Opportunistic Spectrum Access in Multiple Primary User Environments Under the Packet Collision Constraint

Eric Jung
University of California, Davis
Davis, CA 95616
Email: eajung@ucdavis.edu

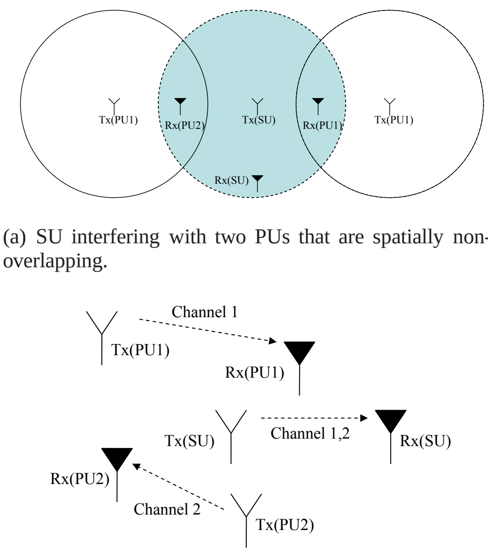
Xin Liu
University of California, Davis
Davis, CA 95616
Email: liu@cs.ucdavis.edu

***Abstract*—Cognitive Radio (CR) technology has great potential to alleviate spectrum scarcity in wireless communications. It allows secondary users (SUs) to opportunistically access spectrum licensed by primary users (PUs) while protecting PU activity. The protection of the PUs is central to the adoption of this technology, since no PU would accommodate SU access to its own detriment. In this paper, we consider an SU that must protect multiple PUs simultaneously. We focus on the PU packet collision probability as the protection metric. The PUs are unslotted and may have different idle/busy time distributions and protection requirements. Under general idle time distributions, we determine the form of the SU optimal access policy, and identify two special cases for which the computation of the optimal policy is significantly reduced. We also present a simple algorithm to determine these policies using principles of convex optimization theory. We then derive the optimal policy for the same system when a SU has extra "side information" on PU activity. We evaluate the performance of these policies through simulation.**

I. Introduction

The static nature of spectrum regulatory policy in the U.S. has led to an artificial scarcity of available spectrum. The FCC has estimated that 60% of the spectrum below 6 GHz is underutilized under the current allocation policy [1]. *Cognitive radio* technology has been considered to mitigate this problem [2]. It enables a secondary user to sense channel conditions and change its operating characteristics to opportunistically access unoccupied primary spectral bands. This new paradigm is typically referred to as *dynamic spectrum access* (DSA).

In the "hierarchical model" of dynamic spectrum access summarized in [3], users in the system are divided into a multi-tiered hierarchy where certain users have priority of channel access over others. Cognitive radio is conceived as a way for unlicensed "secondary users" (SU) to opportunistically access licensed spectral bands if "primary user" (PU) activity is protected from interference [4], [5]. This model is necessary because users will not agree to accommodate secondary networks to their own detriment. Therefore, a design imperative for a SU opportunistic access strategy is to minimize the SUs' effect on PU transmissions. For example, in the DARPA XG project [6], one of the three major test criteria in a cognitive radio prototype field test is "to cause no harm" [7]. Predictably, this is also one of the main bottlenecks of SU performance.



(a) SU interfering with two PUs that are spatially non-overlapping.

(b) SU accessing 2 PU channels simultaneously.

Fig. 1. SU in environments with multiple non-interfering PUs.

In this work, we consider packet collision probability as the PU protection requirement. Under this requirement, the SU must guarantee that the packet collision probability of a PU packet is less than a certain threshold specified by the PU. This type of constraint has already received some attention [8], [9], [10], [11].

In particular, we investigate SU performance in a system with multiple PUs with different packet collision probability constraints and usage patterns. There are common situations where multiple PU systems may need to be protected which may have different owners, priorities, and usage patterns. Two simple examples are cases where the SU interferes with multiple PUs which are *spatially* or *spectrally* non-overlapping, as depicted in Figure 1(a) and 1(b).

Our paper makes the following contributions.First, we derive the optimal policy for SU access in multiple-PU systems with stationary general idle time distributions. We focus on a class of *stationary* access policies, i.e. the same policies are applied every time the channel becomes idle. We also look specifically at multiple PU systems with exponential idle time distributions, as well as idle time distributions that result in "time-threshold" optimal policies. Finally, we extend our model to include two "side information" cases where extra information of PU activity is available to the SU. In the first case, the SU knows which PU is the last to become idle before an SU can transmit. In the second, the SU knows how long every PU has been idle before the channel becomes available. We investigate how such knowledge can affect the optimal policy and performance of the SU.

The paper is organized as follows. In the next section, we present some related work. We present our system model in Section III and define the objective function in Section IV. In Section V we derive an optimal transmission scheme for a channel with multiple PU constraints, where each PU has a general idle time distribution, and show that for certain idle time distributions, the optimal SU access policy can be found in closed form. The system model is extended in Section VI such that the SU has "side information" on PU activity, and the optimal policies with this new information are derived. In Section VII, we compare and analyze the performance of our different policies under the time capacity metric, and conclude our paper in Section VIII.

II. Related Works

In recent years, there has been an explosion of research in cognitive radio. A large portion of this research has been in spectrum overlay, where protocols are devised to maximize SU spectrum utility when PUs are idle and protect PU communication when they become busy. Within this paradigm, there are two focuses, spatial and temporal domain research [3].

In the former, SU activity is assumed to occur in a much faster timescale than the PU activity, and hence the spectral environment (i.e. PU channel occupancy) is treated as static. The main problem then becomes channel allocation between multiple SUs given certain topologies, different channel availabilities, and interference between SUs. In [13], [14], [15], [16], the interference between SU nodes is modeled as a conflict graph, with varying methods and metrics used to allocate channels. In [17], [18], the authors formulate channel selection as a mixed integer linear programming problem, under constraints on both power and channel availability. In all these works, PU activity is protected since each SU node only has access to idle PU channels. Our work does not fall in this category since we assume that PUs change states in a timescale similar to that of SU activity.

In the temporal domain category, PU activity varies quickly in the time domain and SUs within interference range must devise sensing and access schemes in concert to avoid significantly harming PU communication. As such, the metric used to measure interference to PUs is crucial. Several papers consider this power, referred to as *interference temperature*, as the key metric. For example, in [19], the authors consider multiple SUs operating in a multi-PU system where each PU has an average rate requirement and outage probability constraint, both functions of the interference power caused by SUs. Power control for different states of PU activity is considered in [20]. However, in 2007 the FCC officially ended consideration to establish an interference temperature standard for cognitive radio [21]. Their decision was based on several comments predicting a likely increase in interference with PUs in bands where it is used, stemming from the technical difficulties of implementing such an approach. Since we consider packet collision probability as our main constraint, we assume that *any* overlap of SU and PU activity in the same band results in collision with the PU, which is a more conservative measure of PU interference.

Like this work, other works have also considered packet collision probability. Several works have developed medium access schemes for SUs under this protection requirement [8], [9], [10]. One common formulation assumes a slotted system and is formulated using the Partially Observable Markov Decision Process (POMDP). For example, [22], [23], consider a slotted structure network with a single PU protection metric. Optimal access decisions are made by considering long observation history. Likewise, in [24], the authors consider a slotted system with a single CR, and model the problem as a multi-armed bandit problem to decide the best channel(s) to sense and access. In [11], the authors consider an overlay SU network on a multiple PU network with slotted structure, where PU access depends on Markovian evolution. Our work is different from all of these because we consider multiple different PU constraints that must be satisfied *simultaneously*. Many of these works also assume slotted activity, while our model is more general and can accomodate slotted and unslotted systems.

