Opportunistic Spectrum Access in Heterogeneous User Environments

Eric Jung
University of California, Davis
Davis, CA 95616
Email: eajung@ucdavis.edu

Xin Liu
University of California, Davis
Davis, CA 95616
Email: liu@cs.ucdavis.edu

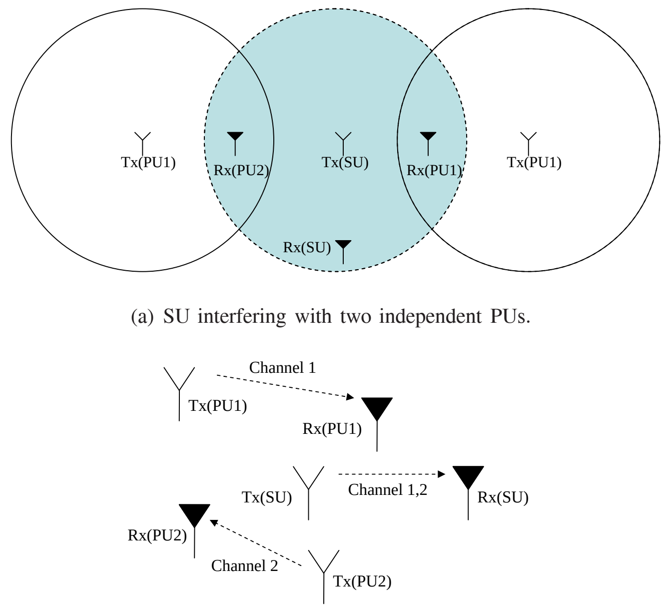
***Abstract*—Cognitive Radio (CR) is an attractive technology to alleviate spectrum scarcity in wireless communications. It allows secondary users (SUs) to opportunistically access the licensed spectrum of primary users (PUs) so long as these users' communications are protected. Therefore, protection of the PUs is central to the adoption of this technology. In this paper, we consider an SU that must protect multiple non-interfering PUs simultaneously. These PUs may be heterogeneous in terms of their activity statistics and protection requirements. Assuming exponentially distributed idle times, we determine the limits on successful SU transmission time in such cases, and demonstrate the impact of these results on the channel selection problem. We also derive a time-threshold optimal policy for PUs with general idle time distributions and determine the conditions necessary for this policy to hold. We then discuss the implications of our results on related issues, in particular the information that the SU needs to determine transmission policies satisfy PU constraints.**

I. Introduction

Cognitive radio is a promising technology to mitigate spectrum inefficiency in wireless communications. It enables secondary users (SUs) to opportunistically access low-occupancy primary spectral bands assuming the primary user (PU) access is protected. Because legacy users have access priority, a design goal of any opportunistic access strategy is to minimize the SUs' effect on licensed users' transmissions. For example, in the DARPA XG project [1], one of the three major test criteria in the field test is "to cause no harm" [2]. This goal has strong implications for both SU performance and incentive to implement such schemes, as licensees will not agree to accommodate secondary cognitive networks to their own detriment. This is also one of the primary bottlenecks of SU network performance. The spectrum access strategy of the SU should aim to maximize the performance of SUs while operating under the protection guarantee of PUs.

One often used protection metric is packet collision probability. In previous work, the researchers have developed medium access schemes for secondary users under this type of protection requirement, e.g., in [3], [4], [5]. While most work has focused on the performance of SUs when interacting with a single PU system, we focus on the case where an SU may interfere with multiple PUs, where those PUs do not interfere with each other in either the spatial or frequency domain.

Consider two motivating examples. Figure 1(a) illustrates an SU transmitter-receiver that interferes with two primary receivers whose transmitters are independent and non-interfering. Both PUs transmit on a common channel, but are sufficiently separated spatially. In the second example, pictured in figure 1(b), we have PU1 and PU2 transmit on different non-overlapping channels so that they are do not interfere with each other. Under some circumstances the SU may have to access some fraction of channel 1 and 2 simultaneously. In both cases, the SU needs a transmission policy that protects the activity of both PUs. The SU must determine its transmission strategy based on the union of both PUs' activities.



(a) SU interfering with two independent PUs.

(b) SU accessing 2 PU channels simultaneously.

Fig. 1. SU in environments with multiple non-interfering PUs.

These examples raise some pertinent questions regarding SU networks:

- What impact do multiple PUs have on SU performance?
- Is information of the unionized behavior of PUs enough to determine sufficiently protective SU access strategies?
- What is the impact of multiple PUs on SU channel selection?

Our paper addresses these questions. We show that for multiple PUs with exponential idle time distributions, an SU's successful transmission time is only limited by the minimum successful transmission time possible for a single PU or the actual idle time of the channel. We use these results to demonstrate the impact of multiple PUs on the channel selection problem. We then derive the optimal policy for general idle distributions under certain conditions. We also investigate the impact of imperfect knowledge of the PUs in determining policies that sufficiently protect each PU.

The paper is organized as follows. After discussing some related work in Section II, we present our system model in Section III. In Section IV we provide brief overview of the results in [6] for single-PU systems. In Section V, assuming PU exponential idle times, we derive an optimal transmission scheme for a channel with multiple PU constraints, and investigate the implications of these results on channel selection. In Section VI, we derive a time-threshold policy for a 2-PU system with general idle time distribution, and present the conditions necessary for such a policy to be optimal. In Section VII, we discuss our results, and in particular how the amount of knowledge of individual PUs affects SU transmission policies. We conclude our paper in Section VIII.

II. Related Works

Much work has been focused on packet collision probability as the protection requirement for PUs. In [3], the authors introduce time capacity, the proportion of time that a DSAN can transmit without violating the PU's packet collision constraint, and determine the time capacity in a single-PU system with exponentially distributed idle time under perfect sensing assumption. In [6], the authors generalize their results in [3] to consider PU general idle time distribution with both perfect and imperfect sensing. Our work can be seen as a direct extension of these works with multiple PU protection constraints.

In [7], the authors formulate the PHY-MAC design of an SU access scheme with imperfect sensing as a constrained partially observable Markovian Decision Process which uses the entire observation history to make optimal decisions. They prove that their optimal access policy obeys a separation principle which simplifies the joint design of the spectrum sensor and MAC protocol. However, this work focuses on a single PU protection requirement in a slotted structure, whereas our work considers a single SU under multiple PU protection constraints in an unslotted system.

Another related issue is how SUs find and choose channels from a pool of potential spectrum bands to transmit on to optimize throughput. In [8], the authors devise an iterative optimization algorithm for max-min fair allocation of channels in a multi-hop CR network. However, their model assumes a static but spatially heterogeneous channel environment such that SUs are not concerned with PU protection requirements. In [9], the authors model the problem as a multi-armed bandit problem to decide the best channel(s) to sense and access. In the special case where all bands are statistically identical, the authors show that the myopic policy is optimal and is equivalent to Whittle's index policy. On the other hand, our channel selection work focuses on the implications of time capacity on selecting multiple bands for simultaneous use by an opportunistic user. In [5], while the authors propose a medium access control scheme for multi-channel parallel transmissions, our work provides explicit performance guarantees while we do not provide a protocol.

III. System Model

Consider an SU that operates within the interference range of two or more non-interfering PU networks. We assume the PUs transmit on a channel or set of channels which the SUs are trying to opportunistically access. Because the PUs are non-interfering, simultaneous transmission of two PUs *does not result* in collision between them.

