Joint Scheduling and Power-Allocation for Interference Management in Wireless Networks

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***Abstract*— Interference management is crucial in wireless communication systems because interference ultimately limits the system capacity. Opportunistic scheduling and power allocation are effective interference management mechanisms. In this paper, we present joint scheduling and power-allocation schemes to alleviate intercell interference. First, we study the problem with the objective to minimize the average transmission power, and thus interference to other cells, while maintaining the required data-rate for each user within the cell. Then we study the problem to maximize the net utility, de£ned as the difference between the value of throughput and the cost of power consumption, with the same data-rate requirements. We establish the optimality of our joint scheduling and power-allocation schemes for both problems in the paper.**

I. Introduction

Wireless spectrum ef£ciency is becoming increasingly important because of the expanding demand for wireless services, especially for wide-band data-communication services. Interference management is a crucial component of ef£cient spectrum utilization in wireless systems. Power allocation and opportunistic scheduling are effective mechanisms for interference management and ef£cient spectrum utilization. Power allocation has been well studied and widely used in wireless systems to maintain desired link quality, alleviate interference to others, and minimize power consumption [5]. In this paper, we introduce the idea of combining "opportunistic" scheduling with power allocation. Because users experience time-varying and location-dependent channel conditions in wireless environments, we can schedule users "opportunistically" so that a user can exploit more of its good channel conditions and avoid (as much as possible) bad times, at least for applications (e.g., data service) that are not time-critical. Hence, joint scheduling and power-allocation scheme should be able to further improve the spectrum ef£ciency and decrease power consumption.

This research is supported in part by the National Science Foundation through grants NCR-9624525, ANI-9805441, 0099137-ANI, ANI-0207892, and 0098089-ECS.

In this paper, we study the intercell-interference-alleviation problem using joint scheduling and power-allocation mechanisms. We study two different versions of this problem. In the £rst problem, the objective is to minimize the total transmission power, and thus interference to other cells, subject to a minimum-data-rate requirement for each user within the cell. In the second problem, the objective is to maximize the system net utility, which is de£ned as the value of the throughput minus the power cost, with the same minimum-data-rate constraints. In both problems, we have joint scheduling and power-allocation decisions.

II. System Model

We consider a cell in a time-slotted system; i.e., time is the resource to be shared among users, and users transmit one at a time. Examples of such a system includes TDMA systems and time-slotted CDMA systems (e.g., IS-856/Qualcomm HDR system). We consider the downlink of a cell, and use a stochastic model to capture the time-varying channel condition of a user. To elaborate, let α_i^k be a random variable representing the received SIR for user i at time k given that the transmission power is 1. For simplicity of the analysis, we assume that the stochastic process $\{\alpha_i^k, k = 1, 2, \cdots\}$ is stationary and ergodic. Hence, we drop the time index k. Let $\vec{\alpha} = \{\alpha_1, \cdots, \alpha_N\}$, where N is the number of users in the system. Basically, $\vec{\alpha}$ indicates the channel conditions of users at a generic time-slot. Let $f_i(c)$ be the required SIR for reliable transmission at data-rate c for user i, which is an increasing function of c, and $f_i(0) = 0$. Different users may have different forms of $f_i(c)$. Given α_i, which indicates the channel condition, the minimum required transmission power to support a data-rate c for user i is $f_i(c)/\alpha_i$. Let P_{max} be the maximum transmission power, which represents the restriction on the transmission power of a practical system. Let R_i denote the required average data-rate for user i.

