Per User Throughput in Large Wireless Networks

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***Abstract*— Previous results show that a node's throughput scales poorly as the network size increases when every node has traffic. However, in many cases, only a fraction of nodes in large networks have data to send or receive at any given time, while other nodes can act as relays/routers. Therefore, in this paper, we study the scaling behavior from a user's viewpoint (a user is a node with traffic). We first derive an upper bound on per user throughput. To derive the lower bound, we propose a simple scheduling scheme that enables users to cooperate with relay nodes and fully utilize the networks capacity. We show that per user throughput depends on the network size, the number of users, and the node deployment schemes, and is in general much more optimistic. Our scheme also sheds light on designing efficient cooperation protocols in heterogeneous networks and cognitive radio networks.**

I. INTRODUCTION

The seminal work of Gupta and Kumar's [1] shows that per node throughput scales as $\Theta(\frac{1}{\sqrt{N \log N}})$ when every node is a source/destination in a randomly deployed wireless networks with N nodes. Following the work, researchers have studied capacity scaling laws under different assumptions or applications, e.g. in [2][3][4][6][8][15][16].

In most previous work, it is either assumed that all nodes are source/destination nodes or there is only one source-destination (s-d) pair. However, it is possible that only a fraction of nodes have traffic at any given time, such as in many applications of wireless sensor networks or mesh networks. Therefore, in this paper, we study the following problem. When there are n s-d pairs in a network of N nodes, what is the capacity of each s-d pair given that all nodes without traffic act as relays/routers, and how to achieve it?

The results in this paper also answer the following question: given n users and m ($m = N - n$) relay nodes, what is the per user capacity[1] with relay node cooperation? In practice, we do not advocate deploying extra relay nodes to enhance the capacity of a wireless network. However, such relays may exist nonetheless. Because wireless networks are being deployed with an increasing density, multiple wireless networks may co-locate in the same geographic area. It is possible for nodes from different networks to cooperate with each other. Such a cooperation is beneficial when networks do not have traffic simultaneously.

Dynamic spectrum access has been considered as an option to mitigate spectrum scarcity and to satisfy the increasing demand for spectrum. In this case, secondary users coexist with primary users and opportunistically access the unused spectrum vacated by idle primaries. It is often assumed that primary users are legacy users and do not cooperate with secondary users. Then, if primary users have heavy traffic, poor performance is expected for secondary users. However, secondary users are often more densely deployed than primary users because of the fast proliferation of wireless devices. In this case, making primary users cooperate with secondary users may result in a significant throughput gain for primary users. This can serve as a strong incentive for primaries to cooperate. With cooperation, secondary users can improve the per user throughput of primaries and achieve much higher throughput themselves. With the advances of radio technologies, cooperation between primary users and secondary users is increasingly feasible. This paper lays a theoretical foundation on designing efficient cooperative protocols between primary and secondary users.

A. Main results

In this paper, we derive upper and lower bounds on the per user capacity in a network of size N with n users. (A user is a node with traffic to send or receive.) The rest m ($m = N - n$) nodes do not have their own traffic and thus serve as relay nodes. The upper bound is a simple extension of existing results on transport capacity. The main contribution of the paper is the achievability result (lower bound). We propose a three-step scheduling scheme for users to fully utilize the network transport capacity. The proposed scheme achieves the upper bound asymptotically in most cases. Achievable per user capacity depends on the network size, the number of users, and the node deployment.

For example, in a random deployed network, on one extreme, when $n = N$, i.e., every node is a source/destination, per user capacity scales as $\Theta(\frac{1}{\sqrt{n \log n}})$. When $n = o(N)$ and $n = \Omega(\sqrt{N})$, per user capacity increases roughly as a squareroot function of the network size. On the other extreme, when n is small (roughly speaking $n = O(\sqrt{N})$), per user capacity is upper bounded by $\Theta(1)$, and lower-bounded by $\Omega(\frac{1}{\log n})$. We show that the gap only exists for a small fraction of users. In other words, an arbitrary large fraction of users can achieve $\Theta(1)$, while others can achieve a throughout $\Omega(\frac{1}{\log n})$ when $n = \Theta(\sqrt{N})$. To guarantee per user capacity of $\Theta(1)$ for all users, we need a much larger network size.

To be more precise, we summarize the results where n users and m ($m = N - n$) relay nodes are randomly deployed. When $m = \Omega(n)$ and $\frac{\sqrt{m}}{\sqrt{\log m}} = O(\frac{n}{\log n})$, per user throughput $\lambda(n)$ is $\Theta(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. When $\frac{\sqrt{m}}{\sqrt{\log m}} = \Omega(\frac{n}{\log n})$, the lower bound of $\lambda(n)$ is $\Omega(\frac{1}{\log n})$, while the upper bound of $\lambda(n)$

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[1] It is used interchangeably with per user throughput.

is $\min\left(\mathrm{O}(1), \mathrm{O}(\frac{1}{n}\sqrt{\frac{m}{\log m}})\right)$. When $m = \Omega(n^{2+\varepsilon})$, $\varepsilon > 0$, given any threshold $P_{th} < 1$, there are at least $P_{th}n$ users that can achieve a throughput of $\Theta(1)$, while all other users can achieve a throughput of $\Omega(\frac{1}{\log n})$.

In this paper, we consider both regularly and randomly deployed nodes. Because the users and relay nodes may come from different networks, we consider four different network deployment schemes. The results in the each case are presented in Section IV.

B. Related work

Following Gupta and Kumar's seminal work, much effort has been made to study capacity bounds of wireless networks. Different models have been considered. For example, in [8], Li et al. investigate the capacity of ad hoc networks with 802.11MAC deployed. A capacity of $\frac{1}{\sqrt{n}}$ per node is verified through extensive simulations. The effect of channel fading is considered in [2], [3] and [4]. Another branch of work focuses on different traffic patterns in wireless networks. In [5], the authors study a many-to-one traffic pattern and show the aggregate network capacity is $\Theta(1)$, when the number of users n goes to infinite. In [6], the authors investigate the multicast capacity of ad hoc networks. The broadcast capacity is studied with the consideration of power constraint in [7].

Because the capacity of wireless networks does not scale well when the number of users increases, much work focuses on finding effective ways to improve the network capacity. Some of them study how physical layer technologies, such as directional antennas [9], UWB [10], and multi-channel multi-interface [11], can improve network capacity.