A. Primary Users

Our system model is similar to [6] but extended to include multiple PUs. The PUs are assumed to be non-cooperative with the SUs and with each other, and access the channel without sensing. PUs are also non-interfering, i.e. they can transmit at the same time. This can be as a result of spatial reuse as in figure 1(a), or because they communicate on separate non-overlapping channels, as in 1(b). For our analysis, we assume packet-based transmission for all PUs.

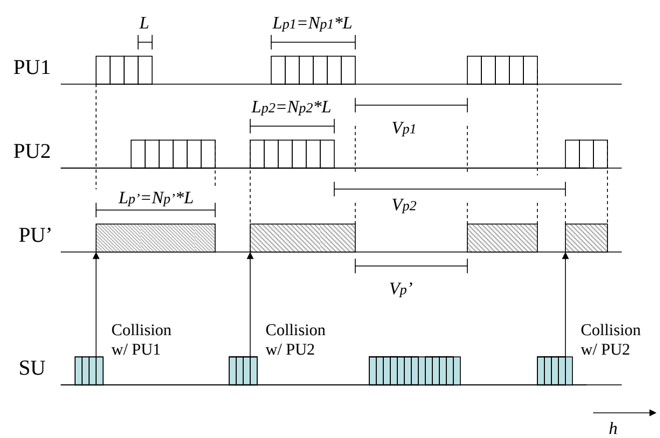


Fig. 2. Timing diagram for PU/SU system model.

Figure 2 illustrates the activity of each user. All PUs are assumed to be working under capacity, such that each PU's idle and active time durations are random and governed by random distributions. The PUs' activity is assumed to be stationary and ergodic. Each PU i has an idle time denoted by V_{pi}, along with an idle probability distribution function of $f_{V_{pi}}(\cdot)$, and cumulative distribution function of $F_{V_{pi}}(\cdot)$. Let $E[V_{pi}] = v_{pi}$.

The number of packets transmitted per active period is also random and denoted as N_{pi}, with $E[N_{pi}] = n_{pi}$ and packet length L for all users. We define an active interval of the ith PU as $L_{pi} = N_{pi}L$, with $E[L_{pi}] = l_{pi} = n_{pi}L$. We can then calculate the idle probability of each PU as $\alpha_i = \frac{v_{pi}}{v_{pi}+l_{pi}}$.

We denote the mean idle and active times of the union of the PU activity as v'_p and $l'_p = n'_p L$. We note that n'_p is the mean number of L length durations in an average active period of the unionized user as shown in figure 2. The idle probability of the unionized activity is then $\alpha' = \frac{v'_p}{v'_p + l'_p}$. The relationship between the individual PUs' activity and their union will be investigated in greater detail in section IV.

Each PU i also has a packet collision probability requirement denoted η_i, defined as the probability of SU collision with the ith PU. Over a time interval $[0, T]$, we denote the number of packets transmitted by the ith PU as $\mathcal{N}_{pi}$, and the number of collisions experienced by that user as $\mathcal{N}_{ci}$. The probability of one of the ith PUs' packets experiencing collision is denoted:

$$p_{pi}^c = Pr[\text{packet collision of } i\text{th PU}]$$

where:

$$p_{pi}^c = \lim_{T\to\infty} \frac{N_{ci}}{N_{pi}}$$

and the PU protection requirement is thus:

$$p_{pi}^c \le \eta_i, \quad \forall i \tag{1}$$

We assume that the collision constraints for all PUs are known to the SUs *a priori*. These constraints must be satisfied for all PUs regardless of the number of SUs wishing to access the channel.

B. Secondary User Transmission

The access behavior of SUs is similar to [6], with the key difference being that the SU must protect multiple heterogeneous PUs. A synopsis of the model in [6] is as follows:

- *Packet Length:* The SU packet length is denoted Δ, and we assume $\Delta \ll v_{pi}$, $\Delta \ll L_{pi}$, and $\Delta \le L$.
- *Sensing:* We assume perfect sensing by the SU, such that any time an SU is sensing the channel while any PU is transmitting, the SU will always detect the presence of the PU and stay idle. We assume that SU sensing follows the listen-before-talk (LBT) principle, where the SU senses the channel before allowing transmission.
- *Collision Detection:* Whenever an SU and PU transmit simultaneously, we assume that both experience collision and the SU can detect the collision. Because we have assumed that $\Delta \le L$, we are always able to detect collision if a PU arrives during an SU transmission. Perfect sensing combined with LBT ensures that packet collisions occur *only* if a PU accesses the channel while an SU is already transmitting. *This ensures that for any single PU active period, at most each PU will only experience one packet collision.* Some typical collisions with a PU are shown in Figure 2.
- *Knowledge of Individual PUs:* We assume that the SU has knowledge of the idle/active distributions of each individual PU *a priori*, i.e. $f_{V_{pi}}$, n_{pi}, η_i for all PUs, indexed by i.
- *Performance Metric:* The SU's performance metric is the *time capacity*, the percentage of time that the SU can transmit successfully under the collision probability constraint. This metric is defined as below:

$$C_s = \lim_{T\to\infty} \frac{\text{SU's successful access time in } [0, T]}{T} \tag{2}$$

Since the channel observed by the SU has idle probability α', clearly $C_s \le \alpha'$

IV. Optimal Policy for Single-PU System

In [6], the authors determine an optimal SU transmission strategy in a single PU system with exponential idle time distribution. Specifically they showed that the SU's optimal transmission policy depends on how long the PU has been idle for the current idle period. Let t be the amount of time that has elapsed since the beginning of the latest idle period, and $\Phi(t)$ be the channel state at time t:

$$\Phi(t) \in \{Idle, Busy\}. \tag{3}$$

The authors also define a time-related decision metric

$$g(t) = \frac{1 - F_{V_p}(t)}{f_{V_p}(t)}. \tag{4}$$

This can be considered the inverse of the likelihood of the conditional collision probability if the SU transmits at t.

The optimal transmission policy $q^*(t)$ is defined as the maximum probability that the SU can transmit at a particular instance t in the current idle period. For a single PU system with general idle time distribution:

$$q^*(t) = \begin{cases} 1, & \text{if} \quad g(t) > \gamma^*, \Phi(t) = Idle \\ p^*, & \text{if} \quad g(t) = \gamma^*, \Phi(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where γ^* and p^* are determined from the collision probability requirement:

$$\int_{\tau: g(\tau) > \gamma^*} f_{V_p}(\tau) d\tau + p^* \int_{\tau: g(\tau) = \gamma^*} f_{V_p}(\tau) d\tau = \min(n_p \eta, 1). \tag{6}$$

It is further shown that when $g(t)$ is monotonically decreasing over t, the optimal policy reduces to:

$$q^*(t) = \begin{cases} 1, & \text{if} \quad t < T^*, \Phi(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

where T^* is determined by $\int_0^{T^*} f_{V_p}(\tau) d\tau = \min(n_p \eta, 1)$.

When the PU has exponentially distributed idle time, $g(t)$ is constant for all t, resulting in an optimal transmission policy:

$$q^*(t) = \begin{cases} p^*, & \text{if } \Phi(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{8}$$

where p^* is the optimal probability of transmission that can be achieved without violating the collision constraint. From (6), $p^* = n_p \eta$, where n_p is the average number of packets per transmission and η is the packet collision constraint. Notice that p^* is the probability that a single collision occurs during an idle/busy cycle of the PU's activity without violating the PU packet collision constraint.

