Distributed Power Control for Cognitive User Access based on Primary Link Control Feedback

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***Abstract*—We venture beyond the "listen-before-talk" strategy that is common in many traditional cognitive radio access schemes. We exploit the bi-directional nature of most primary communication systems. By intelligently choosing their transmission parameters based on the observation of primary user (PU) communications, secondary users (SUs) in a cognitive network can achieve higher spectrum usage while limiting their interference to the PU. Specifically, we propose that the SUs listen to the PU's feedback channel to assess their interference on the primary receiver (PU-Rx), and adjust radio power accordingly to satisfy the PU's interference constraint. We investigate both centralized and distributed power control algorithms without active PU cooperation. We show that the PU feedback information inherent in many two-way primary systems can be used as important coordination signal among multiple SUs to distributively achieve a joint performance guarantee on the primary receiver's quality of service.**

I. Introduction

As a potential solution to alleviating spectrum scarcity, the overlay of cognitive radio networks over the spectrum of high priority primary user systems has attracted a high level of research interests. Most existing works apply the listen-before-talk (LBT) concept that relies on spectrum sensing (e.g., [1], [2], [3]). LBT requires secondary users (SUs) to detect the presence or absence of primary user (PU) signals before channel access. The recent TV white space prototype testing by the FCC found that most LBT-based devices do not degrade TV receiver quality [4]. This finding represents a major victory for the cognitive radio concept. Though conceptually simple, LBT relies solely on the sensing of primary transmission signals instead of the actual receiver performance.

Because it focuses on the transmitter rather than the receiver, LBT needs to be conservative to protect PU against SU interference. First, it has to assume the worst case fading environment; For instance, the threshold for the LBT devices was set at 30dB below the DTV reception threshold in the FCC TV white space testing [4]. Second, it has to consider the aggregated interference at the primary receiver (PU-Rx) from multiple potential SU devices. Third, it does not allow SU systems to explore the extra capacity when a PU system, not fully loaded, can tolerate more interference (e.g., through forward error correction, spread spectrum). On the other hand, LBT may also be too aggressive under the well known hidden node environment.

In order to overcome the limitations of LBT, we advocate a different framework that incorporates the inherent feedback information in typical **two-way** PU communication links. Such data link control information is available in many practical systems, e.g., power control feedback in CDMA cellular systems [5], channel quality indicator (CQI) feedback in HSDPA [5], ACK/NACK feedback in cellular and WiFi networks [5], [6], [7], and the carrier-to-interference-and-noise ratio (CINR) report in WIMAX [8]. Such feedback information from the PU receiver can serve as a good indicator of the actual (often aggregated) impact of the SU interference on the reception quality of the PU communication link.

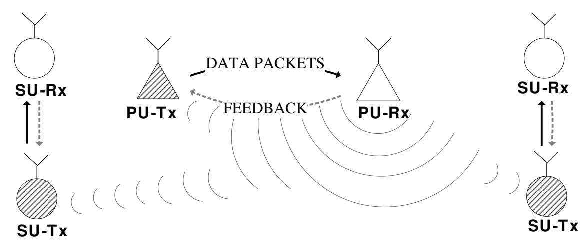

Fig. 1. Using PU feedback in multi-SU systems.

Fig. 1 provides a simple illustration where SUs can overhear the feedback from the PU-Rx to the PU transmitter (PU-Tx). This feedback information enables a SU to observe the performance of PU-Rx (affected by one or more SUs), and adjust its own access parameters accordingly. The benefits of using such link level feedback information are multi-folds: (i) It enables explicit protection on the PU-Rx through feedback monitoring, especially in the presence of multiple SUs; (ii) It facilitates *distributed access control* of multiple SUs; (iii) It permits different levels of interaction between PUs and SUs; (iv) It leads to more efficient spectrum usage through learning; (v) It is robust and adaptive to changes due to network load fluctuation and radio environment dynamics.

The proposed new framework requires that cognitive radio be smart enough to receive and decode link control information from the primary network that has strict interference constraint. Such a framework is particularly suitable for cases where both primary and secondary networks belong to the same operator or interest group. In particular, secondary networks of lower priority opportunistically access spectrum nominally but not fully occupied by (legacy) PUs of higher QoS/access priorities. The DARPA XG project is one such example where secondary cognitive radios access underutilized spectrum of legacy primary users. Another example involves cognitive femto cells attempting to access cellular spectrum (of the same service provider). Such applications offer clear incentives for PUs to conditionally allow cognitive SU access and to permit more flexible and aggressive spectrum sharing, often without regulatory changes. Practically, they require cognitive radio networks to flexibly adapt to dynamic channel environment and to effectively cooperate in spectrum sharing.

We focus on decentralized control of SUs in order to accommodate more general applications for which centralized control may not be feasible (e.g. ad hoc networks). In other cases, the cost to retrofit existing infrastructure, either hardware or software, could be expensive, time consuming, or disruptive. Such situations call for distributed intelligent SU access protocols.

As a concrete step toward this goal, we study a primary system with an outage probability requirement. The PU pairs exchange 1-bit outage feedback information that can be overheard and exploited by (multiple) SUs. The objective of the SUs is to maximize the total spectrum utility while satisfying the outage probability constraint set by the PU operator for allowing SU access. The challenge is to achieve this goal in a distributed manner by utilizing the 1-bit outage feedback, in the absence of PU cooperation, central controller/monitor, or SU information exchange.

Our contributions are as follows: 1) we propose a novel framework for cognitive spectrum access under PU link quality constraint based on PU-Rx feedbacks; 2) we formulate the cognitive spectrum utilization problem as a convex optimization problem through practical approximation, and derive its closed-form solution; 3) we present a distributed access power control algorithm for individual SUs without explicit central control or information exchange; 4) we present performance bounds as well as heuristic algorithms to tackle dynamic changes in the system or the environment; and 5) we show that our scheme provides a decentralized access control alternative to the well known interference temperature control problems.