Other works have considered unslotted systems as well. In [8], the authors introduce the performance metric of *time capacity*, the average proportion of time that a SU can transmit without violating the PU's packet collision constraint, and generalize the results in [25]. The authors also extend their work to examine the impact of imperfect sensing on SU performance as well as access issues between multiple SUs in a single PU system. Our work in [26] generalizes the results in [25] to the multiple PU case for certain distributions, in particular for exponential distributions and distributions which result in time-threshold policies. Our work in this paper presents a framework that can be used for general idle time distributions for multiple PUs, and considers the effects of extra information about the PU system available to the SU. Much of the work in [25] can be generalized in this work as well.

III. System Model

In this section we layout the model for PU and SU activity. Generally, we consider an SU that operates within the interference range and on the same channels as multiple non-interfering PU networks. *Because the PUs are non-interfering,*

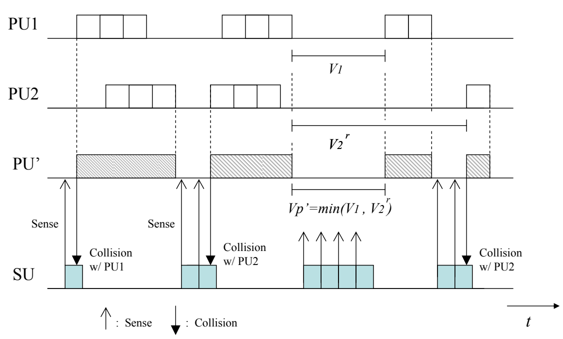


Fig. 2. Timing diagram for PU/SU system model.

simultaneous transmission of multiple PUs does not result in collision between them.

A. Primary User Model

In our system model, we assume there are M PUs that are independent and non-cooperative with the SUs and with each other. A PU accesses its channel without sensing. PUs are non-interfering among themselves, i.e. they can transmit at the same time. This can be due to spatial reuse as in Figure 1(a), or due to communication on separate non-overlapping channels, as in 1(b). We assume packet-based transmission for all PUs in this paper.

Figure 2 illustrates the activity of each user. All PU activity is assumed to be stationary and ergodic. This is reasonable for packet-based data traffic, where the timescale for packet lengths is on the order of milliseconds but arrival rates are commonly stationary on the scale of hours [27]. For PU i, V_i denotes the idle time, which is governed by pdf/cdf $f_{V_i}(\cdot)/F_{V_i}(\cdot)$. Let $E[V_i] = v_i$. The number of packets transmitted per busy period is random and denoted as N_i, with $E[N_i] = n_i$. We assume that the packet length is the same for all users, and normalize all activity to the length of a PU packet. Therefore, a busy period of the ith PU is N_i. The probability that each PU is idle is then $\alpha_i = \frac{v_i}{v_i+n_i}$.

We also define terms related to the union of the activity of all PUs in the system. We refer to this activity as *unionized PU activity* or simply the *unionized PU*. We denote the idle time of the unionized PU as V_p', which is governed by a probability distribution function $f_{V_p'}(\cdot)$, with $E[V_p'] = v_p'$. A busy period of the unionized PU is denoted N_p', with $E[N_p'] = n_p'$.

The probability that the unionized PU is idle is then

$$\alpha' = \frac{v_p'}{v_p' + n_p'} = \prod_{i=1}^{M} \alpha_i, \tag{1}$$

where the final equality results from the independence of the PU activity. The relationship between each PU's activity and the unionized PU activity will be investigated in greater detail in Section IV.

Each PU i also has a packet collision probability requirement denoted η_i, defined as the maximum allowable probability of collision for a packet of the ith PU. Over a time interval $[0, T]$, we denote the number of packets transmitted by the ith PU as $\mathcal{N}_i$, and the number of collisions experienced by that user as $\mathcal{N}_i^c$. The collision probability of the ith PUs' packets experiencing collision is denoted as

$$p_i^c = Pr[\text{packet collision of } i\text{th PU}],$$

where

$$p_i^c = \lim_{T\to\infty} \frac{N_i^c}{N_i},$$

and the PU protection requirement is thus

$$p_i^c \leq \eta_i, \quad i \in \{1, ..., M\}. \tag{2}$$

We assume that the collision constraints for all PUs are known to the SU *a priori*. These constraints must be satisfied for all PUs.

B. Secondary User Model

Throughout this work we assume that there is a single SU, which may be a single CR radio or a SU basestation operating within the range of multiple PU networks [28]. We note that SUs may have access to multiple bands, but low-cost SUs may have a set bandwidth requirement, and may not be able to frequently switch over channels at the timescales we are concerned with [25]. Therefore, we assume that the SU is on a fixed channel and cannot transmit on a sub-band of the channel. We reiterate here that the we are concerned with stationary optimal policies. A stationary policy is one that is applied whenever the channel becomes idle. We have:

a) Packet Length: The SU slot length is denoted Δ, and we assume $\Delta \ll v_i$, $\Delta \ll n_i$, and $\Delta \ll 1$ (i.e. the length of a PU packet). In this paper, we study the extreme case where $\Delta \to 0$. In [8] it was proven that this results in the best SU capacity in the case of no overhead cost for a single PU channel, and overhead techniques used in that paper can be applied to our work as well. This assumption simplifies analysis greatly, and our simulations show that non-zero packet lengths result in negligible differences.

b) Sensing: We assume perfect sensing by the SU, i.e. that the SU can always detect the presence or absence of a PU, and that sensing time is negligible. The SU follows the listen-before-talk (LBT) principle, where the SU senses the channel in each slot before allowing transmission. We assume that sensing occurs over the *entire* band of interest. The multiple channel sensing problem is a significant challenge and beyond the scope of this work [28]. Our previous works consider imperfect sensing [8], [25], and these results apply here as well. We can also consider a fixed non-zero sensing time in the case of non-zero slot lengths. A fixed non-zero sensing time can represent many commonly proposed sensing schemes, such as matched filter, energy, and feature detection. Since we assume a LBT scheme, the performance in non-zero sensing cases is essentially a fixed fraction of the optimal performance.

TABLE I
NOTATION FOR 2-PU SYSTEM

$f_{V_i}(t)$	PDF of PU i idle time
$F_{V_i}(t)$	CDF of PU i idle time
$f_{V_i}^r(t)$	PDF of PU i residual idle time
$F_{V_i}^r(t)$	CDF of PU i residual idle time
$f_{V_p'}(t)$	PDF of unionized PU idle time
$F_{V_p'}(t)$	CDF of unionized PU idle time
n_i	PU i average number of packets per transmission
n_p'	average length of busy period for unionized PU
α_i	Probability PU i idle
α'	Probability all PUs idle
p_{si}	Probability PU i starts the idle period
$f_{si}(t)$	Idle time PDF given PU i starts the current idle period
$F_{si}(t)$	Idle time CDF given PU i starts the current idle period
$h_i(t)$	Portion of $f_{V_p'}(t)$ due to PU i ending current idle period of length t
$\phi_{ij}(t)$	Portion of $f_{si}(t)$ due to PU j ending current idle period of length t

c) Collision Detection: Whenever an SU and PU transmit simultaneously, we assume that both experience collision and the SU can detect the collision after the transmission. Perfect sensing, the SU packet length, and the LBT assumption ensure that packet collisions occur *only* if a PU accesses the channel while an SU is already transmitting. *This ensures that for any single PU busy period, at most one PU will experience packet collision.* Assuming perfect sensing and collision detection allows us to focus on investigating optimal capacity. Collisions with PUs are demonstrated in Figure 2.

d) Knowledge of Individual PUs: We assume that the SU has knowledge of the collision constraint, the idle time distribution, and the mean busy time of each individual PU *a priori*, i.e. $f_{V_i}(\cdot)$, n_i, η_i for all PUs, which are indexed by i. This knowledge can be obtained from the network usage histories obtained from network operators [27]. This is feasible depending on the the conditions of SU deployment, where SU and PU network operators may cooperate during initial SU network planning and deployment. For example, in the DARPA program both PUs and SUs are military, so planning of SU networks could conceivably include knowledge of PU network statistics [25].

e) Performance Metric: The SU's performance metric is the *time capacity*, the percentage of time that the SU can transmit successfully under the collision probability constraint. This metric is defined as below:

$$C_s = \lim_{T\to\infty} \frac{\text{SU's successful access time in } [0,T]}{T}. \quad (3)$$

Since the channel observed by the SU has idle probability α', clearly $C_s \le \alpha'$. We show through simulations later that a system with non-zero slot-length and sensing time results in a fraction of the optimal C_s corresponding to the fraction of the slot time where transmission occurs.

