W_i^B = k) = \hat{P}(1-\hat{P})^{k-1}, k \geq 1$. Then we can write $E(W_i^I) = 1/(1-\hat{P})$ and $E(W_i^B) = 1/\hat{P}$. Therefore we have $E(T_M) \approx M/(1/(1-\hat{P}) + 1/\hat{P})$.

Now, we calculate $E(\lfloor \frac{W_i^I}{N} \rfloor)$. In the event that $\{\lfloor \frac{W_i^I}{N} \rfloor = k\}$, meaning that idle channel block $i$ can support no more than $k$ users, we can then write that $\{kN \leq W_i^I < (k+1)N\}$. Based on this relation, we can calculate the expected value of users that idle channel block $i$ can support:

$$\begin{aligned} E\left(\lfloor \frac{W_i^I}{N} \rfloor\right) = &\sum_{i=1}^{\lfloor \frac{M}{N} \rfloor - 1} \sum_{j=i*N}^{(i+1)N-1} i\hat{P}^{j-1}(1-\hat{P}) \\ &+ \lfloor \frac{M}{N} \rfloor \sum_{j=\lfloor \frac{M}{N} \rfloor N}^{M} \hat{P}^{j-1}(1-\hat{P}). \end{aligned} \tag{18}$$

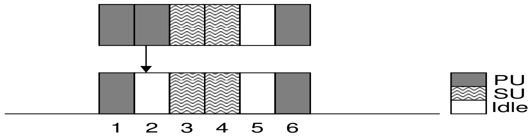



Fig. 6: An example of the opportunity for channel reconfiguration. In this case, $N^*$ for a SU is 2.

We again ignore the boundary effect of $M$. With that approximation, (18) becomes $E\left(\lfloor \frac{W_i^I}{N} \rfloor\right) = \hat{P}^{N-1}/(1-\hat{P}^N)$.

Now we have

$$E(K_M) \approx \frac{M}{\frac{1}{1-\hat{P}} + \frac{1}{\hat{P}}} \frac{\hat{P}^{N-1}}{1-\hat{P}^N}. \quad (19)$$

Finally, we have

$$K_{avg}(N) \approx \min\left(\frac{M}{\frac{1}{1-\hat{P}} + \frac{1}{\hat{P}}} \frac{\hat{P}^{N-1}}{1-\hat{P}^N}, K\right). \quad (20)$$

Combining this result with (11), we derive the closed-form approximation of per SU throughput in the CONC case. Per SU throughput is a function of $N$, i.e., the number of channels each SU uses. The optimal number of channels, $N^*$, can be determined numerically from $N = 1$ to $N_{max}$.

### B. Channel Reconfiguration

The concept of Channel Reconfiguration (CREC) arises out of the CONC scenario. Although we have derived the optimal number of channels in the CONC case, there is no guarantee that the SUs will access the spectrum in an optimal configuration due to the random nature of PU behavior. Fig. 6 is a simple example of this situation. In the figure, channels 1, 2 and 6 are initially occupied by PUs, which forms an idle channel block from channels 3 to 5. A SU occupies channels 3 and 4, which is optimal in this case since we assume $N^* = 2$ and there are only 3 channels available. If channel 2 becomes idle, however, this channel configuration becomes suboptimal. In the resulting configuration, despite having 2 channels which remain available, channels 2 and 5 will remain unused because of the CONC requirement. This is clearly not optimal. If the current SU reconfigures to use channels 2 and 3 (or 4 and 5), another SU could fit into the idle block. This is the basic idea of CREC: rearrange the SUs in an idle block to make room for more SUs.

There are several issues to consider when developing a CREC scheme that maximizes the SUs' overall throughput. First, there is an inherent tradeoff to CREC. Although reconfiguration allows more SUs to be accommodated, it also harms the SUs already transmitting by forcing them to reconfigure to new channels. The unpredictable nature of PU activity can affect the scheme as well. In Fig. 6, if channels 1 or 6 were to become idle, no reconfiguration would be required to add another SU to the resulting idle block.