II. Related Work

Our work differs from preceding works on power control in at least one of the following aspects: 1) our system does not require a centralized control or an interference monitor; Instead, it depends explicitly on the inherent PU link feedback; 2) SUs are not required to exchange information among themselves, thereby significantly reducing the overhead cost and protocol level complexity; 3) our goal is to maximize total SU spectrum utility instead of each SU utility while jointly and distributively satisfying the outage requirement of the PU.

Distributed power control for cellular systems has been studied extensively. For example, in [9], the authors studied the convergence of a simple distributed power control algorithm to a feasible solution that satisfies the target SIR requirement for each user. The authors of [10] proposed a framework for the joint optimization on cell selection and power control algorithm for cellular uplinks. For wireless multihop networks, the authors in [11] proposed using both power price and external interference price/compensation to adjust transmit power in a distributed way for nodes in multihop wireless networks. They also provided convergence analysis through super-modular game theory. In [12], a joint optimization problem on flow rate at transport layer and transmit power at physical layer is studied, advocating the use of layering as optimization decomposition to achieve maximum network utility. However, message (either Lagrange multiplier on the resource constraints or external interference price) passing among different nodes is generally required for both works [11], [12]. The above works assume static channel models, and thus should be performed in the same time scale as the channel fading process.

There also exist works with random channel models. For example, centralized power control with outage probability requirement under Rayleigh and log-normal fading channels has been considered in [13], [14]. In [15], various QoS optimization problems were formulated and solved using geometric programming for Rayleigh fading models for multiple users. However, the distributed algorithm therein requires message passing among users.

Most related is the work using power control to mitigate the interference on PUs while maximizing the spectrum usage of SUs. For example, in [16], the authors applied dynamic programming to allocate transmit power over multiple primary channels to maximize the average sum-rate of SUs. The PUs considered therein are either busy or idle. In [17], the authors proposed to adjust the SU transmit power based on the spectrum sensing results. In essence, these works fall into the LBT category. In [18], the authors proposed an auction based power allocation framework for a group of spread spectrum user to share spectrum with an interference temperature constraint at a fixed measurement point, in which a central controller (manager) is required to collect bids from distributed users. In [19], Centralized joint beamforming and power control was studied for SUs with multiple antennas. In [20], the authors formulated a power allocation game which considers both the interference temperature constraint at the PUs and the QoS requirement at the SUs. Monitoring stations are required to report the value of the dual variable at every iteration for their proposed distributed algorithms. In our preliminary work [21], we presented results on using the PU ACK/NACK information to maximize the utility of a single SU. In this paper, we focus on multiple SUs.

There are also power control algorithms for cognitive radio networks to achieve efficient and fair usage on the common spectrum resources without explicit protection constraint on the PU QoS (e.g., [22]). In our problem setting, the PUs have stringent requirement and do not participate in the power control algorithm.

III. System Model and Basic Assumptions

We consider the scenario in which multiple SUs are allowed to share the spectrum designated for a PU network in a non-intrusive manner. The non-intrusiveness has two-fold meanings. First, PU, with a higher spectrum access priority, is allowed to set the permissible level of interference or disruption from SU transmissions. This level of interference is controllable, which can be used by the PU networks to set a price for SU access. Second, the legacy PUs do not actively cooperate with SUs, i.e., such a SU overlay requires no change to PU system software/hardware. Deployment of such SU networks is faster, less disruptive, and less costly.

We consider a PU link comprising two network nodes that communicate using a forward link and reverse link. The forward link delivers primary traffic data from the PU-Tx to the PU-Rx, while the reverse link carries feedback control information from the PU-Rx to the PU-Tx. We denote the transmit power of the PU-Tx as P_0. For the cognitive network, we consider M secondary transmitters (SU-Tx's), that are geographically spread around the PU nodes. Let $\mathbf{P} = [P_1, \cdots, P_M]$ denote the transmit powers of the M SUs. The SUs adapt their access of the shared spectral band on the forward link through power control. For convenience here, we further assume that the primary channel utilization is slotted. The intelligent SUs have synchronized their spectrum access to the time-slot clock by, e.g., listening to the timing pilot in the broadcast control channel of the PUs (e.g., [23]).

To determine the interference level at the PU-Rx due to SU transmissions, let G_i be the large-scale processing gain between the SU-Tx i and the PU-Rx. Let F_i denote the random small-scale channel fading gain due to multipath and mobility. Note that the special index of $i = 0$ denotes the transmission from the PU-Tx. We consider cases in which G_i remains almost unchanged while F_i varies from slot to slot. We assume a non-line-of-sight radio transmission environment among all transmitters and the PU-Rx. For example, the PU-Rx is a mobile device located in a hot spot in urban areas. In this case, we can apply a Rayleigh fading channel model, where F_i follows independent exponential distribution with unit mean. Thus, the received power at the PU-Rx from SU i and its average are, respectively,

$$P_i G_i F_i, \qquad \text{and} \qquad \mathbf{E}[P_i G_i F_i] = P_i G_i. \tag{1}$$

Let N_0 be the white Gaussian noise power at the PU-Rx. The (random) signal-to-interference-noise-ratio (SINR) at the PU-Rx as a result of the fading channel environment is:

$$\gamma = \frac{P_0 G_0 F_0}{N_0 + \sum_{i=1}^{M} P_i G_i F_i}. \tag{2}$$

Even in the absence of SUs, random channel fading renders zero outage impossible. In this case, based on the exponential distribution of F_0, the outage probability of the PU-Rx is

$$\eta_0 = 1 - \exp(-\frac{N_0 \gamma_{th}}{P_0 G_0}). \tag{3}$$

where γ_{th} is the desired SINR value.