IV. OBJECTIVE FUNCTION

The SU objective is to maximize its time capacity while satisfying all PU's collision constraints,

$$\begin{array}{ll} \max & C_s \\ \text{subject to} & p_i^c \le \eta_i, \quad i \in \{1,...,M\}. \end{array} \quad (4)$$

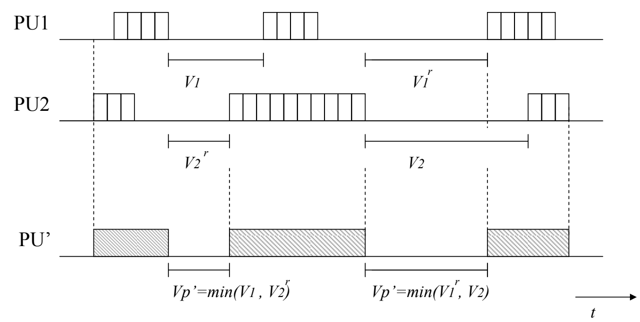


Fig. 3. Timing diagram showing two types of idle periods.

Based on the system model, we now derive the presentation of the objective function. The derivation assumes the system consists of a single SU co-existing with M non-interfering PUs. All notation is summarized in Table I.

The objective function requires the derivation of the idle time distribution for the unionized PU activity. For brevity, we defer this derivation to Appendix A, and define the necessary terms here.

Let t be the time elapsed since the beginning of the most recent idle period. We denote $f_{si}(t)$ as the idle time PDF of the unionized PU given that PU i is the last to stop transmission. In this case we say that PU i *starts* the idle period that follows. Then the unionized PU idle time distribution $f_{V_p'}(t)$ can be rewritten as

$$f_{V_p'}(t) = \sum_{i=1}^{M} p_{si} f_{si}(t), \quad (5)$$

where p_{si} is the probability that PU i starts an idle period.

From Figure 3, we can see that the residual idle time V_j^r of PU j is also important to the system. The residual idle time distribution $f_{V_j^r}(t)$ is

$$f_{V_j^r}(t) = \frac{1 - F_{V_j}(t)}{E[V_j]} = \frac{1 - F_{V_j}(t)}{v_j}. \quad (6)$$

From the same figure, we observe that for M users, if PU i starts the idle period,

$$V_p' = \min(V_1^r, V_2^r, ..., V_{i-1}^r, V_i, V_{i+1}^r, ..., V_M^r).$$

We define $\phi_{ij}(t)$ as

$$\phi_{ij}(t) = f_{V_j^r}(t)[1 - F_{V_i}(t)] \prod_{\substack{k=1 \\ k\neq i,j}}^{M} [1 - F_{V_k^r}(t)], \quad (7)$$

which can be thought of as the portion of the conditional PDF $f_{si}(t)$ due to PU j transmitting first after an idle period of length t. We note that $f_{si}(t)$ can now be defined as

$$f_{si}(t) = \sum_{j=1}^{M} \phi_{ij}(t). \quad (8)$$

We also define $h_i(t)$ as

$$h_i(t) = \sum_{j=1}^{M} p_{sj}\phi_{ji}(t), \tag{9}$$

which can be thought of as the portion of the idle time PDF $f_{V'_p}(t)$ that accounts for PU i transmitting first after an idle period of length t. We can rewrite $f_{V'_p}(t)$ as

$$f_{V'_p}(t) = \sum_{i=1}^{M} h_i(t). \tag{10}$$

We now derive the objective function for our system. We define $\Phi'(t)$ as the channel state of the unionized PU:

$$\Phi'(t) = \begin{cases} Idle, & \text{if all } M \text{ PUs idle} \\ Busy, & \text{otherwise.} \end{cases}$$

We also define the general form of the SU policy $q(t)$ as the probability that the SU policy transmits at time t,

$$q(t) = \begin{cases} p(t), & \text{if } \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases}$$

where $0 \leq p(t) \leq 1$. We note that this is a stationary policy, where the same policy is applied in every idle/busy cycle. Within each idle/busy cycle, the policy is dependent on t, which represents the time since the beginning of the most recent idle period. The structure of the optimal policy $q^*(\cdot)$ is the main focus of our work.

For an SU policy $q(\cdot)$, the time capacity equation (3) can now be defined as

$$C_s(q) = \frac{\int_0^\infty f_{V'_p}(t) \cdot \int_0^t q(\tau) d\tau dt}{v'_p + n'_p} = \frac{G_s(q)}{v'_p + n'_p}, \tag{11}$$

where $G_s(q) = \int_0^\infty f_{V'_p}(t) \cdot \int_0^t q(\tau) d\tau$.

The SU can collide with PU i only when that PU is the first to transmit following an idle period. Therefore, from (9) constraint η_i is satisfied for a policy $q(t)$ if:

$$\int_0^\infty q(t) h_i(t) dt \leq \min\left(n_i\eta_i \frac{v'_p + n'_p}{v_i + n_i}, 1\right). \tag{12}$$

The $n_i\eta_i$ term results from the fact that only one collision can occur in a single PU idle/busy cycle, and PU i transmits n_i packets on an average busy cycle. This term can also be interpreted as the maximum probability that the SU can collide with PU i in any given idle/busy cycle of PU i, $v_i + n_i$. However, the integral in (12) calculates the collision probability under policy $q(\cdot)$ with PU i *for an average cycle of the unionized PU*, $v'_p + n'_p$. Therefore, the right side of the equation must account for this difference by multiplying $n_i\eta_i$ by the cycle-time scale of the unionized PU over PU i.

We define $\eta'_i = n_i\eta_i \frac{v'_p + n'_p}{v_i + n_i}$, we now restate the objective function defined in (4) for the SU as

$$\begin{aligned} \max_{q(t):0 \leq q(t) \leq 1} \quad & C_s(q) \\ \text{subject to} \quad & \Psi(q) \leq \eta'_i, \quad i \in \{1, ..., M\}. \end{aligned} \tag{13}$$

V. Optimal Policy for Multiple PUs with General Idle Time Distributions

We now derive the optimal stationary SU access policy in an M-PU environment with general idle time distributions. We first derive the most general form of the optimal policy $q^*(\cdot)$. This derivation applies to any combination of PU general idle time distributions. We then detail two special cases: when all PUs have exponentially distributed idle times, and when PU idle distributions result in a time-threshold policy. The special cases are of interest because their optimal policies have a simpler structure. We then present a search algorithm for determining the optimal policy based on principles of convex optimization.

A. Deriving the General Optimal Policy

We consider the following policy $q(\cdot)$, defined as

$$q(t, \boldsymbol{\mu}) = \begin{cases} 1, & \text{if } \frac{1 - F_{V'_p}(t)}{\sum_{i=1}^{M} \mu_i h_i(t)} > 1, \Phi'(t) = Idle \\ p, & \text{if } \frac{1 - F_{V'_p}(t)}{\sum_{i=1}^{M} \mu_i h_i(t)} = 1, \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

where $\boldsymbol{\mu} = [\mu_1, \mu_2, ..., \mu_M]^T$. The conditions for each section of $q(t, \boldsymbol{\mu})$ resemble an inverted hazard function, where the pdf $f_{V'_p}(t)$ is replaced by the sum of $h_i(t)$ weighted by $\boldsymbol{\mu}$. Each μ_i term can be thought of as the importance given to each PU's collision constraint.