Infrastructure support has been considered to improve the capacity. In [12], when both nodes and base stations are randomly deployed, per node capacity of $\Theta(\frac{1}{\log n})$ is achievable. However, they only consider the case that the number of BS is on the same order of the number of nodes. In [13], the authors treat the number of BS m as a parameter. They show if m grows slower than $\sqrt{n}$, the capacity scales as $\Theta(\sqrt{\frac{n}{\log\frac{n}{m^2}}})$. Only when base stations are deployed at a rate larger than $\sqrt{n}$, capacity of the ad hoc networks can scale linearly with the number of BS.

In [14], Liu et al. study the bounds on throughput gain achieved by network coding in wireless networks. They show that the benefit of network coding with broadcasting is upper bounded by a constant.

In [15], Grossglauser and Tse show a mobile relaying strategy can improve per user throughput to $\Theta(1)$. Their basic idea is to explore short range communications by distributing a source's packets to many adjacent nodes which serve as relays. With mobility, the probability that at least one relay node gets close to the destination is significant. Meanwhile the one-hop relay strategy ensures that every packet transmitted at most twice. Therefore, per user throughput is improved to $\Theta(1)$. They also show that only exploring mobility but without using relay nodes cannot achieve a throughput of $\Theta(1)$.

In [17], Gastpar and Vetterli also consider the relay node cooperation. They consider a network with only one pair of source and destination, and all the rest nodes act as relays. They show that capacity scales as $\Theta(\log n)$.

C. Roadmap

The rest of paper is organized as follows. In Section II, we describe the system model. In Section III, we present the general idea of our scheduling algorithm that is used to derive low bounds. We then proceed in Section IV to derive the upper and lower bounds under different deployment policies. Discussion is in Section V, followed by conclusions in Section VI.

II. SYSTEM MODEL

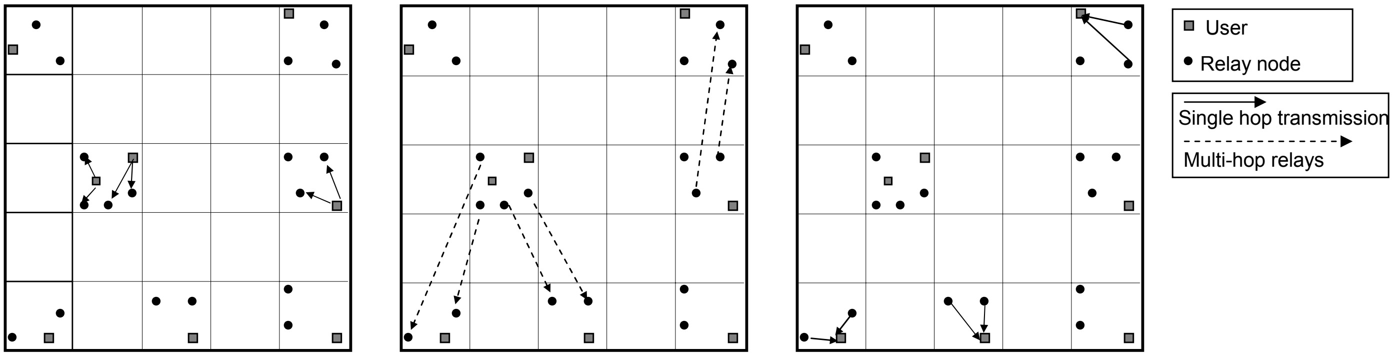
We consider a unit area where all nodes are located. We use n to denote the number of source/destination nodes, which are called users. We use m to denote the number of other nodes that do not have their own traffic and serve as relays. We call them relay nodes. Users and relay nodes can belong to the same or different networks. In the case of a homogeneous network of N nodes, n out of N nodes are randomly chosen to be source/destination nodes. Other nodes serve as relays. We have $m = N - n = \Theta(N)$ since we are interested in the case where $m = \Omega(n)$. In case of overlayed heterogeneous networks, n can be regarded as the number of active (or primary) users and m as the number of inactive (or secondary) nodes serving as relays. We use n and m in the rest of the paper.

We assume all the users and relay nodes share the same spectrum. We use the protocol model proposed in [1] as our interference model. To be more specific, if the distance between a transmitter i and a receiver j is r, the transmission is successful if the distance from all other active transmitters to the receiver j is no less than $(1+\Delta)r$. Here Δ is a parameter that determines the guard zone to prevent interference. We assume that 1 unit of data is transmitted in 1 unit of time if a transmission is successful. In this paper, 1 unit of data is also referred to as a packet.

Definition 1 *(Achievable per user throughput)*: In a wireless network with n users and m relay nodes, per user throughput $\lambda(n)$ (*bit/second*) is achievable if there exists a spatial and temporal cooperation scheduling scheme that every user can send traffic at rate $\lambda(n)$ to its destination almost surely (a.s.).

In Definition 1, almost surely means that the throughput $\lambda(n)$ should be achieved with probability 1 as n goes to infinite. In addition, $\lambda(n)$ is achieved through the cooperation of relay nodes and is a function of both n and m. Definition 1 requires that every user achieve the throughput $\lambda(n)$.

We note that the achievable network transport capacity can be obtained using the per user throughput result easily. In [18], it is shown that for randomly selected source and destination, the average distance between source and destination nodes is $\Theta(1)$ almost surely. This result can also be applied in our paper. Therefore, the aggregate network transport capacity can be obtained by multiplying per user throughput, the number of users n, and average distance $\Theta(1)$, with the unit of *bit-meter/second*. On the other hand, it is not guaranteed that



(a) Step 1 (b) Step 2 (c) Step 3

Fig. 1: An example illustrating the cooperation scheduling algorithm. Each user randomly selects a destination. Every user, either as a source or as a destination, cooperates with the same number of relay nodes.

a user can achieve $\frac{1}{n}$th of the achievable network transport capacity. It is possible that a majority of nodes can achieve it while a small fraction cannot. Details will be discussed in Section IV.

In this paper, relay nodes cooperation simply means that users utilize relay nodes to forward their traffic to destinations. No broadcasting, multiple access, or network coding is considered. From the viewpoint of network layer, using relay nodes reduces single hop transmission range and thus increases spatial reuse. In wireless networks, the number of simultaneous transmissions is limited by interference. Using relay node cooperation can better utilize network resource and thus leads to a higher throughput.

We consider both randomly deployed and regularly deployed networks. Randomly deployed means that nodes are randomly and uniformly deployed in the unit area. Regularly deployed means that nodes are deployed as a regular grid network in the unit area. Because users and relay nodes may belong to the same or different network, we consider the combinations of four possible deployments.