To maintain its quality of service (QoS), the PU would require its outage probability in the presence of SUs to stay below a certain threshold, denoted by η, where $\eta \geq \eta_0$ to allow secondary access. This constraint is expressed as:

$$\Pr[\gamma \leq \gamma_{th}] \leq \eta. \tag{4}$$

We assume that the PU protection requirement η was declared *a priori* to the SUs. In the optimum closed form solution, η_0 is assumed to be known. However, in our distributed algorithm, η_0 is unknown to SUs.

There clearly exists a trade-off between the signal quality at the PU-Rx characterized by its outage probability and the spectrum utilization of the SUs. A desired trade-off can be achieved by choosing an appropriate value for η to allow satisfactory QoS at the PU-Rx while still receiving maximum possible compensation from SUs that were permitted to access the bandwidth for communications.

To enable PU QoS protection, we exploit the feedback from the PU-Rx to the PU-Tx. In particular, the PU-Rx sends 1-bit feedback to the PU-Tx, indicating whether an outage (i.e., $\gamma < \gamma_{th}$) happened in the last time slot. This information can be overheard (with potential error) by all SUs. Based on such feedback information, SUs then make learned decisions on their transmit powers in a distributed manner to satisfy the PU outage probability.

We assume that SUs are deployed without coordination. They may not be aware of one another and cannot exchange information. This scenario is likely, especially when $P_i \ll P_0$, which is typical in cognitive radio networks. Therefore, we focus on cases where SUs are separated widely, as shown in Figure 1. SUs cannot communicate with each other and also do not generate interference to each other, although their interference at the PU-Rx aggregates. When SUs can interfere/communicate with each other, existing literature on access and power control can be applied.

To quantify the spectral utility of SUs, we assume that the utility function of the SU pair i can be characterized by $\log(1+h_i P_i)$, where h_i is a given constant reflecting the impacts such as modulation, interference level, and transmission range of the SU pair. It is clear that this utility definition is related to the information rate that can be reliably conveyed on the ith SU link. We also impose a physical limit on the transmission power, $P_i \leq P_{\max}, i = 1, \cdots, M$.

IV. Secondary Power Control Algorithms

For the SUs, the objective here is to maximize the total utility of all SU pairs while satisfying the PU outage requirement. Challenges are multi-folds: 1) PU is oblivious to SU activities and only reports its own outage; 2) SUs do not exchange information among themselves; 3) Dynamic PU/SU traffic activities require adaptive SU access algorithms.

To achieve our objective, we first formulate our problem as a convex optimization problem and derive its closed-form solution. We then propose a distributed algorithm which can approximate the optimal solution without the aid of either a central controller or information exchange among SUs. We present heuristic schemes to address the dynamics of SU systems. Finally, we study the connection between the traditional interference temperature constraint and our outage probability constraint.

A. Constrained Optimization for SU Power Control

Our objective is to maximize the total SU utility under the PU outage probability constraint. The formal description is

$$\begin{aligned} &\text{maximize} && f_0(\mathbf{P}) = \sum_{i=1}^{M} \log(1 + h_i P_i) \\ &\text{subject to} && \Pr[\gamma \le \gamma_{th}] \le \eta. \end{aligned} \tag{5}$$

For the Rayleigh fading channel, the outage probability at the PU-Rx for a given SU transmit power vector $\mathbf{P}$ is [13]:

$$\Pr[\gamma \le \gamma_{th}] = 1 - \exp(-\frac{N_0 \gamma_{th}}{P_0 G_0}) \prod_{i=1}^{M} \left(1 + \frac{P_i G_i \gamma_{th}}{P_0 G_0}\right)^{-1}. \tag{6}$$

To simplify the notations, we define: $\mu = (1 - \eta_0)/(1 - \eta)$, which can be interpreted as the relative margin to accommodate SU transmissions. Clearly, we expect $\mu \ge 1$. We also define $b_i = \frac{G_i \gamma_{th}}{P_0 G_0}$, which quantifies the unit interference effect from SU-Tx i to the PU-Rx. Then, we have the following constraint on $\mathbf{P} = [P_1, \cdots, P_M]$:

$$\prod_{i=1}^{M} (1 + b_i P_i) \le \mu, \tag{7}$$

which is an upper bound on a posynomial function of $\mathbf{P}$. However, the feasible set defined by (7) is non-convex. Thus, the optimization problem in (5) is nonlinear and non-convex.

When $h_i P_i \gg 1$ (at high SINR), we can approximate the utility function by $\log(h_i P_i)$. Therefore, the objective function is $\sum_i \log(h_i) + \log(P_i)$. Henceforth, without loss of optimality, we can ignore constants $\{h_i\}$. Adopting the technique of geometric programming [24], we can perform a variable transformation, $y_i = \log(P_i), i = 1, \cdots, M$. Denoting $\mathbf{Y} = [y_1, \cdots, y_M]^T$, the resulting optimization becomes

$$\begin{aligned} &\underset{\mathbf{Y}}{\text{minimize}} && -\sum_{i=1}^{M} y_i \\ &\text{subject to} && \sum_i \log\left(1 + b_i e^{y_i}\right) \le \log \mu. \end{aligned} \tag{8}$$

The objective function in (8) is linear in $\mathbf{Y}$, and the constraint function is convex in $\mathbf{Y}$ (its Hessian matrix is a diagonal matrix of positive elements). As a result, we now have (8) as a convex optimization problem which can be solved numerically with efficiency in a centralized manner and may be amenable to a distributed implementation.