We then define the optimal policy $q^*(t)$ as

$$q^*(t) = q(t, \boldsymbol{\mu}^*), \tag{15}$$

where $\boldsymbol{\mu}^* = [\mu_1^*, \mu_2^*, ..., \mu_M^*]^T$. Both $\boldsymbol{\mu}^*$ and p are chosen such that the following conditions are satisfied for $i \in \{1, ..., M\}$:

$$\mu_i^* \geq 0, \tag{C1}$$

$$\int_0^\infty q^*(t) h_i(t) dt \leq \eta'_i, \tag{C2}$$

$$\mu_i^* = 0 \text{ if } \int_0^\infty q^*(t) h_i(t) dt < \eta'_i. \tag{C3}$$

Note here that μ_i^* can be explained as the Lagrange multiplier for the ith constraint, and $q^*(\cdot)$ is the function to be optimized. We now state the following theorem.

Theorem 1: Policy $q^*(\cdot)$ is an optimal policy that maximizes the SU throughput while satisfying the collision constraints of all PUs.

Proof: Consider any feasible policy $\tilde{q}(\cdot)$ that satisfies the collision probability constraints of all PUs. We now prove that

$G_s(\tilde{q}) \leq G_s(q^*)$, and therefore $C_s(\tilde{q}) \leq C_s(q^*)$. From (11),

$$\begin{aligned} G_s(\tilde{q}) &= \int_0^\infty f_{V_p'}(t) \cdot \int_0^t q(\tau) d\tau dt \\ &\overset{(a)}{\leq} G_s(\tilde{q}) - \sum_{i=1}^M \mu_i^* \left(\int_0^\infty \tilde{q}(t) h_i(t) dt - \eta_i' \right) \\ &= \int_0^\infty \tilde{q}(t) \left[1 - F_{V_p'}(t) - \sum_{i=1}^M \mu_i^* h_i(t) \right] dt \\ &+ \sum_{i=1}^M \mu_i^* \eta_i' \\ &\overset{(b)}{\leq} \int_0^\infty q^*(t) \left[1 - F_{V_p'}(t) - \sum_{i=1}^M \mu_i^* h_i(t) \right] dt \\ &+ \sum_{i=1}^M \mu_i^* \eta_i' \\ &= G_s(q^*) - \sum_{i=1}^M \mu_i^* \left(\int_0^\infty q^*(t) h_i(t) dt - \eta_i' \right) \\ &= G_s(q^*). \end{aligned}$$

The inequality (a) is true because $\tilde{q}(\cdot)$ must be a feasible policy, which results in a collision probability less than or equal to the constraint. The inequality (b) results because $q^*(\cdot)$ follows conditions (C1), (C2), and (C3), and $q^*(\cdot)$ is positive whenever $1 - F_{V_p'}(t) - \sum_{i=1}^M \mu_i^* h_i(t)$ is positive. ■

Therefore, the optimal policy search is in fact a search for vector $\boldsymbol{\mu}^*$ such that the optimality conditions (C1), (C2), (C3) are satisfied. Requirement (C1) can be imposed by searching over only non-negative values for $\boldsymbol{\mu}$, but the others are dependent on $q(t, \boldsymbol{\mu})$.

B. Optimal Policy for M PUs with Exponential Idle Times

It is a common assumption that channel idle time is distributed exponentially. For example, in [27], the authors perform a measurement study that shows that the exponential call arrival model is adequate for cellular networks. It was also shown in [25] that the exponential case also provides a lower bound to the achievable time capacity of any PU system. The same result, as we will show, applies here also. Therefore, in a system where the PUs' average idle times are known but the idle time distributions are not, the policy derived here can be used while guaranteeing the packet collision probability constraints of all users.

When all M PUs have exponential idle time distributions, we can show that the optimal policy $q^*(\cdot)$ is

$$q^*(t) = \begin{cases} p^*, & \text{if } \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

where $p^* \in [0, 1]$ is determined by the constraint conditions. In such a system the SU has equal probability of transmitting any time the channel is idle, so the time capacity is $C_s = \alpha' p^*$.

This comes from the idle time distributions. Since all distributions are memoryless, $f_{V_i^r}(t) = f_{V_i}(t)$, and $F_{V_i}(t) = F_{V_i^r}(t) = 1 - e^{-\lambda_i t}$ for all i. Since $V_p' = \min(V_1^r, V_2^r, ..., V_i, ..., V_M^r)$,

$$F_{V_p'}(t) = 1 - e^{-(\sum_{i=1}^M \lambda_i) t}. \tag{17}$$

This also means that

$$h_i(t) = C_i e^{-(\sum_{i=1}^M \lambda_i) t}, \quad i \in \{1, ..., M\}, \tag{18}$$

where C_i is some constant.

From (17) and (18), we observe that for any $\boldsymbol{\mu}$, there are three possibilities:

1. $1 - F_{V_p'}(t) < \sum_{i=1}^M \mu_i h_i(t)$ for $t \geq 0$.
2. $1 - F_{V_p'}(t) > \sum_{i=1}^M \mu_i h_i(t)$ for $t \geq 0$.
3. $1 - F_{V_p'}(t) = \sum_{i=1}^M \mu_i h_i(t)$ for $t \geq 0$.

If the first is true, then from (15) we have $q(t) = 0$ for all t, while if the second is true, $q(t) = 1$ for all t, implying that the collision constraints are trivial. Therefore, assuming non-trivial collision constraints, an appropriate $\boldsymbol{\mu}$ satisfies condition three, and (15) reduces to (16). Therefore, determining $\boldsymbol{\mu}^*$ is not necessary, because we only need p^*.

We now determine a p^* that satisfies (2) for all PUs. We note that p^* can be found by applying (16) and (12), so we present an intuitive explanation here. Observing that p^* is the probability that a single collision occurs during a single cycle of the unionized PU activity, this means a collision with PU i only occurs if the other $M-1$ PUs are idle when PU i begins transmitting. Therefore:

$$\prod_{\substack{j=1 \\ j \neq i}}^M \alpha_j p^* = \frac{\alpha'}{\alpha_i} p^* \leq n_i \eta_i,$$

where the right side results from the fact that only one packet collision can occur when any PU becomes busy again, and on average PU i transmits n_i packets per busy period. It follows that p^* can be written:

$$p^* = \min_{i \in \{1...M\}} (\frac{\alpha_i}{\alpha'} n_i \eta_i, 1). \tag{19}$$

Given this result for p^*, we obtain the following theorem.

Theorem 2. The maximum time capacity of the SU in a channel with the unionized activity of M PUs with exponential idle time distributions is:

$$C_s = \min_{i \in \{1...M\}} (C_{si}, \alpha'). \tag{20}$$

where $C_{si} = \alpha_i n_i \eta_i$, which is the time capacity that can be achieved in the single-PU system consisting of the ith PU with exponential idle time.

Proof: This results directly from (19) and (11). Another proof of this theorem is also available in our earlier work [26]. ■

This result has the following intuitive explanation. As Figure 4 demonstrates, the PUs protect each other from collision.

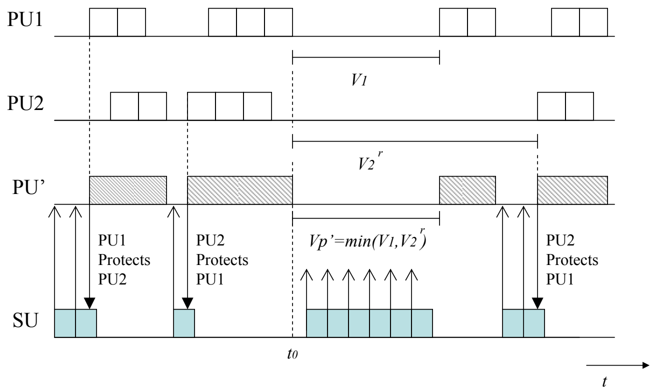


Fig. 4. Collision Protection with 2 PUs.