In this paper, $f(n) = \mathrm{O}(g(n))$ if there is a positive constant c such that $f(n) \leq cg(n)$ for n large enough, $f(n) = \Omega(g(n))$ if $g(n) = \mathrm{O}(f(n))$, and $f(n) = \Theta(g(n))$ if both $f(n) = \mathrm{O}(g(n))$ and $f(n) = \Omega(g(n))$ hold. In addition, $f(n) = o(g(n))$ if $\lim\limits_{n\to\infty} f(n)/g(n) = 0$ and $f(n) = \omega(g(n))$ if $g(n) = o(f(n))$.

III. GENERAL IDEA

In this paper, the upper bound of per user throughput is derived using an extension of existing results on the transport capacity. Our main contribution is to design a cooperation scheduling algorithm to derive the low bound of per user capacity. We introduce the general idea of the scheduling algorithm in this section.

A. *General steps of the cooperation scheduling algorithm*

We divide the unit area into a set of squarelets with the same size. Each squarelet has a certain number of users and a set of relay nodes. Every user is a source with one randomly chosen destination. Meanwhile it is also a destination that receives traffic from only one source[2]. A source and its destination may or may not be in the same squarelet. We have the following three steps in the scheduling algorithm.

Step 1: Users send packets to relay nodes that lie in the same squarelet. Each user selects a disjoint set of relay nodes and sends one packet to each of them as shown in Fig. 1(a). Each relay node serves at most one user and thus receives at most one packet[3].

Step 2: A packet is delivered to the squarelet of its destination hop-by-hop by relay nodes as Fig. 1(b) illustrates. The packet arrives at a relay node in the squarelet of the destination user. Each relay node in a destination squarelet has at most one packet (This can be guaranteed by choosing an appropriate number of relay nodes per user and will be discussed later).

Step 3: A relay node with a packet originated from a source sends the packet to the destination, which lies in the same squarelet. There may be multiple relay nodes each of which sends a packet to the destination.

The three steps are the general procedure of our cooperation scheduling algorithm, which will be examined in detail based on the following black box model.

B. *Black box model*

As shown in Fig. 2, we consider relay nodes as a black box that provide users a (larger) transport capacity. It does not matter how relay nodes achieve that. They can explore mobility or use infrastructure support. In our paper, for simplicity, we only consider the static pure ad hoc model as in [1] [18]. Results under other models can be extended based on the proposed scheduling algorithm. Source users input packets into the black box by sending them to relay nodes in the same squarelet. The output of the black box is the packets sent by relay nodes to the destination users in the same squarelet.

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[2] Our scheduling algorithm also works when a source has multiple destinations and a destination serves multiple sources. To avoid confusion, we only consider the case that a source has a destination and a destination serves a source.

[3] We make it be a requirement here for easy explanation. It holds in most cases in this paper. In the case that a smaller number of relay nodes can achieve a larger capacity than users, a relay node can serve multiple users. We will discuss a case that one relay node can serve multiple users in Section IV.C.

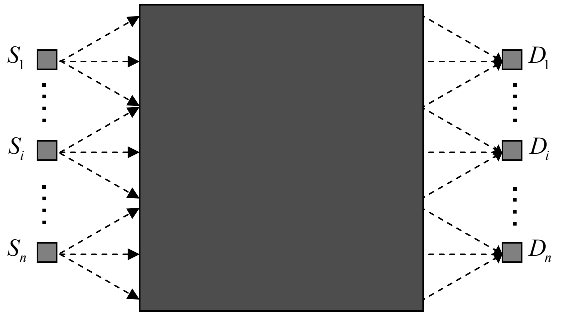

Fig. 2: Illustration of relay nodes cooperation using black box theory

First, let us examine the input process. A source injects a packet to the black box by sending it to a relay node through a one-hop transmission in its squarelet in step 1. To achieve a larger end-to-end capacity, a source user needs to input more packets to the black box. In addition, the input process should evenly distribute packets among relay nodes. Otherwise, a relay node with a large number of packets will become the bottleneck. This is the reason that we let a source user send one packet to each of its relay nodes which serve only one user. The argument also applies to the output process where a destination user receives one packet from each of its relay nodes. We note that a source and its destination should cooperate with the same amount of relay nodes. For example, if a source has a larger pool of relay nodes than its destination, the source should not send a packet to every relay node in the pool. Otherwise the black box cannot send all the packets to the destination.

As in Definition 1, every user should achieve the same throughput. Therefore, in our scheduling algorithm, each s-d pair should cooperate with the same number of relay nodes in both the source's squarelet and the destination's squarelet. Since we consider various deployment schemes of users and relay nodes, in each squarelet, the numbers of users and relay nodes maybe random. Therefore, we need to determine the appropriate squarelet size and the number of relay nodes that can be utilized by each s-d pair. Intuitively, the number of relay nodes that a user can cooperate can be derived by dividing the minimum number of relay nodes by the maximum number of users in a squarelet. We will address this issue under different deployment policies in Section IV.

The capacity of the black box depends on the number of relay nodes. A larger number of nodes result in a higher transport capacity. After a source inputs packets to the black box, more relay nodes result in better spatial reuse and more parallel transmissions of the packets. Although a single packet needs to traverse more hops of transmission by relay nodes, processing multiple packets in parallel will lead to less time for them to arrive at the destination. In the next subsection, we discuss how to determine the time needed in step 1 (input process), step 2 (blackbox), and step 3 (output process) in our scheduling algorithm, respectively.

C. Time needed by the scheduling algorithm

To determine per user throughput, we need to determine how much time is needed by each step in our scheduling algorithm. We divide time into time slots. A time slot is one unit of time during which one unit of data (i.e., a packet) can be transmitted.

In the first step, the number of time slots needed by a squarelet is upper bounded by the product of the number of users and the number of relay nodes per user (same for every user). However, a squarelet cannot use all time slots because it interferes with other squarelets. Therefore we should determine the number of time slots out of which one slot can be scheduled to a squarelet. Consider Fig. 3. Let $l(n)$ denote the edge length of a squarelet. To ensure that a user can transmit to any relay node in its squarelet and vice versa, we choose the transmission radius as $r = \sqrt{2}l(n)$. A successful reception is guaranteed if all other transmitters are at least $(1+\Delta)r$ away from the receiver. Then if there are i squarelets at the vertical or horizontal interval between two squarelets, as long as $i * l(n) \geq (1+\Delta)r$, each of the two squarelets can have a transmission without interfering with the other. Then the two squarelets can be scheduled at the same time slot. Since $r = \sqrt{2}l(n)$, we get the minimum value of the integer i as $i = \lceil\sqrt{2}(1+\Delta)\rceil$. As Fig. 3 shows, the dark squarelets can be scheduled in the same time slot. For a dark squarelet, at most $(i+1)^2 - 1$ squarelets cannot use the same time slot. Therefore, a squarelet can be scheduled every I time slots periodically, where $I = (i+1)^2$. Then the number of time slots needed by all the users in the input process is I times of what is needed by a squarelet. The number of time slots needed by the output process is the same as the input process because a source and its destination cooperates with the same number relay nodes.