It was also proven that as SU packet length Δ decreases, throughput increases. As $\Delta \to 0$, with a given SU transmission policy q, the time capacity equation $C_s(q)$ can be written as:

$$C_s(q) = \frac{\int_0^\infty f_{V_p}(t) \cdot \int_0^t q(\tau) d\tau dt}{v_p + l_p} = \frac{G_s(q)}{v_p + l_p}, \quad (9)$$

where $G_s(q) = \int_0^\infty f_{V_p}(t) \cdot \int_0^t q(\tau) d\tau dt$. For the exponential case, $C_s = \alpha p^*$.

V. Optimal Transmission Policy for Heterogeneous PUs with Exponential Idle Times

In this section, we determine the optimal transmission policy in an environment with heterogeneous, non-interfering PUs. We assume that all PUs have exponential idle and general busy time distributions, and that the SU knows the channel statistics and collision constraints of each individual PU. We first discuss this model for two PUs, and then extend it to M users. Using these results we then investigate the channel selection problem.

A. 2 PUs with Exponential Idle Times

Our goal for the 2 user case is to determine the SU transmission policy that maximizes throughput while ensuring that the packet collision probabilities of PU1 and PU2 satisfy their collision constraints.

The objective function is as follows:

$$\begin{aligned} \max_{q(t):0 \le q(t) \le 1} \quad & C_s(q) \\ \text{subject to} \quad & p^c_{pi}(q) \le \eta_i, \quad i \in \{1, 2\} \end{aligned} \quad (10)$$

where $p^c_{pi}(q)$ is the packet collision probability of PU i under policy $q(t)$.

Since we are now dealing with the union of two PUs activities, with idle distribution $f_{V'_p}(\cdot)$ and average active time l'_p, we rewrite equation (9) as:

$$C_s(q) = \frac{\int_0^\infty f_{V'_p}(t) \cdot \int_0^t q(\tau) d\tau dt}{v'_p + l'_p} \quad (11)$$

We define $\Phi_1(t)$, $\Phi_2(t)$ as the activity indicator for PU1 and PU2, and $\Phi'(t)$ as the indicator for activity observed by the SU, such that:

$$\Phi'(t) = \begin{cases} Busy, & \text{if } \Phi_1(t) \text{ or } \Phi_2(t) = Busy \\ Idle, & \text{otherwise,} \end{cases} \quad (12)$$

As defined previously, the two primary users have idle times V_{p1} and V_{p2}, with means v_{p1} and v_{p2}. They are assumed to have general active time distributions, with n_{pi} being the average number of packets/transmission for user i, and both have packet length L. We note that this can easily be generalized for non-equal packet lengths, or even for non-packetized transmission situations.

We first make two observations that aide analysis:

Observation 1: The idle time distribution of the union of the PUs' activity is also exponential. As shown in figure 3, if the SU senses an idle channel at some time t_0, because of the memoryless nature of the exponential distribution, the time until the channel becomes busy again is $V'_p = \min(V_{p1}, V_{p2})$. Since both V_{p1}, V_{p2} are exponentially distributed, it follows that V'_p is exponentially distributed with rate parameter $1/v'_p$:

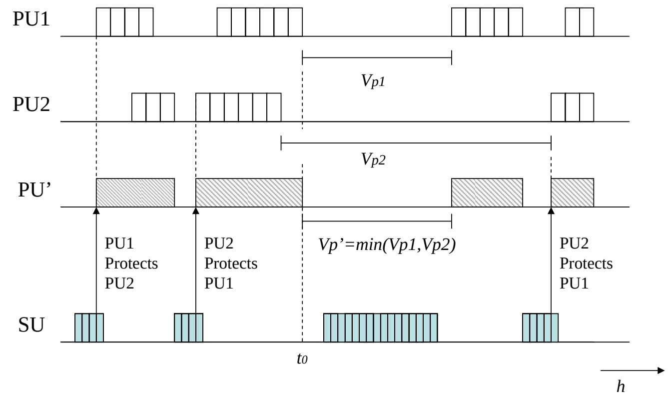


Fig. 3. Collision Protection with 2 PUs.

$$\frac{1}{v'_p} = \frac{1}{v_{p1}} + \frac{1}{v_{p2}}. \quad (13)$$

Observation 2: Since the PUs are independent of each other, the idle probability of the channel is:

$$\alpha' = \alpha_1 \alpha_2. \quad (14)$$

From equations (13) and (14), we can derive an expression for n'_p:

$$\begin{aligned} &\alpha' = \alpha_1 \alpha_2 \\ &\Longrightarrow \frac{v'_p}{v'_p + n'_p L} = \frac{v_{p1}}{v_{p1} + n_{p1} L} \cdot \frac{v_{p2}}{v_{p2} + n_{p2} L} \\ &\Longrightarrow n'_p = v'_p\left(\frac{n_{p1}}{v_{p1}} + \frac{n_{p2}}{v_{p2}} + \frac{n_{p1} n_{p2}}{v_{p1} v_{p2}} L\right) \end{aligned} \quad (15)$$

Since the union of the two PUs results in an activity pattern with exponential idle time, we know that the optimal transmission policy will be:

$$q^*(t) = \begin{cases} p^*, & \text{if } \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \quad (16)$$

Therefore the remaining question is to determine p^* that satisfies (1) for each PU. Here we present an intuitive explanation; a rigorous derivation of p^* can be found in Appendix A.

Notice that p^* is the probability that a single collision occurs during any single idle/busy cycle of the unionized activity. With 2 PUs, a collision with PU i only occurs if the other PU is idle. This means that:

$$\begin{aligned} \alpha_2 p^* &\le n_{p1} \eta_1, \\ \alpha_1 p^* &\le n_{p2} \eta_2. \end{aligned} \quad (17)$$

It follows that p^* can be written:

$$p^* \le \min\left(\frac{n_{p1}\eta_1}{\alpha_2}, \frac{n_{p2}\eta_2}{\alpha_1}, 1\right) \quad (18)$$

Given this result for p^*, we obtain the following theorem.

Theorem 1. The maximum time capacity of the SU in a channel with the unionized activity of two PUs is:

$$C_s = \alpha' p^* = \min(C_{s1}, C_{s2}, \alpha') \quad i \in \{1, 2\} \tag{19}$$

where $C_{si} = \alpha_i n_{pi} \eta_i$, which is the maximum time capacity that can be achieved in the single-PU system consisting of the ith PU.

Proof: The result follows directly from equations (18), (14) and time capacity equation $C_s = \alpha' p^*$. ■

Figure 3 lends intuition to this optimistic result; we see that the PUs mutually protect each other from collision. This protection reduces the collision probability of each PU by a factor that equals the other PU's busy probability, as seen in equation (17), allowing p^* to be increased proportionally. This offsets the lower channel availability caused by the presence of multiple PUs. In other words, the channel availability is reduced but the SU can transmit more aggressively.

We note that in some cases, both PUs will be protected to the point that their collision probability constraint *cannot* be violated. In this case the SU transmits with probability 1 during idle periods, and the time capacity is limited by the idle probability α' of the channel.