There are two components in a joint scheduling and power-allocation scheme: a scheduling policy that decides which user to use the time-slot and a power allocation policy that decides the transmission power of the selected user (and thus its corresponding data-rate). Let Q be a scheduling policy; Q decides which user should transmit at a generic time-slot, given the channel conditions. In general, $Q(\vec{\alpha}) \in \{1, \cdots, N, \text{Null}\}$. If $Q(\vec{\alpha}) = i$, $i = 1, \ldots, N$, then user i is scheduled to transmit. If $Q(\vec{\alpha}) = \text{Null}$, then no user is scheduled to transmit. This may occur if all users experience relatively bad channel conditions. Let $p(\cdot)$ be the power allocation policy, $0 \le p(\vec{\alpha}) \le P_{max}$. If user i is selected to transmit and its transmission power is $p(\vec{\alpha})$, then $c_i(p) = f_i^{-1}(\alpha_i p)$ is its achievable data-rate. In summary, a policy for a joint scheduling and power-allocation scheme is given in the form of $\{Q, p\}$.

III. Minimizing Transmission Power

First, we study the problem where the objective is to minimize the overall transmission power while maintaining the required data-rate for each user within the cell. To achieve this goal, we need to decide which user should be scheduled at a generic time-slot and what should be its transmission power. Let $P(Q, p)$ be the overall transmission power of the system under policy $\{Q, p\}$:

$$P(Q,p) = \sum_{i=1}^{N} E\left(p(\vec{\alpha})\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right).$$

The problem that we are interested in can be formally stated as:

$$\begin{aligned} \underset{Q,p}{\text{minimize}} \quad & P(Q,p) \\ \text{subject to} \quad & 0 \le p(\vec{\alpha}) \le P_{max}, \\ & E\left(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) = R_i, \\ & \quad i = 1, \cdots, N. \end{aligned} \tag{1}$$

Our objective is to minimize the overall transmission power $P(Q, p)$ under two sets of constraints. The first constraint, $0 \le p(\vec{\alpha}) \le P_{max}$, indicates the maximum-transmission-power restriction of the system. The second constraint, $E(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}) = R_i$, is the minimum-data-rate constraint, where $E(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}})$ is the average data-rate of user i given $\{Q, p\}$. Note that we could have written the second constraint in the more general (inequality) form: $E(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}) \ge R_i$. However, because our objective is to minimize the transmission power, a solution to (1) is certainly a solution to the problem with the more general inequality constraints. Hence, without loss of generality, we study the problem with the equality constraints, as defined in (1).

Next, we present our solution to the joint scheduling and power-allocation problem defined in (1). Let

$$\begin{aligned} L(\vec{\lambda}) &= \sum_{i=1}^{N} E\left(p(\vec{\alpha})\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) \\ &\quad - \sum_{i=1}^{N} \lambda_i \left(E\left(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) - R_i\right) \\ &= E\left(\sum_{i=1}^{N} \left(p(\vec{\alpha}) - \lambda_i c_i(p)\right)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) + \sum_{i=1}^{N} \lambda_i R_i. \end{aligned}$$

We define

$$\begin{aligned} l_i(\vec{\lambda}, \vec{\alpha}, p) &= p(\vec{\alpha}) - \lambda_i c(p) \\ p_i^*(\vec{\lambda}, \vec{\alpha}) &= \underset{0 \le p \le P_{max}}{\text{argmin}}\ l_i(\vec{\lambda}, \vec{\alpha}, p) \\ l_i^*(\vec{\lambda}, \vec{\alpha}) &= l_i(\vec{\lambda}, \vec{\alpha}, p_i^*(\vec{\lambda}, \vec{\alpha})). \end{aligned}$$

Note that we have $l_i^*(\vec{\lambda}, \vec{\alpha}) \le 0$ because $l_i(\vec{\lambda}, \vec{\alpha}, 0) = 0$.

Proposition 1: Suppose there exists $\vec{\lambda}^*$ such that

$$E\left(c_i(p^*)\mathbf{1}_{\{Q^*(\vec{\alpha})=i\}}\right) = R_i, \quad i = 1, \cdots, N,$$

where $Q^*(\vec{\alpha})$ is defined as

$$Q^*(\vec{\alpha}) = \underset{i}{\text{argmin}}\ l_i^*(\vec{\lambda}^*, \vec{\alpha}). \tag{2}$$

Then, $\{Q^*, p^*\}$ is an optimal solution to the problem defined in (1).