Next, we consider the number of time slots needed by

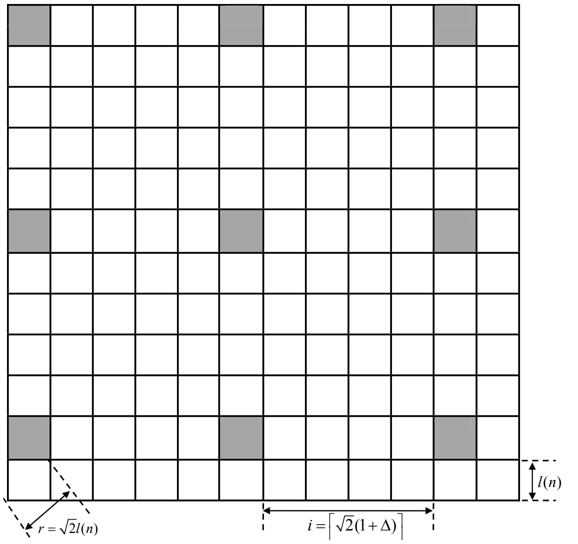

Fig. 3: Squarelets of users, a squarelet can use one out of $I = (i+1)^2$ time slots.

step 2. In our paper, a set of relay nodes serving the same user in a squarelet have to send packets to a set of relay nodes in the squarelet of the destination user. Therefore, in the black box, the source and destination node of a packet are not entirely randomly chosen, which is different from [1]. Fortunately, by the theory of *permutation routing* of processing units (PU) [19][20], which is used in [18] to derive the achievable capacity of wireless networks, no matter how a node determines the destination, only if each source node transmits the same number of packets and each destination receives the same number of packets, the aggregate capacity $\Omega(\sqrt{\frac{m}{\log m}})$ and $\Omega(\sqrt{m})$ hold for uniformly randomly and regularly deployed nodes, respectively. Therefore, we can apply the capacity results in the black box. With the knowledge of the number of packets processed by the black box, we can obtain the number of time slots it needs. Then per user throughput can be derived by dividing the number of packets the user sends to the destination by the sum of time slots needed by the input process, the black box, and the output process.

IV. CAPACITY BOUNDS WITH RELAY NODE COOPERATION

In this section, we derive lower and upper bounds on per user throughput under the following deployment schemes.

- Both users and relay nodes are randomly deployed.
- Users are regularly located and relay nodes are randomly deployed.
- Users are randomly deployed while relay nodes are regularly deployed.
- Both users and relay nodes are regularly deployed.

We note that the first and the last deployment schemes apply to homogeneous networks where a fraction of users have traffic, as well as heterogeneous wireless networks with the same deployment. The second and the third schemes apply to overlayed heterogeneous networks.

A. Randomly deployed users and relay nodes

When users and relay nodes are uniformly and randomly deployed in a unit area, we have the follows results on per user throughput.

Theorem 1: When $m = \Omega(n)$ and $\frac{\sqrt{m}}{\sqrt{\log m}} = \mathrm{O}(\frac{n}{\log n})$, per user throughput $\lambda(n)$ is $\Theta(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. When $\frac{\sqrt{m}}{\sqrt{\log m}} = \Omega(\frac{n}{\log n})$, the lower bound of $\lambda(n)$ is $\Omega(\frac{1}{\log n})$, and the upper bound is $\min\Big(\mathrm{O}(1), \mathrm{O}(\frac{1}{n}\sqrt{\frac{m}{\log m}})\Big)$.

Proof: (**upper bound**) We first derive the upper bound on per user throughput. Based on Gupta and Kumar's results, we know that for n users and m relay nodes, the upper bound of per user throughput is $\mathrm{O}(\frac{1}{n}\sqrt{\frac{n+m}{\log(n+m)}})$. In the case that $m = \Theta(n)$, $\mathrm{O}(\frac{1}{n}\sqrt{\frac{n+m}{\log(n+m)}}) = \mathrm{O}(\sqrt{\frac{1}{n\log n}})$. Therefore, when $m = \Theta(n)$, the capacity gain by using relay nodes is bounded by a constant. We are more interested in the case that $m = \omega(n)$. In this case, the upper bound is $\mathrm{O}(\frac{1}{n}\sqrt{\frac{m}{\log m}})$. Since per user throughput cannot exceed 1, the upper bound of per user throughput is $\min\Big(\mathrm{O}(1), \mathrm{O}(\frac{1}{n}\sqrt{\frac{m}{\log m}})\Big)$. ■

To drive the lower bound on per user throughput, we divide the unit area into a set of squarelets as discussed in Section III. We first divide the unit area into squarelets each of which has a size of $\frac{3\log n}{n}$. We have the following result from [18]:

Result 1: Divide a unit area with n randomly deployed users into squarelets each of which has a size $\frac{3\log n}{n}$. Then no squarelet is empty and the number of users in each squarelet is bounded by $3e\log n$ a.s.

Next, we need to determine the number of relay nodes that each user can cooperate. We have Lemma 1 as follows.

Lemma 1: When $m = \omega(n)$, with a squarelet size of $\frac{3\log n}{n}$, there exists a constant $k^{'} > 0$ such that every user can cooperate with at least $k^{'}\frac{m}{n}$ a.s.