Due to the difficulty in predicting PU activity, we simplify the model by only considering current channel states. We make some key observations on the nature of CREC scheme. The central benefit of CREC is that new SUs can be accommodated. When a new SU is accommodated by CREC, it can then transmit on $N^*$ channels over the duration of a channel holding time with mean $E(T^h(N^*))$. The cost of CREC is the overhead added for relocated SUs, which includes the time to evacuate the channels and set up a new link, $E(C)$. These benefits and costs can be directly mapped to a throughput gain expression, and the objective of the CREC scheme would then be to maximize this expression. We define the optimal CREC scheme as

**Optimal Channel Reconfiguration Scheme:** *A channel reconfiguration scheme $Q^*$ is optimal when it satisfies*

$$Q^* = \text{argmax}_Q \left(\pi(Q)E(T^h(N^*)) - \chi(Q)E(C)\right), \quad (21)$$

where $\pi(Q)$ and $\chi(Q)$ refer respectively to the number of added and reconfigured SUs under scheme $Q$. The optimal CREC scheme yields the maximum throughput gain in (21).

To find the optimal CREC scheme, let us first examine the properties of CREC. First, CREC is conducted on a set of blocks each of which is bounded by channels being used by PUs, i.e., idle channel blocks as defined in Section IV-A.2. We refer to these as CREC blocks here, and there are at most $T_M$ CREC blocks. We note a SU does not need to reconfigure to another CREC block since this will not reduce any cost or bring any benefit. Therefore, a CREC scheme $Q$ is a set of CREC schemes w.r.t each CREC block, i.e., $Q = \{Q_1, Q_2, \ldots, Q_r\}$, where $r$ denotes the number of CREC blocks and we have $r \leq T_M$.

To find $Q^*$, we first focus on a specific CREC block $i$. It is efficient to find all the possible CREC schemes on $i$, due to limited potential idle channel combinations. Based on these CREC schemes, we can set up a table of gains corresponding to the potential number of added SUs. We note that different CREC schemes may result in the same number of added SUs, but they may also result in different throughput gains. We can find the $Q^*$ that results in maximum benefit for the entire system using the following algorithm.

- In a block $i$, $1 \leq i \leq r$, let $\pi^i$ denote the potential number of added SUs. For $\pi^i = 0, 1, \ldots, \pi^i_{max}$, find a $\pi^i$ with maximum gain. Let this be denoted by $\pi^i_{opt}$.
- Let $K_L$ denote the number of SUs that are waiting to obtain channels. If $\sum_{i=1}^r \pi^i_{opt} \leq K_L$, the optimal CREC scheme is to let each block $i$ add extra SUs with an amount of $\pi^i_{opt}$ respectively.
- If $K_L < \sum_{i=1}^r \pi^i_{opt}$, use dynamic programming to calculate how many SUs should be added in each CREC block to maximize the total gain. Add the determined number of SUs to each CREC block.

In Fig. 7, we present a simple example of the optimal CREC scheme algorithm. In the example, $K_L$ is 4. We see that $\pi^i_{opt}$ for the four blocks are 0, 2, 1, 2 from left to right. For each block, the number of SUs that can be added and the associated gains are shown in the $\pi(Q_i)/Gain$. Then the optimal scheme would be to accommodate 2 new SUs in CREC block 2 and 4, which results in a maximum gain of 6. If $K_L \geq 5$, the optimal scheme would be to let each CREC block accommodate new SUs with an amount of 0, 2, 1, 2 respectively.

### C. Evaluation

In this section, we evaluate our proposed solutions on bandwidth configuration in the multiple SUs scenario through

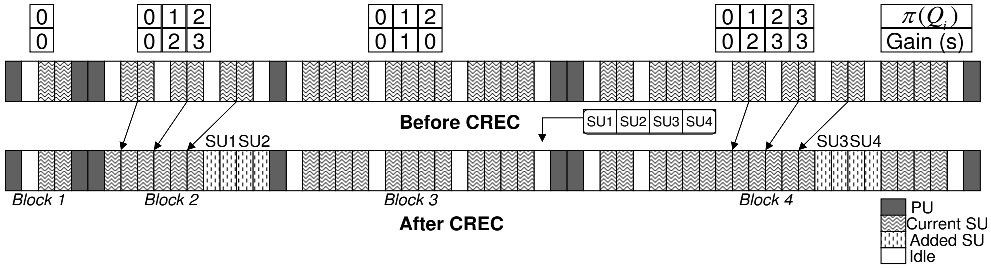



Fig. 7: An example of CREC scheme, $E(T^h(N^*)) = 3$s and $E(C) = 1$s.