The above proposition is valid for all f_is that are increasing functions with $f_i(0) = 0$. Further, if f_i is a strictly convex function, then p^* has a closed-form expression:

$$p^*(\vec{\lambda}, \vec{\alpha}) = \begin{cases} 0 & \text{if } f_i'(0) > \lambda_i \alpha_i \\ \frac{f_i(f_i'^{-1}(\lambda_i \alpha_i))}{\alpha_i} & \text{if } f_i'(0) \le \lambda_i \alpha_i \le f_i'(C_i) \\ P_{max} & \text{if } f_i'(C_i) < \lambda_i \alpha_i \end{cases},$$

where $C_i = f_i^{-1}(\alpha_i P_{max})$ is the maximum data-rate of user i given P_{max} and the channel condition.

From Proposition 1, we observe that a user is chosen to transmit when it is a "relatively-best" user. User i is "relatively-best" if $l_i^*(\vec{\lambda}, \vec{\alpha}) \le \min_j l_j^*(\vec{\lambda}, \vec{\alpha})$ (and hence has the same form as the opportunistic scheduler in [2]. Moreover, the transmission power of the selected user is the power that minimizes $l_i(\vec{\lambda}, \vec{\alpha}, p)$. We can think of λ_i as the *unit reward* (in terms of power/data-rate) to compensate power consumption. It controls the value of transmission power, and in turn the data-rate. The fact that $l_i^*(\vec{\lambda}, \vec{\alpha}) \le 0$ indicates that the transmission power $(p^*(\vec{\lambda}, \vec{\alpha}))$ should be no greater than the reward $(\lambda_i c_i(p^*))$ of the user for transmitting at data-rate $c_i(p^*)$. Also note that the resulting data-rate of a user is an increasing function of its unit reward λ_i. This property enables us to obtain $\vec{\lambda}^*$ iteratively in the implementation.

Proof of Proposition 1: Suppose a policy $\{Q, p\}$ satis£es the maximum transmission power constraint and the data-rate constraint. We will show that

$$P(Q, p) \geq P(Q^*, p^*).$$

Because $\{Q, p\}$ satis£es the rate constraint, we have

$$\begin{aligned} P(Q,p) &= P(Q,p) - \sum_{i=1}^{N} \lambda_i^* \left(E(c_i(p))\mathbf{1}_{\{Q(\vec{\alpha})=i\}} - R_i \right) \\ &= E\left(\sum_{i=1}^{N} \left(p(\vec{\alpha}) - \lambda_i^* c_i(p)\right) \mathbf{1}_{\{Q(\vec{\alpha})=i\}} \right) \\ &\quad + \sum_{i=1}^{N} \lambda_i^* R_i. \end{aligned}$$

We have

$$l_i(\vec{\lambda}^*, \vec{\alpha}, p_i^*(\vec{\lambda}^*, \vec{\alpha})) \leq l_i(\vec{\lambda}^*, \vec{\alpha}, p), \quad 0 \leq p \leq P_{max}$$

by the de£nition of $p_i^*(\vec{\lambda}^*, \vec{\alpha})$. If $Q \neq$ Null, then

$$\begin{aligned} & \sum_{i=1}^{N} \left(p(\vec{\alpha}) - \lambda_i^* c_i(p)\right) \mathbf{1}_{\{Q(\vec{\alpha})=i\}} \\ \geq & \sum_{i=1}^{N} l_i(\vec{\lambda}^*, \vec{\alpha}, p^*(\vec{\lambda}^*, \vec{\alpha}))\mathbf{1}_{\{Q(\vec{\alpha})=i\}} \\ \geq & \sum_{i=1}^{N} l_i(\vec{\lambda}^*, \vec{\alpha}, p^*(\vec{\lambda}^*, \vec{\alpha}))\mathbf{1}_{\{Q^*(\vec{\alpha})=i\}}, \end{aligned}$$