Proof: The probability that a relay node is located in a squarelet is $\frac{3\log n}{n}$. Then the number of relay nodes N_r in a squarelet follows a binomial distribution with the parameter $(\frac{3\log n}{n}, m)$. We need to derive a lower bound on the number of relay nodes, which is denoted as χ. Using Chernoff bound, we have

$$\begin{aligned}
\Pr(N_r < \chi) &= \Pr(-N_r > -\chi)\\
&= \Pr(\exp(-N_r) > \exp(-\chi))\\
&\leq \frac{E(\exp(-N_r))}{\exp(-\chi)}\\
&= \frac{[1 - \frac{3(1-e^{-1})\log n}{n}]^m}{\exp(-\chi)}\\
&\leq \frac{\exp(-3(1-e^{-1})\frac{m\log n}{n})}{\exp(-\chi)}\\
&= \exp(\chi - 3(1-e^{-1})\frac{m\log n}{n}).
\end{aligned} \tag{1}$$

Let $\chi = \frac{3(1-e^{-1})m\log n}{n} - 3\log n$. We have $\Pr(N_r < \chi) < \frac{1}{n^3}$. Since there are totally $\frac{n}{3\log n}$ squarelets, by a simple union bound, the probability that at least one squarelet has less than χ relay nodes is upper bounded by $\frac{1}{n^2\log n}$. Therefore, we have

$$\sum_{n=1}^{\infty} \Pr\{\text{At least a squarelet has less than } \chi \text{ relay nodes}\} < \infty. \tag{2}$$

Use the Borel-Cantelli Theorem, every squarelet has at least $\chi = \frac{3(1-e^{-1})m\log n}{n} - 3\log n$ relay nodes almost surely. Since $m = \omega(n)$, there must exist a constant c such that $\chi = \frac{cm\log n}{n}$. By result 1, there are at most $3e\log n$ users in a squarelet. Therefore, each user can cooperate with at least $\frac{cm}{3en}$ relay nodes. let $k^{'} = \frac{c}{3e}$. Lemma 1 follows. ■

Lemma 1 has two implications: first, every user as a source in a squarelet can have at least $\frac{k^{'}m}{n}$ relay nodes to cooperate. Second, each source user should cooperate with at most $\frac{k^{'}m}{n}$ relay nodes since its randomly selected destination can only guarantee $\frac{k^{'}m}{n}$ relay nodes to cooperate. In others words, at the first step of our scheduling algorithm, each user in a squarelet is allowed to send one packet each of the $\frac{k^{'}m}{n}$ relay nodes in the same squarelet, while these packets are received by $\frac{k^{'}m}{n}$ different relay nodes that are located in the squarelet of the destination. The number of packets injected to the black box is totally $k^{'}m$. With this knowledge, we can derive the lower bound of per user throughput.

Proof of Theorem 1: (**Lower bound**) The number of users in a squarelet is upper bounded by $3e\log n$. Each user cooperates with $\frac{k^{'}m}{n}$ relay nodes. Each squarelet is scheduled once every I time slots. Therefore the upper bound of the number of time slots at step 1 is $I\frac{3ek^{'}m\log n}{n}$. At the second step of the scheduling algorithm, there are $k^{'}m$ relay nodes each with only one packet to send and receive. Recall the achievable capacity of each node in the relay network is $\Theta(\frac{1}{\sqrt{m\log m}})$. In the case there are $k^{'}m$ relay nodes each with one packet where $k^{'}$ is a constant, the achievable capacity of relay network is $\Theta(\frac{\sqrt{m}}{\sqrt{\log m}})$. Therefore the number of time slots is bounded by $k\sqrt{m\log m}$, where $k > 0$. Since each destination cooperates with the same number of relay nodes as its source, step 3 needs the same number of time slots as step 1. After the three steps, for a source user, $\frac{k^{'}m}{n}$ packets are sent to the destination. Then we can use the following equation to derive the lower bound of $\lambda(n)$:

$$\lambda(n) \geq \frac{\frac{k^{'}m}{n}}{2I\frac{3ek^{'}m\log n}{n} + k\sqrt{m\log m}}. \tag{3}$$

In other words, we have

$$\lambda(n) = \Omega(\frac{1}{n}\frac{m}{\frac{m\log n}{n} + \sqrt{m\log m}}). \tag{4}$$

If $\frac{m\log n}{n} < \sqrt{m\log m}$, or more precisely $\frac{\sqrt{m}}{\sqrt{\log m}} = O(\frac{n}{\log n})$, black box needs more time slots than input and output processes and becomes the bottleneck. We have $\lambda(n) = \Omega(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. When $\frac{\sqrt{m}}{\sqrt{\log m}} = \Omega(\frac{n}{\log n})$, black box has a larger capacity and needs less time slots than the input and output processes. We have $\lambda(n) = \Omega(\frac{1}{\log n})$.

The above results hold when $m = \omega(n)$. In the case $m = \Theta(n)$, without cooperation, per user throughput is $\Theta(\frac{1}{\sqrt{n\log n}})$, which is equivalent to $\Theta(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. Therefore, when $m = \Theta(n)$, we also have $\lambda(n) = \Theta(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. Combined with the upper bound we derive before, Theorem 1 follows. ■

Majority rules: In Theorem 1, even for very large m, the achievable per user throughput is $\Omega(\frac{1}{\log n})$, which has a gap with the upper bound $\Theta(1)$. The reason is that each squarelet can only have a throughput of $\Theta(1)$ while there are $\Theta(\log n)$ users sharing it. In addition, aggregate transport capacity is $\Omega(\frac{n}{\log n})$, which also has a gap with the upper bound $\Theta(n)$. The reason is that we divide the unit area into $\frac{n}{3\log n}$ squarelets to guarantee each user has a certain number of relay nodes. From the viewpoint of spatial reuse, there are only $\Theta(\frac{n}{\log n})$ squarelets that can have a transmission simultaneously. Therefore the achievable aggregate transport capacity is limited to $\Theta(\frac{n}{\log n})$.

When m is large enough, we need to improve the spatial reuse in step 1 and step 3 of the scheduling algorithm. If we divide the unit area into n squarelets[4], we can show that an arbitrary large fraction of users can achieve a throughput of $\Theta(1)$. In addition, the aggregate transport capacity of n users can reach $\Theta(n)$. To show that, we first introduce Lemma 2 and lemma 3.

Lemma 2: Divide a unit area with n randomly deployed users into n equal-size squarelets. For any $c > 0$, there are at least one squarelet that has no less than c users a.s., i.e., there exists a squarelet with an infinite number of users. Meanwhile, there are at most $3\log n$ users in any squarelet a.s.

Proof: The proof is omitted here due to page limit. ■

Lemma 3: When the number of randomly deployed relay nodes is $\Omega(n^{2+\varepsilon})$, $\varepsilon > 0$, there exists a $k^{'} > 0$ such that the number of relay node in any squarelet is at least $k^{'}n^{1+\varepsilon}$ a.s.

Proof: The proof is omitted here due to page limit. ■

We have Theorem 2 as follows.

Theorem 2: For randomly deployed n users and m relay nodes, when $m = \Omega(n^{2+\varepsilon})$, $\varepsilon > 0$, given any threshold $P_{th} < 1$, there are at least $P_{th}n$ users that can achieve a throughput of $\Theta(1)$, while the others can achieve a throughput of $\Omega(\frac{1}{\log n})$.