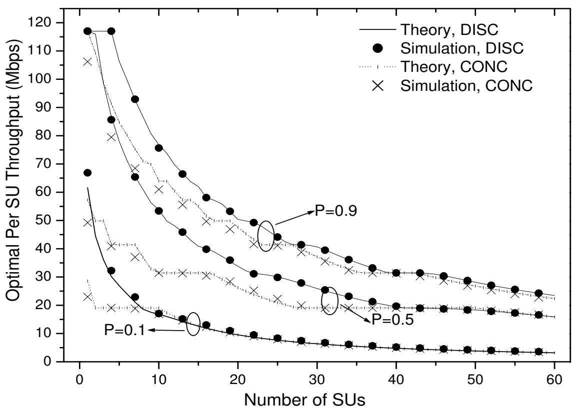



Fig. 8: Optimal performance under theory and simulations.

simulations on both simulated channel activity and real trace data.

First, let us examine the correctness of our derivation on per SU throughput representations. In Fig. 8, we compare the theoretical and numerical results of per SU throughput for both the DISC and CONC cases. Here overhead $C$ is set to 0. We will study $C > 0$ cases later. We consider $K$ from 1 to 60 and $M = 100$.

We observe that for both DISC and CONC cases, under different channel idle probabilities, numerical results match the theoretical results very well. For DISC, the numerical results match theory much better than in the CONC case, since we are able to derive an exact closed-form expression for per SU throughput when $C = 0$. As stated previously, our closed-form expression for the per SU throughput in CONC is actually an upper bound. We observe the theoretical throughput is slightly larger than the numerical result.

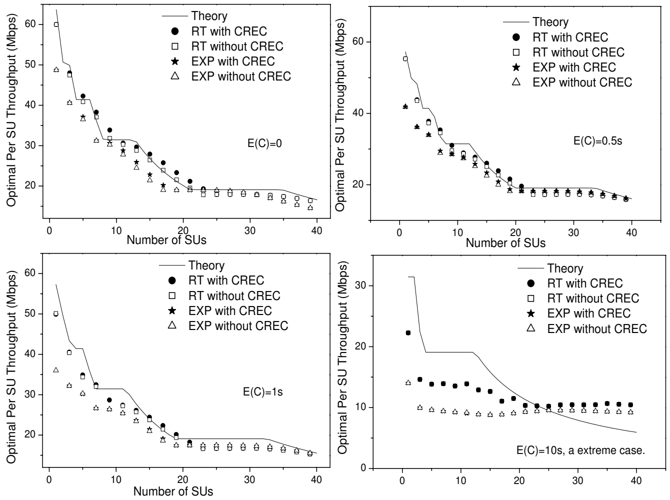
Fig. 8 shows that although DISC generally leads to higher per SU throughput than CONC, this performance difference attenuates as $K$ becomes large, or when $P$ is very small. The key factor that mitigates these differences appears to be the optimal number of channels. When $K$ is large, for example, DISC and CONC exhibit little performance difference because the optimal number of channels approaches 1 in both cases. When $P$ is very small, as in the $P = 0.1$ curves, the optimal number of channels is 1 for both DISC and CONC over a wide range of $K$. When $P$ is very large, however, the number of optimal channels in both cases tends to be rather large because of their high idle probabilities. Therefore the limits imposed by the consecutive channel requirement tend to have less of an effect on CONC per SU throughput because of high channel availability.

In Fig. 9 we compare the results of our optimal bandwidth solution for the CONC case both with and without CREC. In the figure, we plot curves using both real trace (RT) and simulated data traces (EXP). We use the 60 channel real trace data as described in section II.C to test the robustness of our analytical results. Note that in this simulation setting, each channel in the real trace data is assumed to have bandwidth of 5M, which is different from the actual bandwidth of the channels. However, as stated previously, the importance of the real trace data lies in the statistical heterogeneity of PU activity in real environments, such as different idle probabilities, mean idle times, and channel correlations.

For these simulations, we generate $C$ randomly for each SU, but keep the mean of $C$ identical for all SUs. Each graph in Fig. 9 represents the per SU throughput vs. number of SUs in the system for a given mean value of $C$. We also generate simulated channel traces to compare performance over real trace data with known distributions. In the simulated traces, we generate 60 channels with exponentially distributed idle times, with each channel's mean equal to the mean of the corresponding channel in the real trace data. The simulated channels are generated independently. For our theoretical results (labeled ”theory” in the figure), we use the mean of the idle probabilities and the mean of the idle time over all 60 channels as the input parameters to our closed-form expression. The average idle time mean of our real trace data is 13.3s, and the average idle probability is 0.695.