B. Closed-Form Solution

A closed form solution to (8) starts with the Lagrange function with a multiplier $\lambda \ge 0$:

$$\begin{aligned} L(\mathbf{Y}, \lambda) &= -\sum_{i=1}^{M} y_i + \lambda \left(\sum_{i=1}^{M} \log(1 + b_i e^{y_i}) - \log(\mu) \right) \\ &= \sum_{i=1}^{M} \left(\lambda \log(1 + b_i e^{y_i}) - y_i\right) - \lambda \log(\mu) \\ &= -\sum_{i=1}^{M} y_i + \lambda \left(-\log(\eta) + \log(\Pr[\gamma \le \gamma_{th}])\right). \end{aligned} \tag{9}$$

The dual function of the primal problem (8) is:

$$\begin{aligned} q(\lambda) &= \inf_{\mathbf{Y}} L(\mathbf{Y}, \lambda) \\ &= \sum_{i=1}^{M} \inf_{y_i} (-y_i + \lambda \log(1 + b_i e^{y_i})) - \lambda \log(\mu) \end{aligned} \tag{10}$$

For $\lambda > 1$, the minimum of $\inf_{y_i}(-y_i + \lambda \log(1 + b_i e^{y_i}))$ is attained uniquely at:

$$y_i^*(\lambda) = -\log((\lambda - 1) b_i), \qquad i = 1, \cdots, M. \tag{11}$$

On the other hand, if $\lambda \le 1$, we have the minimum as $-\infty$. Therefore, the dual of the primal problem (8) is:

$$\begin{aligned} &\text{maximize} && q(\lambda) \\ &\text{subject to} && \lambda > 1. \end{aligned} \tag{12}$$

Clearly, there exists a $\mathbf{Y}$ such that the constraint is satisfied with strict inequality (by setting $y_i = 0, i = 1, \cdots, M$). Hence, by Slater's condition, the strong duality holds and the duality gap is zero between the dual and primal optimum. The optimal power control can then be expressed as:

$$P_i^* = \frac{1}{(\lambda^* - 1) b_i}, \qquad i = 1, \cdots, M, \tag{13}$$

where $\lambda^* = \text{argmax}_{\lambda > 1} q(\lambda^*)$ is the optimal solution to the dual problem.

From (13), we observe that $P_i^* b_i$ is a constant for $i = 1, \cdots, M$. In other words, at the optimal point, each SU generates the same amount of average interference at the PU-Rx. Because the optimal solution is obtained when the outage probability constraint is tight, we have:

$$P_i^* = \frac{\mu^{1/M} - 1}{b_i}, i = 1, \cdots, M. \tag{14}$$

The corresponding optimal Lagrange multiplier is $\lambda^* = (\mu^{1/M} - 1)^{-1} + 1$. We note that while the above closed-form solution does not consider the maximum transmission power constraint $P_{\max}$, it can be easily incorporated.

C. Distributed Algorithm

To apply the closed-form solution of (14), we need a central controller or a database that tracks and broadcasts the number of active SUs, M, and PU's own outage probability, η_0. Such a system may not be practical because of infrastructure cost, or the difficulty to set up a control channel to exchange channel information among SUs. Hence, we wish to derive a distributed mechanism that can utilize the inherent PU feedback information.

Recall that the PU-Rx transmits a 1-bit indicator to the PU-Tx to signify whether the SINR at the PU-Rx falls below the required SINR threshold in each time slot. Such information reflects the reception quality at the PU-Rx, and can be used to infer the aggregated interference from all SUs to the PU-Rx on the forward link.

Note that the objective function is separable and the optimization variables are only coupled through the outage probability constraint. Therefore, we apply the decomposition method of Lagrange relaxation of the coupling constraint [25], and adopt the subgradient method to solve the dual problem (12). So long as we find the optimal dual variable λ, we can obtain the optimal power control solution through (13). Since $q(\lambda)$ is affine in λ, a subgradient of it at λ is:

$$g = -\log(\eta) + \log(\Pr[\gamma \le \gamma_{th}]), \tag{15}$$

where $\log(\Pr[\gamma \le \gamma_{th}])$ is determined by the primal variable vector $\mathbf{Y}^*(\lambda)$ whose components $y_i^*(\lambda)$s are found from (11). In short, the subgradient is equal to the residual of the constraint expressed in the convex form of the optimization problem (8). We present the distributed power control (DPC) algorithm below.

Algorithm 1 Distributed Power Control (DPC)

1: Initialize: $k = 0$, $\lambda(0) > 1$, $P_i(0) = \frac{1}{(\lambda(0)-1)b_i}$

2: Observe: for the k-th updating period with T time slots, record N_k, the number of outage events during time slot $[(k-1)T + 1, kT]$.

3: Estimate:

$$\hat{\eta}(k) = \begin{cases} 1/T, & \text{if } N_k = 0, \\ \frac{N_k}{T}, & \text{otherwise.} \end{cases} \tag{16}$$

4: Update:

$$\begin{aligned} \lambda(k+1) &= \lambda(k) + a(k)(-\log(\eta) + \log(\hat{\eta}(k))), \\ P_i(k+1) &= \min\left(\frac{1}{(\lambda(k+1)-1)b_i}, P_{\max}\right), \end{aligned} \tag{17}$$

where $a(k)$ is any positive step size satisfying:

$$\sum_{i=0}^{\infty} a(k)^2 < \infty, \quad \sum_{i=0}^{\infty} a(k) = \infty \tag{18}$$

5: Go to Step 2.

One main concern of the DPC algorithm is that $(-\log(\eta) + \log(\hat{\eta}(k))$ is a noisy and **biased** estimate of the subgradient. For convenience, define $g(k) \in \nabla q(\lambda(k))$, as the subgradient at $\lambda(k)$. We next prove the convergence property of the algorithm as $T \to \infty$, as well as provide an error bound on finite T.

Proposition 1: In Alg. 1, $P_i(k) \to P_i^*$ almost surely when $T \to \infty$ and $k \to \infty$.

Proof: We prove this result by showing that the Lagrange multiplier $\lambda(k)$ converges to λ^* almost surely. Given the independent Rayleigh fading assumption, the outage event is independent across time. By the strong law of large numbers, when $T \to \infty$, $\hat{\eta}(k)$ converges to the outage probability almost surely[1]. Consequently,

$$\mathbf{E}[-\log(\eta) + \log(\hat{\eta}(k))] = g(k). \tag{19}$$

In other words, $-\log(\eta) + \log(\hat{\eta}(k))$ is a (noisy) unbiased subgradient of $q(\lambda)$ at $\lambda(k)$ when $T \to \infty$.