Proof: We first show that a user i can achieve a throughput of $\Omega(\frac{1}{\rho+1})$, if there are at most ρ other users that lie in the same squarelet as i and meanwhile there are at most ρ other users that are in the same squarelet as its destination.

According to Lemma 3, there are at least $k^{'}n^{1+\varepsilon}$ relay nodes in each squarelet. Based on our scheduling algorithm, we let users in a squarelet cooperate with $k^{'}n^{1+\varepsilon}$ relay nodes totally (We note here each user does not necessarily cooperate with the same number of relay nodes.). When the squarelets of both user i and its destination have at most $\rho + 1$ users, user i can send at least $\frac{k^{'}n^{1+\varepsilon}}{\rho+1}$ packets to the black box. Its destination can also cooperate with at least $\frac{k^{'}n^{1+\varepsilon}}{\rho+1}$ relay nodes and therefore receive so many packets. Note only relay nodes in non-empty squarelets can receive packets from users. There are at most n non-empty squarelets. Then there are at most $k^{'}n^{2+\varepsilon}$ relay nodes each of which has a packet. Step 2 of the scheduling algorithm needs at most $k\sqrt{n^{2+\varepsilon}\log n^{2+\varepsilon}}$ time slots. The throughput of user i, denoted by λ_i satisfies

$$\lambda_i \geq \frac{\frac{k^{'}n^{1+\varepsilon}}{\rho+1}}{2Ik'n^{1+\varepsilon} + k\sqrt{n^{2+\varepsilon}\log n^{2+\varepsilon}}}. \tag{5}$$

Since for an arbitrary $\varepsilon > 0$, there is $n^{\varepsilon} = \omega(\log n)$, we have $\lambda_i = \Omega(\frac{1}{\rho+1})$.

We next show that we can pick a large enough but finite ρ such that at least $P_{th}n$ users can achieve a throughput of $\Omega(\frac{1}{\rho+1})$.

Let P_ρ be the probability that there are at most ρ other users in the same squarelet as user i. We note that P_ρ is also the probability that there are at most ρ other users in the same squarelet as the destination of user i. The number of users

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[4] We cannot use the squarelet size of $1/n$ in Theorem 1 for the following reason. When $m = \Omega(n)$ and $m = o(n\log n)$, one cannot guarantee that each squarelet has enough relay nodes if the squarelet size is $1/n$.

that are in the same squarelet as user i is a random variable following binomial distribution with the parameter $(\frac{1}{n}, n-1)$. We have

$$P_\rho = \sum_{j=0}^{\rho} C_{n-1}^{j} \left(\frac{1}{n}\right)^{j} \left(1-\frac{1}{n}\right)^{(n-1-j)}. \tag{6}$$

For a user, with a probability ${P_\rho}^2$, it can achieve a throughput of $\Omega(\frac{1}{\rho+1})$. Among all the n users, the number of users that can achieve a throughput of $\Omega(\frac{1}{\rho+1})$ is a binomial random variable with the parameter $({P_\rho}^2, n)$. Let X denote the number of such users. Use Chernoff bound and follow similar steps of the proof of Lemma 1, we have

$$\Pr(X < P_{th} n) \leq [(e^{P_{th}}(1-(1-e^{-1}){P_\rho}^2)]^n. \tag{7}$$

By (6), when n goes to infinite, the binomial distribution converges to a Poisson distribution with mean 1. Therefore, for an arbitrary constant $\sigma < 1$, we can always find a sufficient large but finite value ρ such that ${P_\rho}^2 > \sigma$ is satisfied. That is, we can find a finite ρ making the following equation hold,

$$\Pr(X < P_{th} n) \leq [e^{P_{th}}(1-(1-e^{-1})\sigma)]^n. \tag{8}$$

By (8), as long as $e^{P_{th}}(1-(1-e^{-1})\sigma) < 1$, i.e., $P_{th} < \log\frac{1}{1-(1-e^{-1})\sigma}$, we have

$$\sum_{n=1}^{\infty} \Pr(X < P_{th} n) < \infty. \tag{9}$$

Use the Borel-Cantelli Theorem, we have $X \geq P_{th} n$ almost surely, when $P_{th} < \log\frac{1}{1-(1-e^{-1})\sigma}$, which is equivalent to

$$\sigma > \frac{1-e^{-P_{th}}}{1-e^{-1}}. \tag{10}$$

For any threshold $P_{th} < 1$, we can find a $\sigma < 1$ satisfying (10) and a corresponding ρ. Thus, there are at least $P_{th} n$ users that can achieve a throughput of $\Theta(1)$.

In other squarelets, based on Lemma 2, there are at most $3\log n$ in each squarelet almost surely, then those users can achieve a throughput $\Omega(\frac{1}{\log n})$ a.s. Theorem 2 follows. ∎

When Theorem 2 holds, we have the following corollary.

Corollary 1: When the number of relay nodes m is $\Omega(n^{2+\varepsilon})$, where $\varepsilon > 0$, the aggregate transport capacity of users is $\Theta(n)$ a.s.

Proof: By Theorem 2, there are at least $P_{th} n$ users which have a capacity $\Theta(1)$. Then the aggregate capacity of those users is $\Theta(n)$. Since each source user can randomly select a destination, the average distance between a source and a destination is $\Omega(1)$. Therefore, the aggregate transport capacity of all the n users is $\Theta(n)$. ∎

Till now, we show in the case that the unit area is divided into n squarelets and meanwhile users are allowed to achieve different throughput, a majority users can achieve a throughput of $\Theta(1)$. This policy is acceptable in certain applications where it is more important to let most users have a high capacity, rather than having to cut down the throughput for all users to guarantee that every one achieves the same throughput. We also show although not every user can a throughput of $\Theta(1)$, the aggregate transport capacity of n users can achieve $\Theta(n)$. This means per user throughput cannot be derived by dividing the aggregate transport capacity by the number of users.

The case every user can achieve $\Theta(1)$: If we divide the unit area into squarelets with even smaller size as $o(\frac{1}{n})$, spatial reuse is improved. Intuitively, if there is no crowded squarelets (with infinite number of users), with enough relay nodes, every user can achieve a throughput of $\Theta(1)$. To show that, we need the following lemma first.

Lemma 4: Given a constant $c > 0$, divide the unit area with n randomly deployed users into κ equal-size squarelets, when $\kappa = n^4$, no squarelets has more than c users almost surely.