This reduces the collision probability of each PU, allowing the SU to transmit more aggressively. This offsets the reduced channel availability. In some cases, all PUs will be protected to the point that their collision probability constraints *cannot* be violated, and the SU will transmit whenever the channel is idle, resulting in $p^* = 1$ with time capacity $C_s(q^*) = \alpha'$. In addition, we also note that p^* can be calculated simply using (19), which greatly reduces calculation time. This mutual protection also benefits PU systems with general idle time distributions, but does not result in the same simplified structure as the exponential case.

C. Time-Threshold Policy

There are many cases in which the PU idle distributions result in a time-threshold policy. For example, systems where a PU has a uniform idle distribution or Weibull distributions with shape parameter under unity commonly result in a time-threshold policy. Therefore, we investigate this case here. A time-threshold policy begins transmitting as soon as the channel becomes idle for a length of time T^*, or until the channel becomes busy:

$$q^*(t) = \begin{cases} 1, & \text{if} \quad t \leq T^*, \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

We now state the following theorem.

Theorem 3. If $1 - F_{V_p'}(t)/h_i(t)$ can be shown to be monotonically decreasing for $i = 1, ..., M$, then the optimal policy is a time-threshold policy of the form in (21). Furthermore, T^* is

$$T^* = \min(T_1^*, T_2^*, ..., T_M^*), \tag{22}$$

where T_i^* is determined by

$$\int_0^{T_i^*} h_i(\tau) d\tau = \eta_i'. \tag{23}$$

Proof: The condition for this policy results directly from (15), which can be rewritten as

$$q^*(t) = \begin{cases} 1, & \text{if } \frac{1-F_{V_p'}(t)}{\sum_{i=1}^M \mu_i^* h_i(t)} > 1, \Phi'(t) = Idle \\ p^*, & \text{if } \frac{1-F_{V_p'}(t)}{\sum_{i=1}^M \mu_i^* h_i(t)} = 1, \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases}$$

Clearly, if the stated condition is satisfied, the above equation reduces to (21). T_i^* can be interpreted as the optimal time-threshold for the SU if only PU i is constrained. Since only the minimum T_i^* can guarantee that all PU constraints are satisfied, the optimal policy follows (21) with T^* as defined by (22) and (23). ∎

This leads to significant computation reduction in determining the optimal SU policy, since the policy can now be found through (23). As stated before, several common distributions result in this type of policy.

D. Optimal Policy Search

Because the policy search is essentially an optimization over the Lagrangian multiplier vector $\boldsymbol{\mu}$, we can use convex optimization techniques to search for the optimal policy. Therefore, in our numerical results, we use a descent method with line search to determine $\boldsymbol{\mu}^*$ [29]. The idea of the policy search is that a search or "descent" direction is chosen based on a current choice of $\boldsymbol{\mu}$, a distance to search in that descent direction is chosen based on a "line search" algorithm, and $\boldsymbol{\mu}$ is updated. This process reiterates until a stopping criterion is reached.

For ease of discussion, we define $\Psi_i(q)$ and $L(q, \boldsymbol{\mu})$ as

$$\Psi_i(\boldsymbol{\mu}) = \int_0^\infty q(t, \boldsymbol{\mu}) h_i(t) dt,$$

$$L(\boldsymbol{\mu}) = G_s(q(t, \boldsymbol{\mu})) + \sum_{i=1}^M \mu_i \left(\eta_i' - \Psi_i(\boldsymbol{\mu})\right).$$

$\Psi_i(\boldsymbol{\mu})$ is the collision probability achieved with the PU i given policy $q(t, \boldsymbol{\mu})$ from (12), and $L(\boldsymbol{\mu})$ is essentially the Lagrangian dual for the objective function (13).

Beginning with an arbitrary, feasible choice for $\boldsymbol{\mu}$, the algorithm first determines a search direction $\Delta\boldsymbol{\mu}$ using the gradient descent method,

$$\Delta\boldsymbol{\mu} = [\Psi_1(q(t, \boldsymbol{\mu})) - \eta_1', \ldots, \Psi_M(q(t, \boldsymbol{\mu})) - \eta_M'].$$

This direction is in fact the negative gradient of $L(q, \boldsymbol{\mu})$ with respect to $\boldsymbol{\mu}$. This direction increases the μ_i corresponding to violated constraints under the current policy, and decreases the μ_i for PU constraints which are obeyed. This can be analogized as giving more/less weight to the stricter/looser PU constraints in the search.

Next, a line search algorithm is used to determine the step size ν, the distance to increment in the search direction. In our numerical results we use a modified backtracking line algorithm [29]. It is essentially the same as the standard algorithm, but with conditions that prevent the resulting ν

from searching into negative μ_i values which would violate condition (C1).

Finally, $\boldsymbol{\mu}$ is updated as

$$\boldsymbol{\mu} \leftarrow \boldsymbol{\mu} + \nu \Delta \boldsymbol{\mu}.$$

If the updated $\boldsymbol{\mu}$ satisifes conditions (C2) and (C3) to within a reasonable limit, the algorithm is terminated with the current iterate as $\boldsymbol{\mu}^*$.

In practice, this algorithm runs quickly. For all results in Section VII, each 2-PU-system policy is found in under .3 seconds (and in many cases, significantly less) running on a Pentium 4 3.2 GHz processor with 1 GB of RAM. Given that any policy is meant to be operative on the order of hours, this time is negligible.

VI. PU Side Information

We now investigate how extra information of PU activity can affect the optimal policy, and ultimately the time capacity of the SU. In addition to the original system model assumptions, this information is assumed to be available to the SU through augmented sensing capabilities from the original system model. These two cases of extra PU information are as follows:

- *Side Information 1:* The SU knows which PU was the last to transmit.
- *Side Information 2:* The SU knows which PU was the last to transmit and how long the other PUs have been idle prior to the unionized channel going idle.

We use SI-1 and SI-2 to describe these two cases, and No-SI to refer to the original policy laid out in Section V. We note that in general the sensing capability of SI-1 would seem to imply the capability of SI-2, i.e. if a sensor can differentiate PUs, it is easy to assume it can also keep track of how long each has been idle. SI-1 and SI-2 can achieve significantly better performance than the original case. Through our derivations we show that SI-2 is computationally intractable. We now derive the new optimal policy for each case.

A. Optimal Policy for SI-1

We wish to leverage the new information when determining the optimal policy $q^*(t)$ of the SU. Naturally, the SU can act differently based on which PU ends transmission last. Therefore, an SU policy $q(t)$ can be written as

$$q(t) = \begin{cases} q_i(t), & \text{if PU } i \text{ is last to transmit, } \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

where $q_i(t)$ is the SU access policy when PU i is the last to transmit.

From (12) and (13), the time capacity and the collision constraint can be rewritten as

$$C_s(q) = \frac{\sum_{i=1}^{M} p_{si} \int_0^\infty q_i(t)[1 - F_{si}(t)]dt}{v'_p + n'_p}, \tag{25}$$

$$\sum_{j=1}^{M} p_{sj} \int_0^\infty q_j(t)\phi_{ji}(t)dt \leq \min(\eta'_i, 1). \tag{26}$$

where $F_{si}(t)$ is the cdf of the idle time distribution given that PU i is the last to transmit. Using these more specific equations, the objective function still follows the form of (13).

We can see that the optimal policy $q^*(t)$ is similar to (14), except that there are now M policies for when each PU is the last to transmit. Therefore, we now define the optimal policy $q_i^*(t, \boldsymbol{\mu})$ given that PU i is the last to transmit for a given multiplier vector $\boldsymbol{\mu}$:

$$q_i^*(t, \boldsymbol{\mu}) = \begin{cases} 1, & \text{if } 1 - F_{si}(t) > \sum_{j=1}^{M} \mu_j \phi_{ij}(t), \Phi'(t) = Idle \\ p^*, & \text{if } 1 - F_{si}(t) = \sum_{j=1}^{M} \mu_j \phi_{ij}(t), \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

We note that this equation is quite similar to (15). $F_{V'_p}(t)$ is replaced with $F_{si}(t)$, and $h_i(t)$ is replaced with $\phi_{ij}(t)$, since the policy being defined is only operative when PU i is the last to transmit. We now define the optimal policy $q^*(t)$ as

$$q^*(t) = \begin{cases} q_i^*(t, \boldsymbol{\mu}^*), & \text{if PU } i \text{ last to xmit}, \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{28}$$

where $\boldsymbol{\mu}^*$ satisfies the conditions laid out in Section V-A. The proof of the optimality of the policy is analogous to that of Theorem 1. This policy can be obtained using a slightly modified version of the algorithm presented in Section V-D.