[1]When $T \to \infty$, $P(N_k = 0) \to 0$, and thus the truncation in (16) is immaterial.

In addition, because of the maximum power constraint on SUs, we have

$$|g(k)| \le \sum_{i=1}^{M} \log(1 + P_{\max} b_i) - \log(\eta),$$

i.e., the subgradient of $q(\lambda)$ is bounded. Furthermore, for a system with a finite number of SUs, i.e., $M < \infty$, the distance between the initial value $\lambda(0)$ and λ^* is bounded by

$$|\lambda(0) - \lambda^*| \le \lambda(0) + \frac{1}{\mu^{1/M} - 1} + 1 < \infty. \tag{20}$$

According to Theorem 46 of [26], we have, for arbitrary $\delta > 0$,

$$\lim_{k \to \infty} \Pr[|\lambda(k) - \lambda^*| > \delta] = 0.$$

With this convergence result, and the fact that the minimizer of $L(\mathbf{Y}, \lambda^*)$ is unique, the convergence of $P_i(k)$ to P_i^* follows. □

In practical systems, we need to choose a finite T to obtain a good estimate on the subgradient of $q(\lambda)$. We provide a bound on T such that the gap to $q(\lambda^*)$ from the noisy subgradient update method is within a given range. First, we need the following lemma.

Lemma 1: Let $\tilde{g}(k)$ be a noisy unbiased subgradient of $q(\lambda)$ at $\lambda(k)$, where $\mathbf{E}|\tilde{g}(k)|^2$ is bounded. Let a random sequence $\hat{g}(k)$ satisfy the following conditions:

1) There exists a G such that $\mathbf{E}|\hat{g}(k)|^2 \le G$;
2) $|\mathbf{E}((\hat{g}(k) - \tilde{g}(k))|\lambda(k))| \le \varepsilon$;
3) There exists a R such that $|\lambda(k) - \lambda^*| \le R$ almost surely.

Then, for the following update:

$$\lambda(k+1) = \lambda(k) + a(k)\hat{g}(k),$$

where $a(k) > 0$, and $\sum_{i=0}^{\infty} a(k)^2 < \infty$, $\sum_{i=0}^{\infty} a(k) = \infty$, we have the following error bound holds for its value $q(\lambda(k))$:

$$\lim_{k \to \infty} \max_{i=0,\cdots,k} (q^* - \mathbf{E} q(\lambda(i))) \le \varepsilon R.$$

Proof: From conditions 2) and 3), we have the inequality

$$\left(\mathbf{E}\left(\tilde{g}(k) - \hat{g(k)}|\lambda(k)\right)\right)(\lambda(k) - \lambda^*) \le \varepsilon R. \tag{21}$$

Then we have

$$\begin{aligned} &\mathbf{E}\left(|\lambda(k+1) - \lambda^*|^2 |\lambda(k)\right) \\ =&\mathbf{E}\left(|\lambda(k) + a(k)\hat{g}(k) - \lambda^*|^2 |\lambda(k)\right) \\ =&|\lambda(k) - \lambda^*|^2 + 2a(k)\mathbf{E}\left(\hat{g}(k)|\lambda(k)\right)(\lambda(k) - \lambda^*) \\ &+ a(k)^2 \mathbf{E}\left(|\hat{g}(k)|^2 |\lambda(k)\right) \\ \le&|\lambda(k) - \lambda^*|^2 - 2a_k(q^* - q(\lambda(k))) + \varepsilon R \\ &+ a(k)^2 \mathbf{E}\left(|\hat{g}(k)|^2 |\lambda(k)\right), \end{aligned}$$

where the last inequality follows from (21) and the definition of the subgradient $\mathbf{E}\left(\tilde{g}(k)\right)(\lambda(k) - \lambda^*) \le q(\lambda(k)) - q(\lambda^*)$.

Take expectation with regard to $\lambda(k)$ and apply recursion on k. Using $\mathbf{E}|\hat{g}(k)|^2 \le G$, we get

$$\begin{aligned}&\mathbf{E}\left(|\lambda(k+1)-\lambda^*|^2\right)\\ \le&\mathbf{E}\left(|\lambda(0)-\lambda^*|^2\right) - 2\sum_{i=0}^{k} a(i)\left(q^* - \mathbf{E}q(\lambda(i))\right)\\ &+2\sum_{i=0}^{k} a(i)(\varepsilon R) + G^2\sum_{i=0}^{k} a(i)^2.\end{aligned}$$

Then we have

$$\sum_{i=0}^{k} a(i)\left(q^* - \mathbf{E}q(\lambda(i))\right) \le R^2 + 2\sum_{i=0}^{k} a(i)\varepsilon R + G^2\sum_{i=0}^{k} a(i)^2.$$

Therefore,

$$\max_{i=0,\cdots,k}\left(q^* - \mathbf{E}q(\lambda(i))\right) \le \frac{R^2 + G^2\sum_{i=0}^{k} a(i)^2}{2\sum_{i=0}^{k} a(i)} + \varepsilon R.$$

When the step size $a(k)$ is chosen such that it is square summable but not summable, the lemma stands. □