B. M PUs with Exponential Idle Times

We again assume the SU has perfect knowledge of the M PUs' mean idle/busy times v_{pi}, n_{pi}, $i \in \{1, ...M\}$, where the PUs have exponentially distributed idle time and generally distributed busy time, and that all PUs transmit using the same packet length L. We also assume the SU knows the corresponding η_i for all each PU i. Observation 1 still holds in this case with $V'_p = \min(V_{pi})$, $i \in \{1...M\}$, so the optimal transmission probability follows (16), where $\Phi'(t) = Idle$ when all M users are idle.

We can rewrite (13) and (14) as follows:

$$\frac{1}{v'_p} = \sum_{i=1}^{M} \frac{1}{v_{pi}}, \tag{20}$$

$$\alpha' = \prod_{i=1}^{M} \alpha_i. \tag{21}$$

Derivation of n'_p is analogous to (15), but with new α':

$$n'_p = \frac{v'_p}{L}[(\prod_{i=1}^{M} \frac{1}{\alpha_i}) - 1] \tag{22}$$

With M PUs, the SU collides with PU i only if the other $M-1$ PUs are idle. Therefore:

$$\prod_{j=1}^{M} \alpha_j p^* \leq n_{pi}\eta_i, \quad \forall i, \ j \neq i. \tag{23}$$

As a result, p^* follows the same format as (18), with

$$p^* = \min_{i \in \{1...M\}} (\frac{\alpha_i}{\alpha'} n_{pi}\eta_i, 1). \tag{24}$$

Corollary: The time capacity for M PUs follows Theorem 1:

$$C_s = \min_{i \in \{1...M\}} (\alpha_i n_{pi} \eta_i, 1). \tag{25}$$

Proof: The proof follows directly from Theorem 1. ■

Since each PU in the system reduces the α', with larger M there is less chance of actually being limited by a PU instead of the idle probability. However, the reduction of each PU's collision probability due to protection still holds, and is stronger due to the addition of more and more PUs.

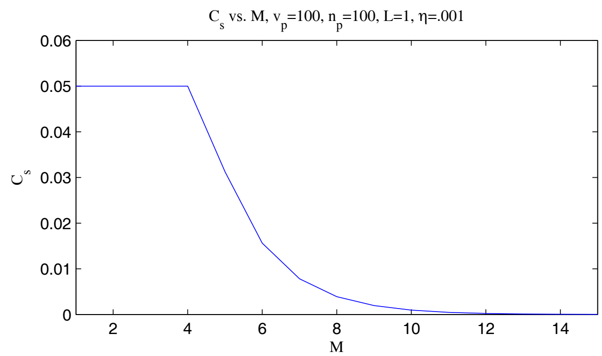


Fig. 4. C_s vs. M Homogenous Users ($v_p = 100, n_{p1} = 100, L = 1$)

In figure (4), we can see this relationship between the SU's time capacity C_s and the number of PUs. We consider homogenous PUs for simplicity. We can see that for up to 4 users, C_s stays constant. This is because the SU can increase p^* without reaching one. We emphasize that C_s is the *percentage* of time that the SU can transmit. With 4 users or more, however, α' becomes the limiting factor (i.e. $p^* = 1$ for $M \geq 5$).

C. Channel Selection

We now investigate the implications of our results on channel selection. Consider a scenario where the SU can choose from a set of licensed independent channels. The SU's performance may benefit by choosing and operating in several channels simultaneously.

Theorem 1 provides insights on how to select channels to maximize the total throughput. That is if an SU can choose a set of PU channels to access simultaneously with the goal of maximizing throughput, it should choose PU's with similar single user capacities. However, attention must also be paid to the idle probabilities as well, since as the number of users grows, the significance of the channel idle probability α' grows also. Thus, we see that channel selection must consider the interaction between the single-PU time capacities C_{si} and their idle probabilities α_i.

To investigate these factors' interaction in channel selection, we consider an SU that chooses from a pool of M channels. Each of the M channels is licensed to a PU with exponential idle time, with v_{pi} and n_{pi} as described before, and we assume that the PUs are independent of each other. Each channel has bandwidth of B_0, and gaussian noise with distribution $\mathcal{N}(0, \sigma^2)$. We define the performance metric R_s of our channel selection algorithm as the average throughput, which is the product of the time capacity C_s and the Shannon capacity of the channels selected:

$$R_s = C_s(\mathcal{S})|\mathcal{S}|B_0 log_2(1 + \frac{P}{|\mathcal{S}|B_0\sigma^2}), \quad \mathcal{S} \subseteq \{1...M\}. \tag{26}$$

Our objective function is:

$$\mathcal{S} = \arg\max_{\mathcal{S}\subseteq\{1...M\}} C_s(\mathcal{S})|\mathcal{S}|B_0 log_2(1 + \frac{P}{|\mathcal{S}|B_0\sigma^2}), \tag{27}$$

where $C_s(\mathcal{S})$ is the time capacity of the channels in subset $\mathcal{S}$.

Because of the interaction between time capacity and α', one may need exhaustive search to consider all possible combinations. The complexity of exhaustive search is exponential, so we consider a suboptimal heuristic algorithm. We define a myopic algorithm that greedily takes the channels with highest C_{si} until throughput R_s begins to decrease. We define the algorithm as follows:

1. The SU orders the M channels by their individual capacities. Therefore:

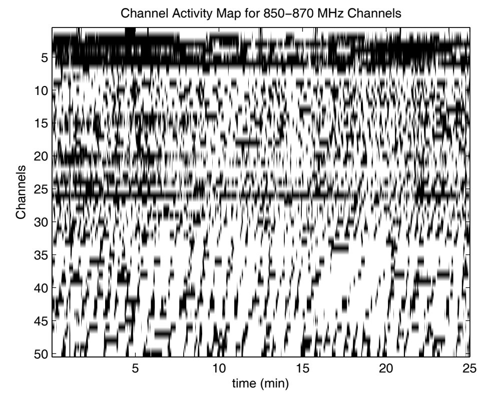
$$\alpha_1 n_{p1}\eta_1 \geq \alpha_2 n_{p2}\eta_2 \geq \ldots \alpha_M n_{pM}\eta_M.$$

2. The SU adds the channels in that order to its pool until the R_s value obtained is less than the previous.

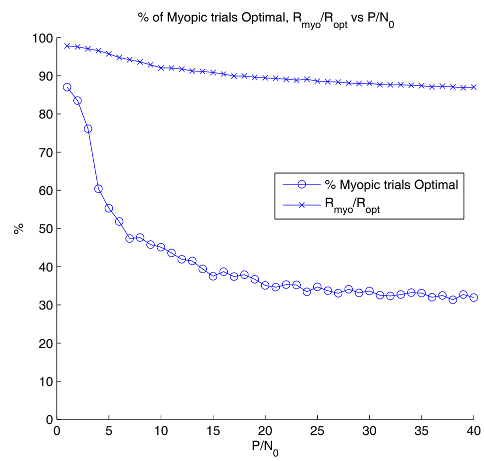
Numerical Comparison

To generate our numerical results, we use statistics collected from real trace data taken in the 850-870MHz band. The spectral data is collected in 0.01s snapshots, with a DFT frequency resolution of 8.333kHz, taken over a duration of 100 minutes. Within the measured band, we select 50 channels with different idle probabilities, shown in Figure 5(a), where black ticks represent PU activity and white ticks represent idle time. Due to very high power transmitters used, PU activity can be determined by the application of a simple energy threshold method, with low-level processing to eliminate false alarms caused by noise. We note that each of the channels only has a bandwidth of 25kHz; these channels are mainly of interest because they exhibit a large number of on/off cycles, and statistical heterogeneity.