B. Optimal Policy for SI-2

We now derive the optimal policy for the SI-2 case. As stated previously, this case is computationally intractable and primarily serves as an upper bound for possible performance for stationary policies under our model. In the SI-2 case, the SU knows how long every PU has been idle prior to the channel going idle. We call this time the *lost time*, denoted by X_i for the ith PU, with x_i as a realization of X_i. By definition, at least one PU's lost time will be zero at the start of any idle period, since at least one PU is transmitting prior to the moment the unionized PU goes idle. We define the vector $\boldsymbol{X}_i$ as the vector containing all of the lost times given PU i is the last to return:

$$\boldsymbol{X}_i = \{X_1, ..., X_{i-1}, 0, X_{i+1}, ..., X_M\}$$

and $\boldsymbol{x}_i = \{x_1, ..., x_{i-1}, 0, x_{i+1} ... x_M\}$ as a particular instantiation of $\boldsymbol{X}_i$. We note that the ith element is always zero for $\boldsymbol{x}_i$.

The SU policy depends on both the PUs' idle time distributions and the vector $\boldsymbol{X}_i$. We denote the SU access policy given that PU i is the last to transmit with lost time vector $\boldsymbol{X}_i$ as $q(t|\boldsymbol{X}_i)$, and a SU policy $q(t)$ as

$$q(t) = q(t|\boldsymbol{x}_i), \text{ if } X_i = 0, \boldsymbol{X}_i = \boldsymbol{x}_i, \Phi'(t) = Idle. \tag{29}$$

The time capacity and constraint equations (11) and (12) must be re-derived using the idle time distribution conditioned on the values of the lost time vector $\boldsymbol{X}_i$.

To obtain the time capacity and constraint equations, we must determine several related pdfs conditioned on the lost time vector. We first define $f_{si}(t|\boldsymbol{x}_i)$, the conditional pdf of the idle time given that PU i is the last PU to transmit with lost time vector $\boldsymbol{x}_i$,

$$f_{si}(t|\boldsymbol{x}_i) = \sum_{j=1}^{M} \phi_{ij}(t|\boldsymbol{x}_i). \quad (30)$$

In this case, $\phi_{ij}(t|\boldsymbol{x}_i)$ is the portion of $f_{si}(t|\boldsymbol{x}_i)$ due to PU j ending an idle period of length t, given that PU i is the last to transmit and a lost time vector $\boldsymbol{x}_i$. We refer the reader to Appendix A for the derivation $\phi_{ij}(t)$. If $j = i$ we have

$$\begin{aligned}\phi_{ii}(t|\boldsymbol{x}_i) &= Pr[V_i = t] \prod_{\substack{j=1\\ j\neq i}}^{M} Pr[V_j > t + x_j | X_j = x_j] \\ &= Pr[V_i = t] \prod_{\substack{j=1\\ j\neq i}}^{M} \frac{Pr[V_j > t + x_j]}{Pr[V_j > x_j]} \quad (31) \\ &= f_{V_i}(t) \prod_{\substack{j=1\\ j\neq i}}^{M} \frac{1 - F_{V_j}(t + x_j)}{1 - F_{V_j}(x_j)}.\end{aligned}$$

If $j \neq i$, we have:

$$\begin{aligned}&\phi_{ij}(t|\boldsymbol{x}_i) \\ &= Pr[V_j = t + x_j | X_j = x_j] \prod_{\substack{k=1\\ k\neq j}}^{M} Pr[V_k > t + x_k | X_k = x_k] \\ &= \frac{Pr[V_j = t + x_j]}{Pr[V_j > x_j]} \prod_{\substack{k=1\\ k\neq j}}^{M} \frac{Pr[V_k > t + x_k]}{Pr[V_k > x_k]} \\ &= \frac{f_{V_j}(t + x_j)}{1 - F_{V_j}(x_j)} \prod_{\substack{k=1\\ k\neq j}}^{M} \frac{1 - F_{V_k}(t + x_k)}{1 - F_{V_k}(x_k)}.\end{aligned} \quad (32)$$

We must also determine the distribution of the lost time vector X_i. The distribution for X_i is the same as the residual time distribution for the ith user:

$$f_{X_i}(x_i) = f_{V_i^r}(x_i) = \frac{1 - F_{V_i}(x_i)}{v_i}, \quad (33)$$

and the distribution of $\boldsymbol{X}_i$ is the joint distribution of all of the lost time distributions of the other variables. Since all PUs act independently, this distribution amounts to

$$f_{\boldsymbol{X}_i}(\boldsymbol{x}_i) = \prod_{\substack{j=1\\ j\neq i}}^{M} f_{X_j}(x_j), \quad (34)$$

where $f_{X_i}(x_i)$ is not included in (34) because in $\boldsymbol{X}_i$, $Pr[X_i = 0] = 1$. With (31), (32), (30), and (34) we can derive the equation for $f_{si}(t)$ as

$$f_{si}(t) = \int_{\boldsymbol{x}_i} f_{si}(t|\boldsymbol{x}_i) f_{\boldsymbol{X}_i}(\boldsymbol{x}_i) d\boldsymbol{x}_i. \quad (35)$$

The equation for $f_{V_p'}(t)$ still follows (5). Integrating (30) over t also allows us to obtain the conditional cdf $F_{si}(t|\boldsymbol{x}_i)$. We also rewrite $h_i(t)$ in terms of $\phi_{ij}(t|\boldsymbol{x}_i)$ as

$$h_i(t) = \sum_{j=1}^{M} p_{sj} \int_{\boldsymbol{x}_i} \phi_{ji}(t|\boldsymbol{x}_i) f_{\boldsymbol{X}_i}(\boldsymbol{x}_i) d\boldsymbol{x}_i. \quad (36)$$

We are now able to derive the time capacity $C_s(q)$ and constraint equations for SI-2:

$$\begin{aligned}C_s(q) &= \frac{\int_0^\infty q(\tau)[1 - F_{V_p'}(\tau)] d\tau}{v_p' + n_p'} \\ &= \frac{\sum_{i=1}^{M} p_{si} \int_0^\infty \int_{\boldsymbol{x}_i} q(\tau|\boldsymbol{x}_i)[1 - F_{si}(\tau|\boldsymbol{x}_i)] d\boldsymbol{x}_i d\tau}{v_p' + n_p'},\end{aligned} \quad (37)$$

$$\sum_{j=1}^{M} p_{sj} \int_0^\infty \int_{\boldsymbol{x}_j} q(t|\boldsymbol{x}_j) \phi_{ji}(t|\boldsymbol{x}_j) f_{\boldsymbol{X}_j}(\boldsymbol{x}_j) d\boldsymbol{x}_j dt \leq \eta_i'. \quad (38)$$

The objective function then follows the form of (13) using the time capacity and constraint equations (37) and 38).