The result described in Lemma 1 shows that with a moderately noisy and biased replacement for the true subgradient, the proposed update method for the dual variable λ finds the dual optimum within a moderate range. For the DPC, with our truncation on the estimate of outage probability in (16), we have $\mathbf{E}\left(\log(\hat{\eta}) - \log(\eta)\right)$ to be always bounded. Thus, the first condition is satisfied. With a practical maximum and a minimum[2] power constraint, together with similar argument in (20), the third condition in Lemma 1 is satisfied. Next we reason that with T sufficiently large, the second condition in Lemma 1 is also satisfied. For notational convenience, define $p = \Pr[\gamma \le \gamma_{th}]$; and let a random variable Z denote the estimated outage probability. First, note that for $T \ge 10$, the distribution of Z, $f_Z(z)$, can be very well approximated by a Gaussian distribution with mean p and variance $\frac{p(1-p)}{T}$. Then, we have for $\delta < p$,

$$\begin{aligned}&\mathbf{E}\left(\log(Z) - \log(p)\right)\\ =&\int_{p-\delta}^{p+\delta}\left(\log(z) - \log(p)\right) f_Z(z)dz\\ &+\int_{|z-p|>\delta}\left(\log(z) - \log(p)\right) f_Z(z)dz\\ \le&\max\left(|\log(\frac{p-\delta}{p})|, |\log(\frac{p+\delta}{p})|\right)\Pr[|Z-p| \le \delta]\\ &+\max\left(|\log(p)|, |\log(\frac{1}{T}) - \log(p)|\right)\Pr[|Z-p| > \delta]\\ \le&\frac{\delta}{p} + |\log(p)|\Pr[|Z-p|>\delta] + O(\delta^2)\\ \le&\frac{\delta}{p} + \exp(-\frac{\delta^2 T}{2p(1-p)}) + O(\delta^2),\end{aligned} \tag{22}$$

[2] A minimum power constraint means that if the required power is smaller than the minimum threshold, the SU does not transmit.

where we apply a well-known upper bound function on the Q-function. To satisfy the accuracy requirement, we let $\frac{2\delta}{p} = \frac{\varepsilon}{2}$, and $\exp(-\frac{\delta^2 T}{2p(1-p)}) \le \frac{\varepsilon}{2}$, leading to an upper bound on required T as:

$$T^u = -\frac{32(1-p)\log(\varepsilon)}{p\varepsilon^2}.$$

Hence, with $T \ge T^u$, we can guarantee that of our algorithm is as good as q^* within a range of εR. The above bound on T provides us an estimate of the scale of its value, though it may be relatively loose. In practice, $T = 100$ appears to be sufficient for obtaining a good error margin, and therefore is used in all our simulations.

D. Dynamic Secondary User Systems

With random arrivals and departures of SUs, different SUs can have different values of λ upon the initialization of its subgradient method. Denote the ith SU's local version of λ as λ_i. When λ_is differ, the observed violation in the outage probability constraint is no longer the subgradient of $q(\lambda)$. In this case, λ_i may be trapped on the boundary of the feasible set, which generates transmit powers that satisfy the outage probability constraint, but not optimal. This problem manifests in cognitive radio networks because the PU does not cooperate and we do not assume a central controller.

To solve this problem caused by the dynamic nature of SUs, we propose to modify the distributed algorithm heuristically to limit the effect of its long term memory (initial point). We assume that SUs follow the same timing update, i.e., they can only join or depart the system at the start or the end of any updating period $[(k-1)T+1, kT]$. This condition can be satisfied by synchronizing with the timing clock of the primary systems. Suppose that the SU-Tx i joins the spectrum sharing network at updating period t_i. At the k-th updating period, it keeps a record on previous observations $\hat{\eta}(j), j = \max\left(k+1-W, t_i\right), \cdots, k$, where W denotes the record length. The modified algorithm can then update

$$\lambda_i(k) = \lambda_0 + \sum_{j=\max(k+1-W,t_i)}^{k} a(j)(\log(\hat{\eta}(j)) - \log(\eta)), \tag{23}$$

where λ_0 is a preset value and $a(j)$ is the step size, both known to all SUs. Note that the algorithm does not require the SU to be aware of other SU activities, or a global clock k. We call this modified algorithm “finite-horizon distributed power control (FDPC)”.

In FDPC update, when all active SUs have remained active longer than W, their local versions of λ become the same. This behavior approximates the true subgradient method while the SUs discard their old observations in updating. In addition, the larger the value of W, the longer time it takes for all SUs to reach an agreement. However, there exists less fluctuation in the updating process.

There is a tradeoff in choosing $a(j)$. On one hand, more recent update provides better information. Hence, $a(j)$ should be an increasing function. One the other hand, too much emphasis on recent information can lead to large update fluctuation and poor convergence. As a practical compromise, we let $a(j)$ be a constant. Simulation results positively affirm such a choice, and we are investigating the convergence property of the proposed algorithm under different choices of step size.

The FDPC algorithm requires a memory at each SU to store its observation record. Alternatively, we can also adopt an iterative algorithm by adding a forgetting factor to remove the storage requirement. Let $0 < \beta < 1$. Each SU updates via

$$\lambda_i(k+1) = \beta\lambda_i(k) + a(\log(\hat{\eta}(k)) - \log(\eta)). \qquad (24)$$

When $k \to \infty$, the effect of the initial discrepancy on the local copies of λ vanishes. Here, $\beta \approx 1$ is a design parameter that balances the convergence speed and the sub-optimality bound when $\lambda_i(k)$ stabilizes. With a properly chosen β, performance results similar to those of the FDPC algorithm have been observed.

V. Interference Temperature

In cognitive radio networks, interference temperature (or interference power) is often used as a PU protection metric (e.g., [18], [20]). In this section, we explore the relationship between the interference temperature and the outage probability for Rayleigh channel model. It serves two goals: 1) it provides upper/lower bounds on the performance of the original optimization problem (5), and thus can be used to evaluate our approximation, and 2) it shows our proposed scheme can be applied to the well-known interference temperature problem.

Define the certainty-equivalent SINR at the PU-Rx as [13]:

$$\gamma_{ce} = \frac{P_0 G_0}{N_0 + \sum_{i=1}^{M} P_i G_i}, \qquad (25)$$

which can be interpreted as the SINR by assuming fading-free channels with fixed $F_i = 1$. As in [13], the certainty-equivalent SINR margin (CEM) is defined as γ_{ce}/γ_{th}.