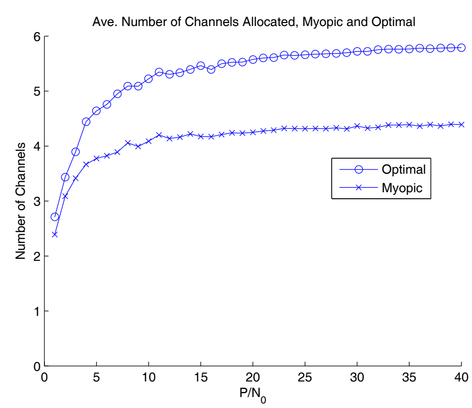
In our simulations, we run 10,000 iterations of the greedy algorithm over varying levels of power P. For each iteration, we randomly select 10 of the 50 channels as our SU channel pool, since an exhaustive search for the optimal allocation takes exponential computation time. For these 10 channels, α_i, v_{pi} and n_{pi} are obtained from the measured data. The v_{pi}s for all 50 channels range from 1 to 90s, and the n_{pi}s range from 1.5 to 12.8s. We use the time resolution as the PU packet length, so $L = .01s$. We set $\eta_i = .001$ for all PUs, $B_0 = 1$ for all channels, and assume that they are AWGN channels with noise density of 1. The optimal channel set $\mathcal{S}$ and capacity is found by exhaustive search using equation (27).



(a) Channel activity maps from 850-870MHz trace data



(b) % of trials optimal, R_{myo}/R_{opt} vs. P



(c) Average number of channels selected by SU vs. P.

Fig. 5. SU in environments with multiple non-interfering PUs.

TABLE I
NOTATION FOR 2-PU SYSTEM

$f_{V_{pi}}(t)$	PDF of PU i idle time
$F_{V_{pi}}(t)$	CDF of PU i idle time
$f^r_{V_{pi}}(t)$	PDF of PU i residual idle time
$F^r_{V_{pi}}(t)$	CDF of PU i residual idle time
$f'_{V_{pi}}(t)$	PDF of PU1$\bigcup$PU2 idle time
$F'_{V_{pi}}(t)$	CDF of PU1$\bigcup$PU2 idle time
α_i	Probability PU i idle
α'	Probability all PUs idle
p_{si}	Probability PU i starts the idle period
$f_{si}(t)$	Idle time PDF given PU i starts the current idle period
$h_i(t)$	Probability that PU i ends current idle period at time t
$\phi_{ij}(t)$	Probability that PU j ends current idle period given PU i starts
$g_i(t)$	Time decision metric for PU i

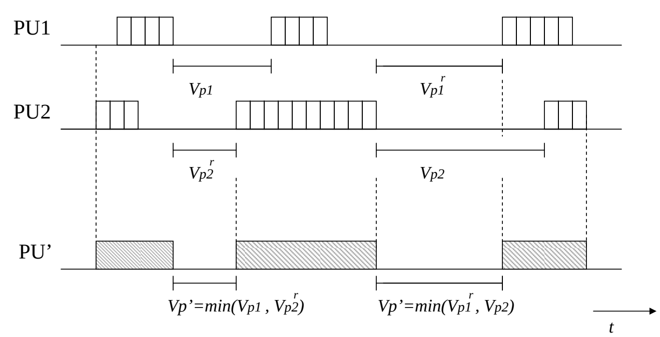


Fig. 6. Timing diagram showing two types of idle periods.

Figure 5 compares the performance of the myopic algorithm with the optimal channel selection over power P. In figure 5(b), when P is low, there is very little difference between the myopic and optimal choices. This is because power is the limiting factor in the throughput in the low-SNR regime. However, as power goes up, the greedy algorithm is much less likely to result in the optimal throughput values. In this case, the idle time probability α' shrinks very quickly as the number of channels goes up, and therefore the time capacity limits R_s. However, the throughput difference between the suboptimal and optimal selections seem quite close even in the high SNR regime. For example, at an SNR of 40, the throughput achieved by the myopic algorithm is only $\tilde{1}3\%$ less than the optimal throughput on average. This implies that the myopic algorithm achieves fairly strong rate performance, although it should be noted that the difference between these two algorithms can be large in degenerate cases.

Figure 5(c) shows that the average number of channels that leads to the optimal capacity also increases as P increases. However, because of the limits of the channel idle probabilities, the number of channels begins to level off for higher P. We also see that the myopic algorithm results in consistently less channels being allocated, particularly as power increases.

This result implies that channel selection is more than a matter of choosing the channels with best time capacities at any time. With higher power, the idle probability of the channels becomes a limiting factor of performance, and therefore must be considered quite carefully when designing channel selection schemes.

VI. OPTIMAL TRANSMISSION POLICY FOR PUS WITH GENERAL IDLE TIME DISTRIBUTIONS

We now derive the optimal transmission strategy for two PUs with general idle time distributions under certain conditions. Table I lists all notations used in the derivation.

A. Framework

In the 2 PU case, we first notice that the idle time distribution depends on which PU finishes transmitting last before a given idle period. Figure (6) demonstrates these two situations. When the ith PU's transmission is the most recent to end before a given idle period, we say that PU i *starts* the idle period that follows. It follows that the unionized PU idle time distribution can be rewritten:

$$f_{V'_p}(t) = p_{s1} f_{s1}(t) + p_{s2} f_{s2}(t) \tag{28}$$

where p_{si} is the probability that an idle time starts with PU i, and $f_{si}(t)$ is the PDF of the unionized PU's idle time given that the idle period starts with PU i.

To determine p_{si}, the activity of the PUs is observed over time period T, where T is the length of the first N' idle/busy cycles of the unionized PU. I'_k, B'_k, I^i_k, B^i_k are the kth idle and busy periods of PUi and the unionized user. We assume that every cycle begins with an idle period for ease of analysis. Therefore:

$$\begin{aligned} T &= \sum_{k=1}^{N'} I'_k + B'_k \\ &= (\sum_{m=1}^{N_i} I^i_m + B^i_m) + R_i \quad i \in \{1, 2\} \end{aligned} \tag{29}$$

where $R_i < I^1_{N_i+1} + B^1_{N_i+1}$, and in general $N_1 \neq N_2 \neq N'$.

Since the PU must end a transmission for every idle/busy cycle, each PU ends N_i transmissions during T. An idle period results if the other user is idle during one of these events. Defining $\mathcal{N}_{si}$ as the number of idle periods started by PU i, we know that over long interval T, $\mathcal{N}_{s1} \to \alpha_2 N_1$, $\mathcal{N}_{s2} \to \alpha_1 N_2$, and their sum is the total number of idle periods of PU' over $[0, T]$. We can now derive p_{s1}:

$$
\begin{aligned}
p_{s1} &= \lim_{T\to\infty} \frac{\mathcal{N}_{s1}}{\mathcal{N}_{s1}+\mathcal{N}_{s2}} \\
&= \lim_{T\to\infty} \frac{\frac{\mathcal{N}_{s1}}{N_1}N_1}{\frac{\mathcal{N}_{s1}}{N_1}N_1 + \frac{\mathcal{N}_{s2}}{N_2}N_2} \\
&= \lim_{T\to\infty} \frac{\frac{\mathcal{N}_{s1}}{N_1}\frac{T}{\frac{1}{N_1}\sum_{m=1}^{N_1} I_m^1+B_m^1+R_1}}{\frac{\mathcal{N}_{s1}}{N_1}\frac{T}{\frac{1}{N_1}\sum_{m=1}^{N_1} I_m^1+B_m^1+R_1} + \frac{\mathcal{N}_{s2}}{N_2}\frac{T}{\frac{1}{N_2}\sum_{m=1}^{N_2} I_m^2+B_m^2+R_2}} \\
&= \lim_{T\to\infty} \frac{\frac{\mathcal{N}_{s1}}{N_1}\frac{T}{v_{p1}+l_{p1}}}{\frac{\mathcal{N}_{s1}}{N_1}\frac{T}{v_{p1}+l_{p1}} + \frac{\mathcal{N}_{s2}}{N_2}\frac{T}{v_{p2}+l_{p2}}} \\
&= \frac{\alpha_2 \frac{1}{v_{p1}+l_{p1}}}{\alpha_2\frac{1}{v_{p1}+l_{p1}} + \alpha_1\frac{1}{v_{p2}+l_{p2}}} \\
&= \frac{v_{p2}}{v_{p1}+v_{p2}}
\end{aligned} \tag{30}
$$

where the 4th step results from the law of large numbers. Similarly, p_{s2} is derived as:

$$p_{s2} = \frac{v_{p1}}{v_{p1}+v_{p2}} \tag{31}$$

To determine $f_{s1}(t)$, $f_{s2}(t)$, from figure (6) we can see that when PU 1 starts the idle period, $V_p' = \min(V_{p1}, V_{p2}^r)$, where V_{p2}^r is a random variable denoting a residual idle period of PU 2. When PU2 starts, $V_p' = \min(V_{p1}^r, V_{p2})$. To determine the unionized idle period distribution, we must obtain the residual idle time distribution of each PU, which can be written as:

$$f_{V_{pi}^r}(t) = \frac{1-F_{V_{pi}}(t)}{E[V_{pi}]} = \frac{1-F_{V_{pi}}(t)}{v_{pi}} \tag{32}$$

It follows that:

$$f_{s1}(t) = f_{V_{p1}}(t)[1-F_{V_{p2}^r}(t)] + f_{V_{p2}^r}(t)[1-F_{V_{p1}}(t)], \tag{33}$$

$$f_{s2}(t) = f_{V_{p1}^r}(t)[1-F_{V_{p2}}(t)] + f_{V_{p2}}(t)[1-F_{V_{p1}^r}(t)]. \tag{34}$$

Both (33) and (34) can be separated into two functions that represent which PU begins transmitting first, terminating the idle period. We say that this PU *ends* the current idle period. We define a set of functions $\phi_{ij}(t)$ as the portion of the idle time distribution that represents the jth user ending the current idle time period given the ith user starts the idle period:

$$
\begin{aligned}
\phi_{11}(t) &= f_{V_{p1}}(t)[1-F_{V_{p2}^r}(t)] \\
\phi_{12}(t) &= f_{V_{p2}^r}(t)[1-F_{V_{p1}}(t)] \\
\phi_{21}(t) &= f_{V_{p1}^r}(t)[1-F_{V_{p2}}(t)] \\
\phi_{22}(t) &= f_{V_{p2}}(t)[1-F_{V_{p1}^r}(t)].
\end{aligned} \tag{35}
$$

We can then rewrite (28) as follows:

$$
\begin{aligned}
f_{V_p'}(t) &= p_{s1}f_{s1}(t) + p_{s2}f_{s2}(t) \\
&= p_{s1}\{f_{V_{p1}}(t)[1-F_{V_{p2}^r}(t)] + f_{V_{p2}^r}(t)[1-F_{V_{p1}}(t)]\} \\
&+ p_{s2}\{f_{V_{p1}^r}(t)[1-F_{V_{p2}}(t)] + f_{V_{p2}}(t)[1-F_{V_{p1}^r}(t)]\} \\
&= p_{s1}[\phi_{11}(t)+\phi_{12}(t)] + p_{s2}[\phi_{21}(t)+\phi_{22}(t)].
\end{aligned} \tag{36}
$$

B. Optimal Policy in a 2-PU System

The derivation of an optimal policy for 2 PUs is based on the single PU case. In this paper we focus on distributions that result in T^* optimal policies. We start by rewriting (36) as:

$$
\begin{aligned}
f_{V_p'}(t) &= p_{s1}[\phi_{11}(t)+\phi_{12}(t)] + p_{s2}[\phi_{21}(t)+\phi_{22}(t)] \\
&= h_1(t) + h_2(t),
\end{aligned} \tag{37}
$$

where $h_i(t)$ represents the probability that the ith user ends the current idle period at time t. They are defined as:

$$h_1(t) = p_{s1}\phi_{11}(t) + p_{s2}\phi_{21}(t), \tag{38}$$

$$h_2(t) = p_{s1}\phi_{12}(t) + p_{s2}\phi_{22}(t). \tag{39}$$

We then define two decision metrics $g_1(t)$, and $g_2(t)$ as:

$$g_1(t) = \frac{1-F_{V_p'}(t)}{h_1(t)}, \tag{40}$$

$$g_2(t) = \frac{1-F_{V_p'}(t)}{h_2(t)}. \tag{41}$$

Similar to the original metric, $g_1(t)$ and $g_2(t)$ are the inverse of the likelihood of the conditional collision probability with user i given that the SU has transmitted at time t. Given these equations, we present the following theorem.

Theorem 2. To satisfy the collision probability constraint of PU i in a 2-PU system, an optimal transmission policy follows the form:

$$q^*(t) = \begin{cases} 1, & \text{if} \quad g_i(t) > \gamma_i^*, \Phi'(t) = Idle \\ p_i^*, & \text{if} \quad g_1(t) = \gamma_i^*, \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{42}$$

with γ_i^* and p_i^* are determined from:

$$
\begin{aligned}
&\int_{\tau: g_i(\tau)>\gamma_i^*} h_i(\tau)d\tau + p_i^* \int_{\tau: g_i(\tau)=\gamma_i^*} h_i(\tau)d\tau \\
&\qquad = \min(n_{pi}\eta_i \frac{v_p'+l_p'}{v_{pi}+l_{pi}}, 1).
\end{aligned} \tag{43}
$$

Proof: We first note that (43) is a modified version of (6). Since we are only interested in protecting PU i, the full idle time distribution f_{V_p} in (6) is replaced with $h_i(t)$. The idle/busy multiplier on the right side of (43) is necessary because the integral calculates the PU i collision probability under policy $q^*(t)$ for an average PU' cycle , whereas $n_{pi}\eta_i$ is the collision requirement for an average cycle of PU i.