Proof: Let N_u denote the number of users in a squarelet. N_u is a binomial random variable with the parameter $(\frac{1}{\kappa}, n)$. The probability that N_u is no more than c is

$$\Pr(N_u \leq c) = \sum_{i=0}^{c} C_n^i \left(\frac{1}{\kappa}\right)^i \left(1-\frac{1}{\kappa}\right)^{n-i} \tag{11}$$

Let P_c denote the probability that at least one squarelet has more than c users, we have

$$\begin{aligned} P_c &= 1 - \left(\sum_{i=0}^{c} C_n^i \left(\frac{1}{\kappa}\right)^i \left(1-\frac{1}{\kappa}\right)^{n-i}\right)^{\kappa} \\ &= 1 - \left(\left(1-\frac{1}{\kappa}\right)^n \sum_{i=0}^{c} C_n^i \frac{1}{(\kappa-1)^i}\right)^{\kappa} \\ &\leq 1 - \left(\left(1-\frac{n}{\kappa}\right)\left(1+\frac{n}{(\kappa-1)}+....+\frac{C_n^c}{(\kappa-1)^c}\right)\right)^{\kappa} \\ &\leq 1 - \left(1-\frac{n^2}{\kappa^2}\right)^{\kappa} \leq \frac{n^2}{\kappa}. \end{aligned} \tag{12}$$

By (12), when $\kappa = n^4$, $P_c \leq \frac{1}{n^2}$. Using the Borel-Cantelli Theorem, we conclude that any squarelet has no more than c users almost surely. ∎

Lemma 4 show that the squarelet size is small enough, every user lies in a squarelet which has constant number of users. Then apply our cooperation scheduling algorithm, we have the following Theorem.

Theorem 3: For randomly deployed n users and m relay nodes, when $m = \Omega(n^{5+\varepsilon})$, $\varepsilon > 0$, every user can achieve a throughput of $\Theta(1)$ almost surely.

Proof: The proof is similar to the proof of Theorem 2 and thus omitted here. ∎

Now we show, every user can achieve a throughput of $\Theta(1)$. However, it needs a large amount of relay nodes. Even we loose the requirement and let every user achieve a throughput of $\Theta(1)$ with high probability (with the probability approaching 1 when n goes to infinite), we still need $\Omega(n^{3+\varepsilon})$ relay nodes.

B. Regularly deployed users and randomly placed relay nodes

Theorem 4: Consider the case that n users are regularly deployed and m relay nodes are randomly deployed. When

$m = \Omega(n)$ and $m = O(n\log n)$, the upper bound is $O(\frac{\sqrt{m}}{n})$, the lower bound is $\Omega(\frac{1}{\sqrt{n}})$. When $m = \Omega(n\log n)$ and $m = \Omega(n^2\log n)$, the upper bound is $\min(O(1), O(\frac{\sqrt{m}}{n}))$, the lower bound is $\Omega\left(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}}\right)$. When $m = \Omega(n^2\log n)$, we have $\lambda(n) = \Theta(1)$.

Proof: **(upper bound)** When there are n users regularly deployed and m relay nodes randomly deployed, the aggregate transport capacity is upper bounded by $O(\sqrt{n+m})$ (We note here the upper bound can be achieved when all $n+m$ nodes are arbitrarily deployed. In our case, this upper bound is likely to be loose, especially when m is increasingly large, which causes the gap in Theorem 4.). Then per user throughput is upper bounded by $\min\left(O(1), O(\frac{\sqrt{n+m}}{n})\right)$, which is also $\min\left(O(1), O(\frac{\sqrt{m}}{n})\right)$ when $m = \Omega(n)$. ■

The capacity of relay nodes is $\Theta(\frac{\sqrt{m}}{\sqrt{\log m}})$, which is larger than $\Theta(\sqrt{n})$ when $m = \Omega(n\log n)$. We focus on this case to derive the lower bound since we have assumed the black box should provide a larger capacity than users. We divide the unit area into a set of squarelets of size $\frac{1}{n}$. Each squarelet has one fixed user. Due to the randomness of relay nodes, the number of relay nodes around each user to cooperate is also random. Following similar steps in the proof to Theorem 1, we need to derive the number of relay nodes per user can cooperate. We have the following lemma.

Lemma 5: Divide a unit area into n squarelets each of which has a size of $\frac{1}{n}$, When $m = \omega(n\log n)$, there exists a constant $k^{'}$, such that the number of relay nodes each user can cooperate is $k^{'}\frac{m}{n}$ a.s.

Proof: Similar to the proof of lemma 1. ■

Proof of Theorem 4: **(low bound)** By Lemma 5, Each source user and destination user can cooperate with $\frac{k^{'}m}{n}$ relay nodes. There is only one user in each squarelet. Therefore the time slots needed for step 1 and step 3 is upper bounded by $2I\frac{k^{'}m}{n}$. After step 1, there are $k^{'}m$ relay nodes which has one packet. Then we have

$$\lambda(n) \ge \frac{\frac{k^{'}m}{n}}{2I\frac{k^{'}m}{n} + k\sqrt{m\log m}}. \tag{13}$$

By (13) we have $\lambda(n) = \Omega(\frac{1}{n}\frac{m}{\frac{m}{n}+\sqrt{m\log m}})$. Then if $\frac{\sqrt{m}}{\sqrt{\log m}} < n$, i.e., $m = O(n^2\log n)$, the black box becomes a bottleneck and takes more time to exchange $\frac{k^{'}m}{n}$ packets, we have $\lambda(n) = \Omega(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. If $\frac{\sqrt{m}}{\sqrt{\log m}} \ge n$, i.e., $m = \Omega(n^2\log n)$, the black box have a larger capacity and need less time slots than the input and the output process, the lower bound of $\lambda(n)$ is $\Omega(1)$.

The above results hold when $m = \omega(n\log n)$. In the case $m = \Theta(n\log n)$, per user throughput without relay nodes cooperation is $\Theta(\frac{1}{\sqrt{n}})$, which is equivalent to $\Theta(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$. This means $\Omega(\frac{1}{n}\frac{\sqrt{m}}{\sqrt{\log m}})$ can be achieved. Combined with the results of the upper bound, Theorem 4 follows. ■

Under the setting of regularly deployed users and randomly deployed relay nodes, there is no gap between the lower and the upper bound when m is roughly n^2. The reason is that we divide the unit area into n squarelets and there can be $\Theta(n)$ squarelets each of which has a transmission at the same time. Also in each squarelet, there is only one user to share the throughput as $\Theta(1)$.