As in the previous cases, we define the optimal policy $q^*(t)$ as the policy that corresponds to the vector $\boldsymbol{\mu}^*$ such that the optimality conditions (C1)-(C3) are satisfied. The optimal policy $q^*(t|\boldsymbol{x}_i)$ for PU i given $\boldsymbol{x}_i$ is:

$$q^*(t|\boldsymbol{x}_i) = \begin{cases} 1, & \text{if } \ 1 - F_{si}(t|\boldsymbol{x}_i) > \sum_{j=1}^{M} \mu_j^* \phi_{ij}(t|\boldsymbol{x}_i), \Phi'(t) = Idle \\ p^*, & \text{if } \ 1 - F_{si}(t|\boldsymbol{x}_i) = \sum_{j=1}^{M} \mu_j^* \phi_{ij}(t|\boldsymbol{x}_i), \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \quad (39)$$

and the optimal policy $q^*(t)$ is

$$q^*(t) = \begin{cases} q^*(t|\boldsymbol{x}_i), & \text{if } X_i = 0 \text{ and } \boldsymbol{X}_i = \boldsymbol{x}_i, \ \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \quad (40)$$

where $\boldsymbol{\mu}^* = \{\mu_1^*, ..., \mu_M^*\}$ satisfies (C1)-(C3). The proof is the same as that presented in theorem 1.

This policy gives better performance than either the No-SI or SI-1 cases because of the additional information. However, in practice, obtaining the optimal policy in (40) is computationally difficult. For any $\boldsymbol{\mu}$, the collision probability depends on an infinite number of policies $q(t|\boldsymbol{x}_i)$ corresponding to all possible lost time vectors, rendering numerical solutions prohibitively expensive.

VII. Numerical Results

We now present numerical results demonstrating how SU performance is affected by the number of PUs in the system and the side sensing information defined in the previous section. The results presented are meant to demonstrate the effect of time capacity in a number of different network scenarios where multiple PUs may need to be protected. First, we investigate how the number of PUs affects the SU performance for the No-SI case. We show that while time capacity generally degrades with the number of users, throughput increases to a

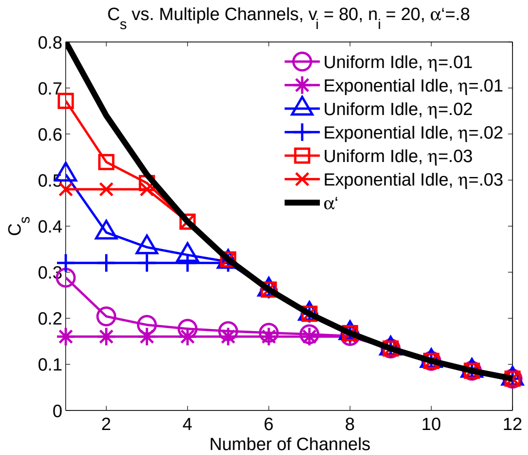


Fig. 5. C_s vs. Multiple PUs, with single PU parameters $v_i = 80, n_i = 20$.

point if the PUs are on different channels. Then we show that in the SI-1 and SI-2 cases the extra PU information available to the SU over the No-SI system model leads to higher time capacity while still satisfying PU packet collision probability constraints.

A. Multiple PUs

We first investigate how multiple PUs affect SU performance in the No-SI system model. In all simulations in this section, we have multiple PUs with the same idle/busy time distributions, $v_i = 80$ and $n_i = 20$, and the same collision constraint η. We compare SU performance for two different idle time distributions, exponential and uniform. The PU packet length is 1, and the SU packet length $\Delta = .01$. Therefore, any SU policy $q(\cdot)$ must assign transmission probabilities for time increments of length Δ, and all idle time distribution functions are calculated with time increment of Δ also.

First, in Figure 5, we study the time capacity performance as a function of the number of PUs to be protected. This is an accurate measure of SU performance in the spatial separation case demonstrated in Figure 1(a), since all PUs and the SU operate on the same channel. In Figure 5, each curve represents a simulation where all PUs have the same protection requirement and idle time distribution, either exponential or uniform.

We first observe that for all cases, the time capacity is eventually limited by the idle probability α'. This corresponds to when the PUs are sufficiently protected such that no PU constraint can be violated. In this case, the SU transmits whenever the channel is idle. We also note that the time capacity for uniform idle distribution cases degrades as the number of PUs grows, with the largest drop between $M = 1$ and $M = 2$. However, the exponential idle time cases experience no degradation until the SU transmits with probability 1, corresponding to a time capacity of α'.

Finally, we observe that the uniform case outperforms the exponential case for all η and M before the time capacity is limited by α'. This is because the memorylessness of the exponential case reduces the SU access policy to a random access scheme with probability corresponding to the collision constraint. On the other hand, in the uniform distribution case, the time that the unionized channel has been idle helps in predicting when the channel will become busy. This results in a policy that exploits this predictability, transmitting more aggressively earlier on in an idle period.

In a case where each PU resides on a different channel as in Figure 1(b), we must also consider the bandwidth of the unionized channel to evaluate SU performance. We consider this in Figure 6. In these simulations, we now assume that in addition to each PU having the same usage statistics and collision constraints as in Figure 5, each PU occupies its own channel of bandwidth B_0. All channels have the same noise power density N_0, and the SU operates only under a maximum power constraint P. We now define the throughput of the SU $R_s(M)$ as the product of the time capacity and the Shannon capacity of an SU operating on M channels. Therefore,

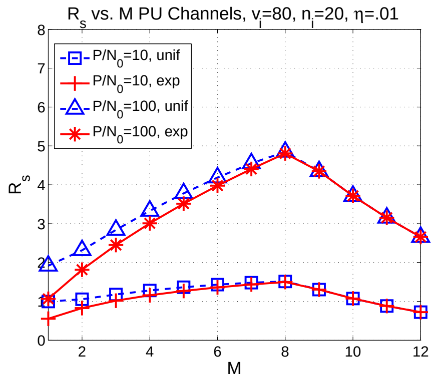
$$R_s(M) = C_s(M) M B_0 \log_2\left(1 + \frac{P}{M B_0 N_0}\right), \qquad (41)$$

where $C_s(M)$ is the time capacity of the SU on M channels. Therefore, for any simulation, $C_s(M)$ always follows one of the curves in 5. For simplicity, we also assume that $B_0 = 1$. In Figures 6(a) and 6(b), η is held constant at .01 and .02 respectively, and each curve represents a different P/N_0 ratio and idle time distribution (exponential vs. uniform). First, observing Figure 5, we see that when $\eta = .01$ and .02, the number of PUs that results in a time capacity $C_s \approx \alpha'$ is 8 and 5, respectively. Now, from 6(a) and 6(b), we see that when $\eta = .01$ and .02, the maximum throughput also occurs at 8 and 5 PU channels respectively, regardless of distribution and SU transmission power. This suggests that a good heuristic to obtain maximum throughput in a multi-PU multi-channel system is to use as many channels as possible, such that the time capacity is not severely limited by the idle probability of the channel.

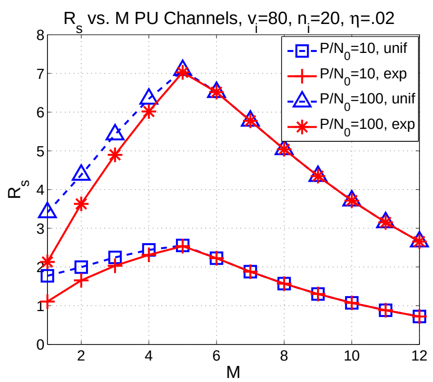
Figures 6(a) and 6(b) also show that the uniform case has a higher throughput than its corresponding exponential case until the maximum throughput is reached. This is again due to the memorylessness of the exponential case, which makes the arrival of the PU unpredictable. However, we also notice that the exponential and uniform cases with the same SU power and η value both result in roughly the same performance at the elbow of each of the curves. The cause of this is obvious: in either case, the SU is transmitting whenever the channel is idle, and both cases have the same idle probabilities. This result suggests that a strong heuristic to obtaining a channel grouping that obtains near optimal performance may be to treat all channels as exponential regardless of their actual distributions. This will significantly reduce computation of the unionized PU idle time distribution as well as the optimal access policy.