Recalling (9), we now prove that the policy $q^*(t)$ presented in (42) is optimal over any other policy $q(t)$. The proof is presented in (44), and is similar to that presented in [6] for the optimality of (5) in a single-PU system. ∎

Using Theorem 2, we can now show that an optimal policy for certain combinations of distributions is a time-threshold policy.

$$
\begin{aligned}
&G_s(q^*) - G_s(q) \\
&= \int_0^\infty [q^*(\tau) - q(\tau)] h_i(\tau) g(\tau) d\tau \\
&= \int_{\tau : g_i(\tau) > \gamma_i^*} [q^*(\tau) - q(\tau)] h_i(\tau) g(\tau) d\tau + \int_{\tau : g_i(\tau) < \gamma_i^*} [q^*(\tau) - q(\tau)] h_i(\tau) g(\tau) d\tau + \gamma_i^* \int_{\tau : g_i(\tau) = \gamma_i^*} [q^*(\tau) - q(\tau)] h_i(\tau) d\tau \\
&\geq \gamma_i^* \{ \int_{\tau : g_i(\tau) \geq \gamma_i^*} [q^*(\tau) - q(\tau)] h_i(\tau) d\tau + \int_{\tau : g_i(\tau) < \gamma_i^*} [q^*(\tau) - q(\tau)] h_i(\tau) d\tau \} \\
&= \gamma_i^* \{ \int_0^\infty q^*(\tau) h_i(\tau) d\tau - \int_0^\infty q(\tau) h_i(\tau) d\tau \} \\
&\geq \gamma_i^* (n_{pi}\eta_i - n_{pi}\eta_i) \frac{v'_p + l'_p}{v_{pi} + l_{pi}} = 0.
\end{aligned} \tag{44}
$$

Theorem 3. When $g_1(t)$ and $g_2(t)$ are monotonically decreasing, an optimal transmission policy is a time-threshold based policy of the form:

$$
q^*(t) = \begin{cases} 1, & \text{if} \quad t < T^*, \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{45}
$$

where T^* can be determined using Theorem 2.

Proof: By Theorem 2 above and Corollary 1 in [6], we know that if only PU i is constrained and $g_i(t)$ is monotonically decreasing an optimal policy is a time-threshold policy of the form:

$$
q^*(t) = \begin{cases} 1, & \text{if} \quad t < T_i^*, \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{46}
$$

with threshold T_i^* determined by:

$$
\int_0^{T_i^*} h_i(\tau) d\tau = \min(n_{pi}\eta_i \frac{v'_p + l'_p}{v_{pi} + l_{pi}}, 1). \tag{47}
$$

Assuming now that both PUs are constrained, we obtain T_1^*, T_2^* by applying Theorem 2 to both PUs. Without loss of generality, assume that $T_1^* = \min(T_1^*, T_2^*)$, so that if the SU transmits with the policy defined in (45) with $T^* = T_1^*$, then the collision constraint of PU1 will be satisfied with equality, and PU2 will satisfy its constraint with inequality. Because no other policy can improve the time capacity while satisfying PU1's constraint, we conclude that the T^* policy is optimal, with:

$$
T^* = \min(T_1^*, T_2^*). \tag{48}
$$

■

Under this policy, from (9) the time capacity equation is:

$$
C_s = \frac{\int_0^{T^*} t f_{V'_p}(t) dt + T^* \int_{T^*}^\infty f_{V'_p}(t) dt}{v'_p + l'_p}. \tag{49}
$$

VII. Discussion and Future Work

We discuss the implications of our results on PU protection, and SU policy making when the SU has imperfect information of the PUs in the system.

A. PU Protection

These implications are self-evident. As Theorem 1 shows, with knowledge of the PUs' transmission behaviors, we can ensure the protection requirements of all PUs while maintaining SU time capacity. The SU time capacity is limited by the lowest time capacity under any single PU or the unionized channel availability with M PUs.

In contrast, in Theorem 2 we have shown that without any knowledge of the individual PU's idle/active time distributions, we cannot determine a p^* that can ensure the PU protection requirements. On the other hand, if we are able to determine the v_{pi} and n_{pi} parameters from scrutiny of the unionized active time distribution, we would essentially see the same situation as in Theorem 1.

B. The Imperfect Information Problem

Although we assume the SU has perfect knowledge of the PUs in the system, in practical systems such knowledge is not guaranteed. We plan to investigate this issue comprehensively in future work, and present a motivating example of this problem here.

To that end, consider a 2-PU system in which the only knowledge available to the SU is the minimum protection requirement η' of all PUs, and the packet length L of PU transmissions. We also assume that the SU can obtain the joint statistics v'_p and n'_p of the system through channel observation. Furthermore, we assume both PUs have exponential idle time distributions for simplicity. The idle time still follows observation 1, so an optimal transmission strategy follows (16).

However, without knowledge of each PU's statistics, an optimal policy of the form in (16) is difficult to obtain. For example, a policy that attempts to treat the system as a single PU:

$$
q^*(t) = \begin{cases} a n'_p \eta', & \text{if} \, \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{50}
$$

cannot guarantee that the collision constraint of each PU is satisfied for any value $a > 0$.

To see this, we assume, without loss of generality, that PU1 is the limiting user of the unionized activity:

$$\alpha_1 n_{p1}\eta_1 \le \alpha_2 n_{p2}\eta_2,$$
$$\frac{n_{p1}\eta_1}{\alpha_2} \le 1.$$

Without knowledge of v_{pi}, n_{pi}, we set $p^* = a n'_p \eta'$ to be less than the optimal value assumed above (due to PU1 being the limiting user). Using (15), it can be shown that for:

$$p^* = a n'_p \eta' \le \frac{n_{p1}\eta_1}{\alpha_1} \quad (51)$$

we need

$$a \le \frac{1}{n'_p \eta'} \frac{n_{p1}\eta_1}{\alpha_1} \quad (52)$$

Therefore, without knowledge of n_{p1}, we cannot find an appropriate value for a.

For example, if an SU observes unionized activity with parameters $v'_p = 50$, $n'_p = 150$, $L = 1$, $\eta' = .001$, this can result from two different pairs of PUs: $v_{p1} = v_{p2} = n_{p1} = n_{p2} = 100$, or $v_{p1} = n_{p1} = 67$, $v_{p2} = n_{p2} = 200$, with $\eta_1 = \eta_2 = .001$ in both cases. However, from equation (52) the former case requires a value of $a \le 1.33$, while the latter requires $a \le .89$. In fact, *for a given set of parameters v'_p, n'_p, and α', a unique set of parameters $v_{p1}, n_{p1}, v_{p2}, n_{p2}$ can be found which violates collision constraints for any value of $a > 0$.*

However, we also note that we have not taken into account what information may be learned from the distribution of N'_p. Since it is assumed that the SU can measure the channel's idle/active periods, which then determines v'_p and n'_p, scrutiny of the *distribution* of N'_p may yield close estimates of the distributions of N_{p1}, N_{p2}, which can be used to determine n_{p1}, n_{p2}.

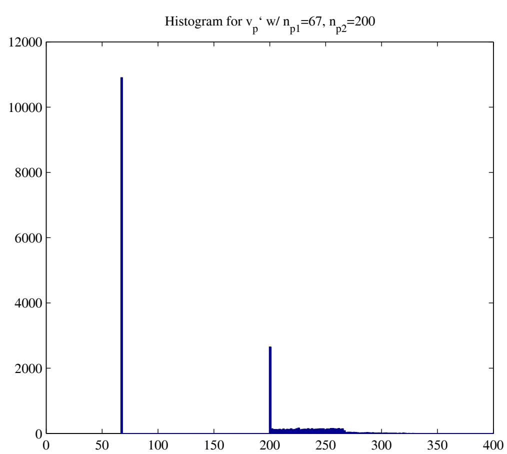


Fig. 7. Histogram of N'_p when busy periods of PUs are constant. ($v_{p1} = n_{p1} = 67, v_{p2} = n_{p2} = 200$, L=1)

For example, consider the case where $v_{p1} = n_{p1} = 67$, $v_{p2} = 200 = n_{p2} = 200$, $L = 1$, $\eta_1 = \eta_2 = .001$, and assume the PUs transmit a constant number of packets in each busy period. The resulting distribution for N'_p is shown in figure 7. It is obvious that $\{n_{p1}, n_{p2}\} = \{67, 200\}$. With equations (13), (15), and (18), one can determine a transmission probability that is non-trivial.