where the last inequality is because of the de£nition of Q^*. If $Q =$ Null, because $l_i(\vec{\lambda}^*, \vec{\alpha}, p^*(\vec{\lambda}^*, \vec{\alpha})) \leq 0$, we have

$$\begin{aligned} & \sum_{i=1}^{N} \left(p(\vec{\alpha}) - \lambda_i^* c_i(p)\right) \mathbf{1}_{\{Q(\vec{\alpha})=i\}} \\ & = 0 \\ & \geq \sum_{i=1}^{N} l_i(\vec{\lambda}^*, \vec{\alpha}, p^*(\vec{\lambda}^*, \vec{\alpha}))\mathbf{1}_{\{Q^*(\vec{\alpha})=i\}}. \end{aligned}$$

Hence,

$$\begin{aligned} P(Q,p) &\geq E\left(\sum_{i=1}^{N} l_i(\vec{\lambda}^*, \vec{\alpha}, p^*(\vec{\lambda}^*, \vec{\alpha}))\mathbf{1}_{\{Q^*(\vec{\alpha})=i\}} \right) \\ &\quad + \sum_{i=1}^{N} \lambda_i^* R_i \\ &= P(Q^*, p^*). \end{aligned}$$

□

IV. Maximizing Net Utility

Now we study a joint scheduling and power-allocation problem in a different scenario. In [2], we consider a scheduling problem that maximizes the system throughput subject to each user's minimum data-rate requirement. Because a user's throughput is an increasing function of its transmission power and the objective is to maximize the throughput, it is obvious that the base station should always transmit with its maximum power. However, because transmission power causes interference to other cells in wireless systems, we need to take the power consumption into account as well. To achieve this goal, we introduce the notion of "net utility," which is de£ned as the difference between the value of the throughput and the cost of the power consumption [4]. Let $g_i(p)$ be the power cost of user i; then, $c_i(p) - g_i(p)$ is de£ned as the net utility of user i. As before, let Q be the scheduling policy and p be the power policy. Let $T_i(Q, p)$ be the average net utility of user i given the joint policy $\{Q, p\}$:

$$\begin{aligned} T_i(Q,p) &= E\left((c_i(p) - g_i(p))\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) \\ C_i(Q,p) &= E\left(c_i(p)\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) \\ T(Q,p) &= \sum_{i=1}^{N} T_i(Q,p). \end{aligned}$$

We formulate the problem as:

$$\begin{array}{ll} \underset{Q,p}{\text{maximize}} & T(Q,p) \\ \text{subject to} & 0 \leq p(\vec{\alpha}) \leq P_{max} \\ & C_i(Q,p) \geq R_i, \quad i = 1, \cdots, N \end{array} \tag{3}$$

In other words, the objective is to maximize the net utility given the maximum power constraint and data-rate requirement constraints. We de£ne

$$\begin{aligned} & b_i(\vec{\eta}, \vec{\alpha}, p) = c_i(p)\eta - g_i(p) \\ & p^*(\vec{\eta}, \vec{\alpha}) = \max_{0 \leq p \leq P_{max}} b_i(\vec{\eta}, \vec{\alpha}, p) \\ & b_i^*(\vec{\eta}, \vec{\alpha}) = b_i(\vec{\eta}, \vec{\alpha}, p^*(\vec{\eta}, \vec{\alpha})). \end{aligned}$$

Let $Q^*(\vec{\alpha})$ be de£ned as

$$Q^*(\vec{\alpha}) = \underset{i}{\text{argmax}}\, b_i^*(\vec{\eta}^*, \vec{\alpha}), \tag{4}$$

where $\vec{\eta}^*$ satis£es:

1) $\min_{i=1,\cdots,N} \eta_i^* = 1$
2) $C_i(Q^*, p^*) \geq R_i$ for all i
3) For all i, if $C_i(Q^*, p^*) > R_i$, then $\eta_i^* = 1$.

Proposition 2: $\{Q^*, p^*\}$ is an optimal solution to the problem de£ned in (3).