From Fig. 9, we can see that when $E(C)$ is small (0 to 1s), our theoretical results are very close to our RT and EXP results. It also appears that RT leads to a higher per SU throughput than EXP. This is most likely caused by the likely correlations that appear in real channel traces. Correlations between channels allow SUs to have more channel access opportunities and longer transmission times, as we discussed in Section III-C.

We also observe that CREC increases the optimal throughput of each SU. However, notable gains are only made in cases where the optimal number of channels is 2 or 3, where we observe frequent reconfigurations of SUs. In fact, generally the gain from CREC is not significant for two reasons. First, opportunities to use CREC are fairly limited. Second, CREC introduces its own overhead since SU reconfiguration disrupts SU communication and requires SUs to undergo communication setup overhead. From the figure, we also see that as $E(C)$ increases, the benefit of CREC decreases, and in the extreme case that $E(C) = 10$s, there is no throughput gain from CREC. This is because when $E(C) = 10$s, the overhead




Average throughput of each SU under different overhead

Fig. 9: Comparison of performance among Theory, Simulation on real channel trace (RT) and channel trace with exponentially distributed idle time (EXP). $P = 0.695$ and $E(T^I) = 13.3$s for the curve of Theory. $M = 60$, $K = 1, \ldots, 40$.

$C$ is too large compared to the expected channel holding time, $E(T^h(N^*))$. In this case, our channel holding time is generally much smaller than 13.3s, the average idle time mean.

Finally, the figure also shows that in cases when $E(C)$ is very large, our analytical model appears to be inapplicable. This is expected, since our model is based on the assumption that overhead $C$ is much smaller than average channel holding time. As we argued before, it is impractical for a SU network to operate in a given spectrum if its average overhead is comparable to the channel holding times. However, we also note that the optimal number of channels $N^*$ is very similar in the analytical model, the RT, and EXP simulations. This indicates that in practical systems, our model can be used to derive the optimal bandwidth, which leads to optimal throughput.

## V. Related Work

Research in cognitive radio networks has spanned in many areas, including spectrum pooling [5], channel sensing [11], and coexistence of SUs [12] [13]. Centralized and distributed schemes for spectrum allocations have been considered. In [14], the authors propose a distributed algorithm to share spectrum among users with various fairness considerations. In [15], the authors propose a multi-armed bandit model to address the spectrum access problem of the SU. In [16], the authors consider the capability of cognitive radio in determining bandwidth and frequency location. The authors of [20] propose b-smart, a spectrum-time block allocation scheme targeted for TV band unlicensed usage. Neither of these works addresses the tradeoff between instantaneous rate and switching overhead. Our work differs because it focuses on the impact of PU behavior on SU performance and addresses the optimal channel bandwidth allocation issue considering switching overhead and multiple SU competition.

There are significant advances in channel allocation schemes in multi-channel scenarios, e.g., in [17] [18] [19]. These works handle channel allocation dynamically based on traffic conditions, but cannot be applied in cognitive radio networks directly, because they do not explicitly address the challenge of channel dynamics due to PU activity.

## VI. Conclusions

In this paper, we study optimal bandwidth selection for SUs in multi PU bands. We present optimal bandwidth selection in both the single SU and multi-SU cases, which depends on various factors, including PU channel idle time and probability, PU channel correlation, and SU sensing and access scheme. In both single- and multi-SU cases, the hardware capability of the SU, i.e., whether it has to use a set of consecutive channels or can use discrete channels, has significant impact on the SU performance. In both cases, using discrete channels results in higher performance. In the multi-SU case, the capability allows SUs to better utilize fragmented spectrum holes and thus results in higher performance. In the single SU case, using discrete channels allows low overhead in channel sensing and thus higher throughput. The tradeoff to achieve a higher throughput is that the scheme is more sensitive to the idle time variation and traffic demand.

Motivated by the observation of channel correlation during a measurement study, we also analyze the impact of channel correlation caused by adjacent channel interference based on the *fatal shock* model. We learn that identifying channel correlation is important in optimizing SU performance.

Numerical simulations and real channel activity traces are used to validate our analysis. In general, these simulations show that our analytical models are robust and applicable to real world models. Again, the ability to use discrete channels results in higher throughput for SUs because it allows SUs to better utilize fragmented spectrum holes.

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