For given values for v'_p, n'_p, n_{p1}, and n_{p2}, the values for v_{p1}, v_{p2} are unique. However, p^* still may not be achieved because we do not know which η_i corresponds to which PU. This means we must use η' for both η_i on the right side of (18), to ensure that both users are sufficiently protected. If the actual p^* does include the larger η_i value, we could end up with a suboptimal transmission probability p.

This example demonstrates the challenges involved with SU policy making. Determining the individual statistics of the PUs in the system is of central importance. This can be difficult due to the actual activity of the PUs as well as the limitations of SU sensing. This is an issue which requires further investigation.

C. How Useful is the T^* Optimal Policy?

The most general optimal policy in a system with multiple PUs with general idle time distributions would be a strong result with many implications in real-world scenarios, which we leave for future study. However, our preliminary work indicates that many combinations of practical distributions meet the conditions for an optimal T^* policy in a 2-PU system. For example, a uniform distribution combined with several other distributions (uniform, exponential, Weibull) always results in a T^* optimal policy. Under certain conditions, exponential-Weibull and Weibull-Weibull combinations also result in this policy. In such cases, computation involved in SU policy-making could be greatly reduced. Therefore, we conclude that this result in itself has strong implications in the design of cognitive radio systems.

VIII. Conclusion

In this paper, we study the effect of multiple PU protection requirements on SU policy making and performance under the assumption of both exponential and general idle time distributions of PUs. For exponential idle times, we address two major issues. First, we find that SU access time is only limited by the minimum access time of each individual PU constraint and the unionized idle probability of the joint channel. When there are multiple PUs, they provide mutual protection to each other. In other words, SUs observe less opportunities, but can transmit more aggressively.

We then present a channel selection problem based on our results where the SU can select channels among a set of independent PU channels to maximize SU throughput while protecting PUs.We consider a simple myopic policy that sorts channels based on time capacity. While the policy is suboptimal, it achieves good performance compared to that of the exhaustive search in the low SNR regime.

For PUs with general idle time distributions, we show that if the time-related decision metric devised is monotonically decreasing for all PUs, then an optimal policy is a time-threshold based policy. We then discuss the implications of our results. In particular, we show that knowledge of the statistics of individual PUs is vital to determine policies which guaranteed sufficient protection of all PUs. This finding provides insights on the requirements of sensing techniques.

References

[1] Shared spectrum company: DARPA XG program information. [Online]. Available: http://www.sharedspectrum.com/technology/darpaxg.html

[2] F. W. Seelig, "A description of the august 2006 xg demonstrations at fort a.p. hill," in *Second IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2007, pp. 1–12.

[3] S. Huang, X. Liu, and Z. Ding, "Opportunistic spectrum access in cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, April*, 2008. [Online]. Available: "http://www.ece.ucdavis.edu/~senhua/OSAinfocom08.pdf"

[4] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Optimal dynamic spectrum access via periodic channel sensing," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007.

[5] S. C. Tao Shu and M. Krunz, "Medium access control for multi-channel parallel transmission in cognitive radio networks," *Proceedings of the IEEE GLOBECOM 2006 Conference, San Francisco, CA, Dec.*, 2006.

[6] S. Huang, X. Liu, and Z. Ding, "Optimization of transmission strategies for opportunistic access in cognitive radio networks," UC Davis, Technical Report, 2008, www.cs.ucdavis.edu/∼liu/preprint/huang_tr.pdf.

[7] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," *Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers, November,*, 2006.

[8] Y. Shi and T. Hou, "A distributed optimization algorithm for multi-hop cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, Apr.*, 2008.

[9] Q. Z. T. Javidi, B. Krishnamachari and M. Liu, "Optimality of myopic sensing in multi-channel opportunistic access," *Proc. of IEEE International Conference on Communications (ICC), May*, 2008.

Appendix A: Derivation of Optimal Transmission Policy in 2 PU System with Exponential Idle Time

Referring to (29), it follows that the number of the ith PU's packets $\mathcal{N}_{pi}$, transmitted in $[0, T]$ is:

$$\mathcal{N}_{pi} = \frac{1}{L}\sum_{k=1}^{N_i} B_k^i + N_{R_i}, \quad i \in \{1, 2\}, \tag{53}$$

where N_{R_i} is the number of packets transmitted in the ith user's remaining time R_i.

We define $\mathcal{N}_c'$ as the number of collisions that occur in $[0, T]$. Since there are N' idle/busy cycles of unionized PU activity, we can rewrite p^* as:

$$p^* = \lim_{T\to\infty} \frac{\mathcal{N}_c'}{N'}. \tag{54}$$

The fraction of collisions the ith PU experiences is:

$$\begin{aligned} &Pr[\text{PU } i \text{ transmits before PU } j|\Phi'(t) = Idle] \\ &= Pr[V_{pi} < V_{pj}] \\ &= \int_0^\infty f_{v_{pi}}(t)(1 - F_{v_{pj}}(t))dt \\ &= \frac{1/v_{pi}}{1/v_{pi} + 1/v_{pj}} = \frac{v_p'}{v_{pi}}. \end{aligned} \tag{55}$$

Using (1) and (53) we can derive packet collision probability p_{pi}^c for ith PU:

$$\begin{aligned} p_{pi}^c &= \lim_{T\to\infty} \frac{N_{ci}}{N_{pi}} \\ &= \lim_{T\to\infty} \frac{\frac{v_p'}{v_{pi}}\frac{N'}{N'}\mathcal{N}_c'}{\frac{1}{L}\sum_{k=1}^{N_i} B_k^i + N_{R_i}} \\ &= \lim_{T\to\infty} \frac{v_p'}{v_{pi}} \frac{\frac{T}{\frac{1}{N'}(\sum_{k=1}^{N'} I_k' + B_k')}\frac{\mathcal{N}_c'}{N'}}{\frac{T\cdot\frac{1}{N_i}\frac{1}{L}(\sum_{k=1}^{N_i} B_k^i + N_{R_i})}{\frac{1}{N_i}(\sum_{k=1}^{N_i} I_k^i + B_k^i + R_i)}} \\ &= \lim_{T\to\infty} \frac{v_p'}{v_{pi}} \frac{\frac{T}{v_p' + n_p' L} p^*}{\frac{T}{v_{pi} + n_{pi}} n_{pi}} \\ &= \frac{v_p'}{v_{pi}} \frac{v_{pi} + n_{pi}}{v_p' + n_p' L} \frac{p^*}{n_{pi}} \\ &= \frac{\alpha'}{\alpha_i n_{pi}} p^* \le \eta_i, \end{aligned} \tag{56}$$

where the third step is due to the law of large numbers, since the number of cycles $N_i, N' \to \infty$ as $T \to \infty$.

With (56), we set $p_{pi}^c = \eta_i$ and solve for p^*:

$$p^* \le \frac{\alpha_i}{\alpha'} n_{pi}\eta_i \tag{57}$$

We therefore conclude that:

$$p^* = \min(\frac{\alpha_1}{\alpha'} n_{p1}\eta_1, \frac{\alpha_2}{\alpha'} n_{p2}\eta_2, 1). \tag{58}$$