Proof: Suppose $\{Q, p\}$ satis£es the maximum transmission power constraint and the data-rate constraint. We will show that

$T(Q,p) \leq T(Q^*,p^*)$. We have

$$\begin{aligned} T(Q,p) &\leq T(Q,p) + \sum_{i=1}^{N} (\eta_i^* - 1)(C_i(Q,p) - R_i) \\ &= \sum_{i=1}^{N} E\left((c_i(p)\eta_i^* - g_i(p))\mathbf{1}_{\{Q(\vec{\alpha})=i\}}\right) \\ &\quad - \sum_{i=1}^{N} (\eta_i^* - 1) R_i. \end{aligned}$$

Further,

$$\begin{aligned} &\sum_{i=1}^{N} (c_i(p)\eta_i^* - g_i(p))\mathbf{1}_{\{Q(\vec{\alpha})=i\}} \\ \leq\ &\sum_{i=1}^{N} (c_i(p^*)\eta_i^* - g_i(p^*))\mathbf{1}_{\{Q(\vec{\alpha})=i\}} \\ \leq\ &\sum_{i=1}^{N} (c_i(p^*)\eta_i^* - g_i(p^*))\mathbf{1}_{\{Q^*(\vec{\alpha})=i\}}. \end{aligned}$$

The £rst inequality is due to the de£nition of p^*, and the second to the de£nition of Q^*. Hence,

$$\begin{aligned} T(Q,p) &\leq \sum_{i=1}^{N} E\left((c_i(p^*)\eta_i^* - g_i(p^*))\mathbf{1}_{\{Q^*(\vec{\alpha},p^*)=i\}}\right) \\ &\quad - \sum_{i=1}^{N} (\eta_i^* - 1) R_i \\ &= \sum_{i=1}^{N} T_i(Q^*,p^*) + \sum_{i=1}^{N} (\eta_i^* - 1)\left(C_i(Q^*,p^*) - R_i\right) \\ &= T(Q^*,p^*), \end{aligned}$$

which completes the proof. □

Note that the solutions of the two problems have certain similarities. Both policies choose the "relatively-best" user to transmit and the optimal transmission power maximizes/minimizes b_i/l_i. Here, a user is relatively-best if $b_i^*(\vec{\eta}^*, \vec{\alpha}) \geq \max_j b_j^*(\vec{\eta}^*, \vec{\alpha})$.

The problem studied in [2] can be considered as a special case of the problem de£ned in (3) with $g_i(p) \equiv 0$. In other words, if we do not penalize transmission power at all, then the base station always transmits with its maximum power P_{max}. In this case, the joint scheduling and power-allocation degenerates to a pure scheduling problem.

V. Numerical Results

We only present numerical results of our joint scheduling and power-allocation scheme that minimizes the total transmission power in this section. In the following, we £rst describe brie¤y our simulation model of a cellular system, and then we show the simulation results for the joint policy using the cellular model.

Rate (kbps)	10	20	30	40	50	60	70	80
SIR (dB)	5	10	15	20	25	30	35	40

TABLE I

Achievable data-rate vs. SIR

A. Cellular Model

In our simulation, we consider a multi-cell system consisting of a center hexagonal cell surrounded by hexagonal cells of the same size. The base station is at the center of each cell, and simple omni-directional antennas are used by mobiles and base stations. The frequency reuse factor is 3, and the co-channel interference from the six £rst-ring neighboring cells are taken into account. We assume that each cell has a £xed number of frequency bands, and focus on one frequency band that is shared by 10 users in the central cell. The users have exponentially distributed "on" and "off" periods.

Users move with random speed and direction in the cell. They perceive time-varying and location-dependent channel gains. The channel gains of the users are mutually independent random processes determined by the sum of two terms: one due to path (distance) loss and the other to shadowing. We adopt the path-loss model (Lee's model) and the slow log-normal shadowing model in [3].