Following a similar approach, we obtain the upper and lower bounds on the outage probability that includes the effect of Gaussian noise (ignored in [13]). An upper bound is:

$$\Pr[\gamma < \gamma_{th}] \le 1 - \exp(-\frac{1}{\text{CEM}}); \qquad (26)$$

and a lower bound is:

$$1 - (1-\eta_0)(1 + \frac{1}{\text{CEM}} - \frac{\gamma_{th} N_0}{P_0 G_0})^{-1} \le \Pr[\gamma < \gamma_{th}] \qquad (27)$$

When the PU outage probability constraint is low (e.g., less than 10%), both the lower and upper bounds are fairly tight. Therefore, we can approximate the original problem (5) by replacing the outage probability with its upper/lower bound.

Applying the upper bound on the outage constraint transforms the optimization problem into

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{M} \log(1 + h_i P_i) \\ \text{subject to} \quad & 1 - \exp(-\frac{1}{\text{CEM}}) \le \eta. \end{aligned} \qquad (28)$$

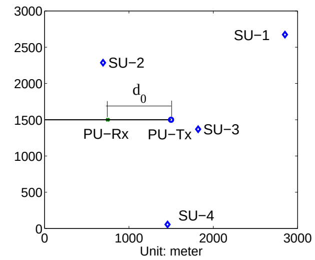

Fig. 2. User locations.

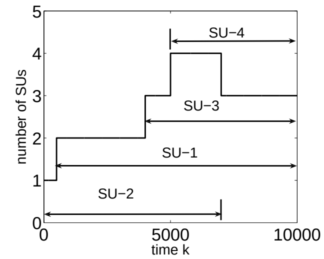

Fig. 3. SU dynamics

The constraint is equivalent to

$$\sum_{i=1}^{M} P_i G_i \le \frac{-P_0 G_0 \log(1-\eta)}{\gamma_{th}} - N_0. \qquad (29)$$

Note that this approximation is conservative in terms of protecting the PU-Rx. This is a convex optimization problem, easy to solve numerically. Similarly, we can obtain a more aggressive SU utility optimization problem when we apply the following lower bound as the constraint:

$$\sum_{i=1}^{M} P_i G_i \le \frac{\eta - \eta_0}{1-\eta} \frac{P_0 G_0}{\gamma_{th}}. \qquad (30)$$

Eq. (29) and (30) are average interference power constraints. Given such a constraint, the existing power control algorithms either require a central controller, or a measurement device co-located with the PU-Rx updates the price of violating the constraint (e.g., [20]) for multiple SUs. In contrast, our outage-based algorithm relies only on the inherent 1-bit feedback from the PU-Rx. Thus, one can transform the average interference temperature constraint into a more conservative outage probability constraint using the CEM model. Our distributed power control algorithm can then be applied in the absence of controller/monitors.

VI. Simulation Results

In this section, we present simulation results for our proposed power control algorithms. We set up a system with multiple SUs and one PU pair with their locations shown in Fig. 2. For SUs, only transmitters are shown. The simulation parameters are set as: $\eta = 10\%$, $N_0 = -100$dBm, $P_0 =$ 33dBm, $P_{\max} = 33$dBm, $M = 4$, $G_i = d_i^{-4}, i = 0, 1, \cdots, 4$, where d_i is the distance from transmitter i to the PU-Rx, and $\gamma_{th} = 6$.

A. Gap to the Optimum

We first compare the total utility achieved by the certainty-equivalent model (both upper bound and lower bound approximations) and the closed-form solution. We vary the location of the PU-Rx by moving it along the line shown in Fig. 2. The value of d_0 varies from 50 meters to 750 meters. The smaller the value of d_0, the stronger the primary signal strength at the PU-Rx, and thus the smaller the value of η_0. With a larger

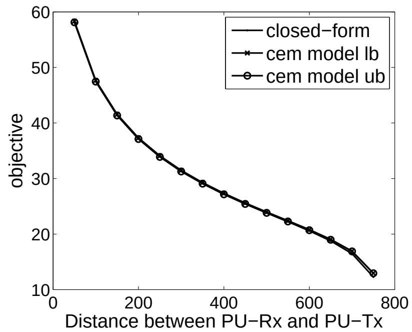

Fig. 4. Gap between the optimal utility and closed-form approach.

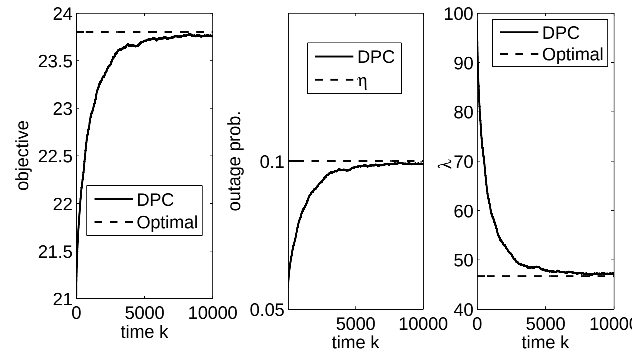

Fig. 5. Simulation results for DPC algorithm

margin on the outage probability $\eta - \eta_0$, the total SU utility increases (as the PU-Rx moves closer to the PU-Tx).

In Fig. 4, we observe that the SU performance obtained using our closed-form solution is almost indistinguishable to those of the upper and lower bound approximation using (29) and (30), respectively. Therefore, our approximation is tight for the original problem (5). In addition, one can convert the interference temperature problem to our problem with litter performance difference (while adopting our distributed scheme).