C. Randomly deployed users and regularly placed relay nodes

Theorem 5: Consider the case that n users are randomly deployed and m relay nodes are regularly deployed. When $m = \Omega(\frac{n}{\log n})$ and $m = O(n)$, the low bound of $\lambda(n)$ is $\Omega(\frac{\sqrt{m}}{n})$, and the upper bound is $O(\frac{1}{\sqrt{n}})$. When $m = \Omega(n)$ and $m = O(\frac{n^2}{\log^2 n})$, $\lambda(n) = \Theta(\frac{\sqrt{m}}{n})$. In the case $m = \Omega(\frac{n^2}{\log^2 n})$, the low bound of $\lambda(n)$ is $\Omega(\frac{1}{\log n})$ and the upper bound is $\min\left(O(1), O(\frac{\sqrt{m}}{n})\right)$.

Proof: Similar to the proof of Theorem 4, in this case, the upper bound of per user throughput with the cooperation of relay nodes is $\min\left(O(1), O(\frac{\sqrt{m+n}}{n})\right)$, which is $\min\left(O(1), O(\frac{\sqrt{m}}{n})\right)$ when $m = \Omega(n)$.

In this case, the capacity of relay nodes is $\Theta(\sqrt{m})$, while the capacity of users is $\Theta(\frac{\sqrt{n}}{\sqrt{\log n}})$. Therefore, even when m is smaller than n (but larger than $\frac{n}{\log n}$), the black box can still provide a larger capacity for users. Then different from the other deployment schemes, when m is smaller than n, we do not restrict that each relay node only serves one user.

We divide the unit area into squarelets each of which has a size $\frac{3\log n}{n}$. The number of relay nodes in a squarelet is fixed as $m\frac{3\log n}{n}$. Since in each squarelet, the number of users is upper bounded by $3e\log n$. When $m < en$, in a squarelet, it is not guaranteed that there is at least one different relay node for each user to cooperate. In this case, we allow multiple users cooperate with the same relay node. We let each user send one packet to a relay node. Since there are at most $3e\log n$ users in a squarelet, each relay node receives at most $\frac{en}{m}$ packets. Therefore, in the black box, m relay nodes process at most en packets. Then we have

$$\lambda(n) \ge \frac{1}{2I(3e\log n) + k\frac{en}{\sqrt{m}}}. \tag{14}$$

When $m > en$, each user can have a disjoint set of relay nodes to cooperate. In this case, each user can cooperate with $\frac{m}{en}$ relay nodes and send one packet to each of them. Each relay node receives at most one packet, and there are $\frac{m}{e}$ relay nodes each of which has a packet. Therefore, we have

$$\lambda(n) \ge \frac{\frac{m}{en}}{2*3I\frac{m\log n}{n} + k\sqrt{m}}. \tag{15}$$

Combine (14) and (15). When $m = O(\frac{n^2}{\log^2 n})$, $\lambda(n) = \Omega(\frac{\sqrt{m}}{n})$. When $m = \Omega(\frac{n^2}{\log^2 n})$, $\lambda(n) = \Omega(\frac{1}{\log n})$.

Combined with the results of the upper bound, we have Theorem 5. ■

Similar to the case that both users and relay nodes are randomly placed, when the number of relay nodes m is $\Omega(n^2)$, Given any threshold $P_{th} < 1$, there are at least $P_{th}n$ users that can achieve a throughput as $\Theta(1)$. Meanwhile the aggregate transport capacity of users is $\Theta(n)$ a.s. When $m = \Omega(n^5)$, every user can achieve a throughput of $\Theta(1)$ a.s.

D. Both are regularly deployed

In this case, divide the unit area into n squarelets with the size of $\frac{1}{n}$, each squarelet has one user and $\frac{m}{n}$ relay nodes. Similar to the above cases, we have Theorem 6.

Theorem 6: When n users and m relay nodes are regularly deployed, in the case $m = \Omega(n)$ and $m = \mathrm{O}(n^2)$, $\lambda(n) = \Theta(\frac{\sqrt{m}}{n})$. In the case $m = \Omega(n^2)$, $\lambda(n) = \Theta(1)$.

V. DISCUSSIONS

The main contribution of the paper is to derive the achievability results through the proposed scheduling scheme. The squarelet size plays an important role. As m increases, the squarelet size decreases, so that spatial reuse increases, when users are randomly deployed. When the squarelet size is $\log n/n$ and $m = \mathrm{O}(n^2)$, users and relay nodes are evenly distributed in each squarelet (i.e., there are $\mathrm{O}(\log n)$ users per squarelet and $\Theta(m/n)$ relay nodes per user.). When m increases, we reduce the squarelet size to $1/n$. However, in this case, some squarelets can be overly crowded, i.e., with an arbitrarily large number of users. In this case, one can guarantee that an arbitrary large fraction of s-d pairs can achieve $\Theta(1)$. If m increases further, we can have even smaller squarelets and guarantee all users a throughput of $\Theta(1)$.

Our results indicates the potential of throughput gain through network cooperation. For example, consider an area with n primary users and m secondary users regularly deployed. We know that without secondary users, per primary user capacity scales as $\Theta(\frac{1}{\sqrt{n}})$ if all users are busy. In this case, without interfering with the transmission of primary users, the potential throughput for secondary users will be extremely low without cooperation. The question is whether there is an incentive for primary users to cooperate. According to our results, if $m = \Theta(n)$, there is at least a constant gain in terms of the aggregate capacity through cooperation which may be divided among primary and secondary users. It is more evident if $m = \omega(n)$. For example, if $m = \Theta(n^2)$, then with cooperation, per primary user capacity can achieve $\Theta(1)$, which is much higher than $\Theta(\frac{1}{\sqrt{n}})$. Meanwhile, per secondary user throughput can achieve $\Omega(\frac{1}{\sqrt{m}})$ by scheduling time slots alternatively for primary users' traffic and secondary users' traffic. This can serve as a strong incentive for primary users to cooperate. Meanwhile, second user can have a much higher throughput than the case that there is no cooperation. Similar conclusions hold for randomly deployed primary and secondary users.

VI. CONCLUSIONS

Previous work suggests that per user throughput scales poorly when the network size increases, e.g., in [1][18]. The assumption is that every node is a source/destination. However, in many cases, only a portion of nodes in a large network have data to send or receive at a given time, while the others can act as routers. Therefore, in this paper, we study the scaling behavior from a user's viewpoint. The scaling behavior of per user throughput depends on the network size, the number of s-d pairs, and the deployment scheme, and is in general much more optimistic.

Our results indicate that increasing density of wireless networks do not necessarily significantly decrease per user throughput if not all wireless nodes have traffic simultaneously. Therefore, per user performance benefits from efficient network cooperation in both homogeneous and heterogeneous networks. Our scheduling algorithm sheds light on the design of such protocols, especially for primary and secondary user cooperation in cognitive networks.

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