Recall that $f(c)$ is the required SIR for reliable transmission at data-rate c. In practice, f is usually not a continuous function because the system only supports discrete data-rates by adapting different coding rates and modulation schemes. Hence, in the simulation, we assume that there are ten discrete data-rates available and the corresponding SIR requirements are listed in Table I. The data is similar to the result presented in [1].

B. Numerical Results

In the following, we show simulation results for the joint scheduling and power-allocation scheme that minimizes the overall transmission power with minimum-data-rate guarantees. First, we set $\vec{\lambda}^0$. Then the system performs the following procedure at every time-slot. Basically, the users measure their channel conditions, and then send the information to the base station ($\vec{\alpha}^k$). The base station decides the user to be scheduled and its transmission power using the joint scheduling and power-allocation policy by substituting $\vec{\lambda}^k$ into (2). Then the base station updates the parameter used in the scheduling policy by

$$\lambda_i^{k+1} = \lambda_i^k + a^k(R_i - \bar{R}_i^k),$$

where a_k is the step size ($a_k = 0.001$), $\bar{R}_i^k$ is the estimated average data-rate of user i at time k, R_i is the required data-rate, and $R_i = 3$ (kbps). The above parameter tuning scheme is similar to the one in [2].

For the purpose of comparison, we also simulate a round-robin policy. In the round-robin scheduling policy, active users follow a predetermined order. When a user's turn comes, if its average transmission rate is lower than its required rate, then we let the user transmit. Its transmission power equals the minimum power required to support the highest data-rate achievable to the user given its channel condition. Otherwise, if the use does not qualify to transmit, then we go to the next active user until we £nd a user to transmit in the current time-slot or all active users have been exhausted.

We compare the average transmission power of our policy with that of round-robin. The simulation is run for 100,000 time-slots. In the simulation, both policies can maintain the required data-rates of users. Figure 1 shows the average transmission power of each user using our policy and using round-robin. The x-axis is the user's ID. The y-axis is the average power consumption, which is the amount of transmission power consumed by a user divided by its throughput. The £rst bar is the average power consumption of a user in the round-robin scheme and the second bar is that in our scheme. Note that the power consumption of each user in our scheme is unanimously lower than that in round-robin. The total transmission power of round-robin is 33% higher than that of our policy.

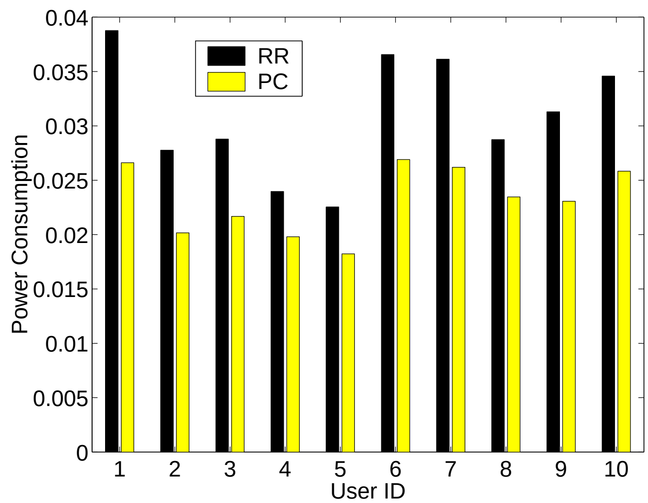

Fig. 1. The average transmission power of round-robin and our policy.

VI. CONCLUSIONS

Opportunistic scheduling exploits time-varying channel conditions to improve spectrum ef£ciency, providing an additional degree of freedom to the system. Its merits also lie in its ability to work in conjunction with other resource management mechanisms. In this paper, we study joint scheduling and power-allocation schemes to alleviate intercell interference. We study two problems with different but related objectives. The £rst objective is to minimize the total transmission power and the second objective is to maximize the net utility, both under minimum-data-rate constraints. We provide optimal solutions to the studied problems and use simulation results to evaluate the power savings compared with a round-robin scheme. Further research includes the study of the system behavior when all cells implement the same scheduling and power-allocation algorithms.

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