B. Distributed Power Control for Static Systems

Next, we present simulation results for the proposed DPC algorithm. The distance from the PU-Rx to its transmitter is set as $d_0 = 500$ meters, with the corresponding value of $\eta_0 = 1.9\%$. In the simulation, we set initial point of λ as $\lambda_0 = 100$, which is large enough such that SUs start with a small transmit power, and thus do not cause abrupt distortion on PU QoS. We choose the step size[3] as $a(k) = 20/\sqrt{k}$, and $T = 100$. Considering a packet length of $1ms$, the value of T means that the frequency of the SU power control is about 10Hz. The simulation results are shown in Fig. 5. We can observe that the algorithm converges well with an acceptable convergence speed. The transmit powers of SUs are $[33.0, 18.0, 17.7, 28.2]$ dBm when $k = 10000$. This shows that the transmit power of a SU depends on its distance to the PU-Rx. Specifically, SU-Tx's that are far away from the PU-Rx can transmit with high powers. The spectrum utility is more efficient than the LBT approach with which SUs only transmit when the PU-Tx transmission signal is sensed to be absent.

C. Distributed Power Control for Dynamic Systems

To study the performance of the proposed algorithm in a dynamic system, we set up a simulation where the number of active SUs varies over time, as shown in Fig. 3. Each SU applies the FDPC update algorithm of Sec. IV-D to adjust its local parameter λ_i, and to control its transmit power accordingly. We set the parameters of the algorithm as $\lambda_0 = 100$, $W = 1024$, and use a constant step size 10. The results are shown in Fig. 6. We can observe that the performance achieved by the proposed FDPC algorithm approaches the optimal total SU utility closely. In addition, the outage probability perceived by the PU-Rx is below the constraint $\eta = 10\%$ most of the time. We also plot the update progress of the local copies of Lagrange multiplier λ in Fig. 6. We can observe that the proposed FDPC algorithm is capable of synchronizing multiple SU updates and track the optimal value given sufficient time. The fluctuations around the optimal line is the result of the noisy observation used to estimate the subgradient and the use of a constant (non-diminishing) step size.

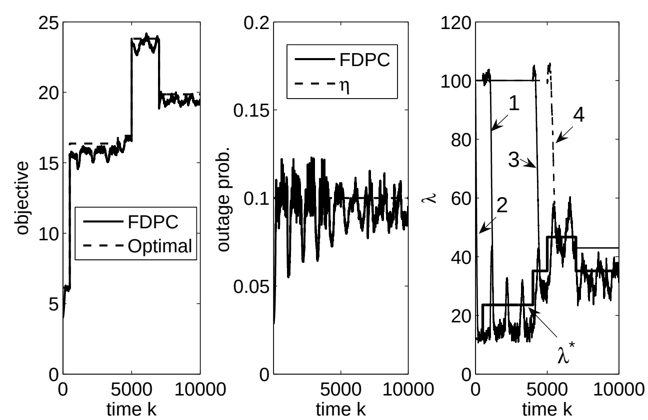

Fig. 6. Performance of FDPC algorithm.

We also test FDPC's performance when SUs cannot correctly decode all feedback packets. In the simulation, we assume that each SU decodes a PU feedback packet with a probability of 0.95 independently. A SU can only estimate the outage probability perceived by the PU-Rx using the correctly decoded feedback packets. In this case, SUs have different estimates of the outage probability. Results are shown in Fig. 7. We can observe that under the imperfect observation of the PU feedback, the updates at difference SUs are not synchronous, and have larger fluctuations. Nevertheless, the proposed algorithm enables the SUs to follow correct updating directions most of the time. We also observe from simulations

[3]Simulation results show better convergence behavior for $a(k) = c/\sqrt{k}$ than $a(k) = c/k$, where c is a positive constant.

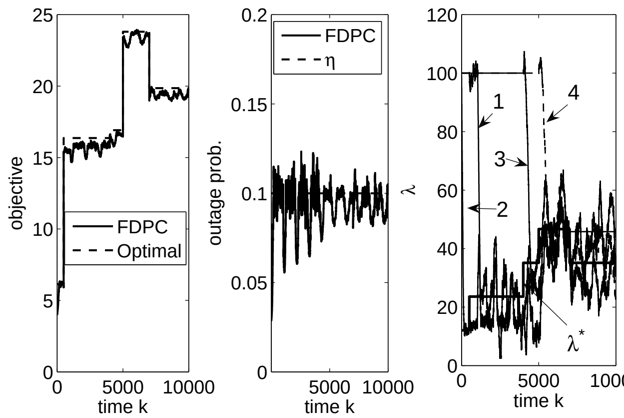

Fig. 7. Performance of FDPC algorithm with control packet loss.

that the total SU utility achieved is close to that without decoding errors. Furthermore, the outage probability constraint is well satisfied.

VII. Conclusions

We presented a novel access framework for cognitive radio networks based explicitly on inherent PU link feedback information. This structure represents a significant shift from the widely studied LBT strategy. Because the PU-Rx link feedback provides (partial) information with respect to the total impact on the PU link quality from the aggregated interference of all SUs, the new framework can provide explicit protection on the PU-Rx in the presence of multiple SUs, which may be geographically separated and do not communicate.

We considered one specific scenario where the SUs perform distributed power control based on the 1-bit outage information from the PU-Rx. We showed that the proposed distributed SU power control mechanism can maximize the total utility while satisfying the predetermined PU-Rx outage constraint. This optimized SU power control is achieved without PU cooperation, central controller/monitor, or inter-SU message passing. We also developed heuristic algorithms that can track dynamic change in the system or the wireless environment. We also provided a decentralized alternative to the well known centralized interference temperature control problem.

In future works, we plan to generalize our framework to include the more general cases in which both PU and SUs are adaptive. Additionally, we would also like to analyze and develop in-depth understanding on the SU performance under different security constraints. We are particularly interested in assessing the tradeoff between the security concerns and the reward of adopting unencrypted link control feedback by the PUs. We shall also investigate different PU system feedbacks and evaluate the convergence property of the proposed heuristic algorithms in a dynamic system.

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