Finally, in Figure 6(c), $P/N_0 = 100$ for all simulations, and each curve represents a different η value and idle time distribution. We note that regardless of the η value, as the

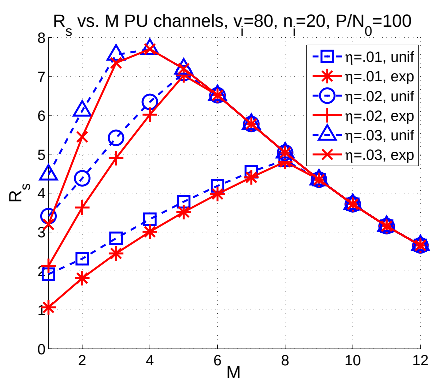
Fig. 6. Throughput vs. Number of PU Channels (M) with $v_i = 80, n_i = 20$.



(a) $\eta = .01$



(b) $\eta = .02$



(c) η varying, $P/N_0 = 100$

number of PU channels goes up, eventually performance is limited by the time capacity.

B. Comparison of No-SI and SI-1

We now compare the performance of the No-SI and SI-1 cases. In these simulations, activity of a two PU system is generated. Both PUs have uniform idle time distributions with $v_1 = v_2 = 80$, and general busy time distributions with $n_1 = 20$, $n_2 = 40$ packets per transmission. We assume that the packets for both PUs have the same length, so that $\alpha_1 = .8$, $\alpha_2 = .6667$, and $\alpha' = .5333$. PU and SU packet lengths are 1 and .01, respectively.

In Figure 7(a), the time capacity C_s of the optimal No-SI and SI-1 policies are plotted as a function of η, where both PUs are assumed to have the same packet collision probability constraint, i.e. $\eta_1 = \eta_2 = \eta$. Table II(a) displays the collision probability values obtained by the No-SI (NS) and SI-1 (S1) policies for each PU and desired η value. In Tables II(a), II(b), and III, the constraint values set by the PUs are labeled as η_i, and the collision probabilities achieved by the optimal policies are labeled as $\hat{\eta}_i$. We observe that the SI-1 optimal policy achieves higher C_s when $\eta < .035$. We also notice that, while both policies are able to achieve the desired η_1 value, the SI-1 policy is able to achieve higher collision rates than No-SI for PU2 without violating the collision constraints. This is because SI-1 exploits the extra information to transmit more aggressively, achieving a higher time capacity.

Finally, at $\eta = .035$, we notice that both policies achieve approximately the same C_s, and that both policies have collision probability rates less than the desired η. This corresponds to the case where the SU transmits whenever the channel is idle: $C_s = \alpha' = .5333$, and neither collision constraint can be violated.

In Figure 7(b), we observe the effect that differing collision constraint values have on the time capacity for No-SI and SI-1. Time capacity C_s is plotted against changing η_2, while PU1's collision constraint is held at $\eta_1 = .01$, with corresponding achieved collision probability values displayed in Table II(b). Again it is clear that the SI-1 policy performs better in general, and similarly, SI-1 achieves a higher collision probability than No-SI without violating the collision constraints.

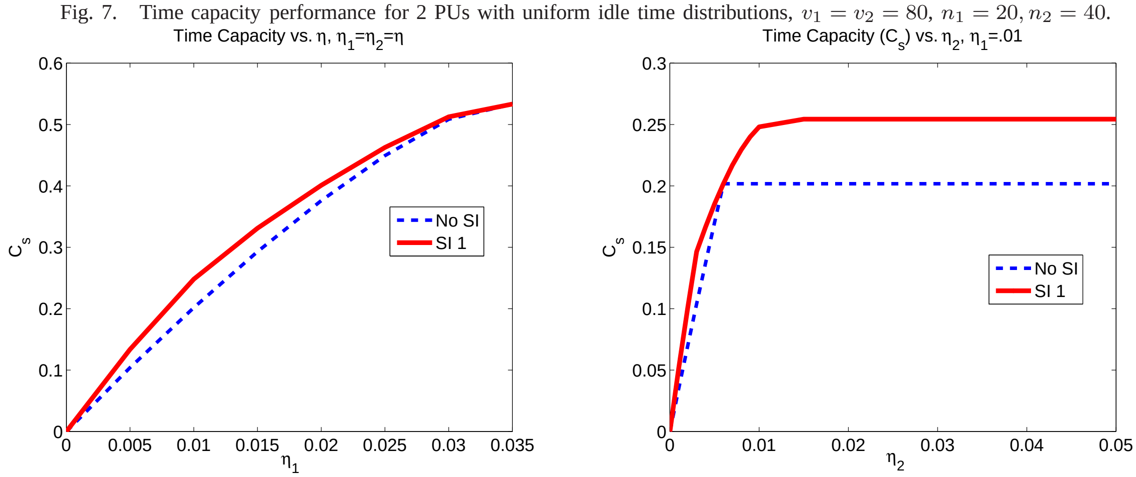
The one exception occurs at $\eta_2 = .006$, where both cases satisfy the collision constraints with equality (bolded in Table II(b)). The corresponding C_s values achieved by No-SI and SI-1 are the same at this point. In general, this observation holds true for all combinations of idle time distributions that result in T^* policies: for any η_1 value, there is a corresponding η_2 value such that both constraints are satisfied with equality by the No-SI and SI-1 optimal policies. In this case, the No-SI and SI-1 policies are both time-threshold policies with the same T^* value, which results in their corresponding C_s values being equal.

We also observe that both C_s curves become constant at higher η_2 values. This is because η_1 becomes the limiting constraint for both policies, such that neither No-SI nor SI-1 can transmit more aggressively to exploit looser η_2 constraints.

Finally, in Table III, we observe the effect of non-zero SU slot lengths and sensing times on performance. We run all policies over the same simulated PU transmission patterns as in the results from Figure 7, with $v_1 = v_2 = 80$, and $n_1 = 20$, $n_2 = 40$. PU slot length is again 1. We assume that the SU senses the channel for a constant amount of time in every slot, with Δ_s denoting the sensing time. The table in this case corresponds only to SI-1 type policies; the results from the No-SI results show similar trends.

From the table, we see that performance is affected very little by the increased packet length. In all cases with zero sensing time, when compared with the theoretical policy per-

Fig. 7. Time capacity performance for 2 PUs with uniform idle time distributions, $v_1 = v_2 = 80$, $n_1 = 20, n_2 = 40$.



(a) Time Capacity vs. $\eta = \eta_1 = \eta_2$

(b) Time Capacity vs. η_2 with $\eta_1 = .01$

TABLE II
ACHIEVED η_1, η_2, C_s FOR FIGURE 7

(a) Achieved values for Figure 7(a), $\eta = \eta_1 = \eta_2$

η	$\hat{\eta_1}$ **(NS)**	$\hat{\eta_2}$ **(NS)**	$\hat{\eta_1}$ **(S1)**	$\hat{\eta_2}$ **(S1)**
.0050	.0050	.0030	.0050	.0050
.0100	.0100	.0060	.0100	.0100
.0150	.0150	.0090	.0150	.0135
.0200	.0200	.0120	.0200	.0153
.0250	.0250	.0150	.0250	.0171
.0300	.0300	.0180	.0300	.0188
.0350	.0333	.0200	.0333	.0200

(b) Achieved values for Figure 7(b), η_2 varying, $\eta_1 = .01$

η_2	$\hat{\eta_1}$ **(NS)**	$\hat{\eta_2}$ **(NS)**	$\hat{\eta_1}$ **(S1)**	$\hat{\eta_2}$ **(S1)**
.0020	.0033	.0020	.0067	.0020
.0040	.0067	.0040	.0100	.0040
.0060	**.0100**	**.0060**	**.0100**	**.0060**
.0080	.0100	.0060	.0100	.0080
.0100	.0100	.0060	.0100	.0100
.0150	.0100	.0060	.0100	.0118
.0500	.0100	.0060	.0100	.0118

formance, there are only slight variations of achieved collision probabilities and time capacity. Generally, higher values of $\hat{\eta_i}$ correspond to slightly higher values of time capacity. These differences are minor, since the differences in $\hat{\eta_i}$ are less than 2% in all cases with respect to the $\Delta = .01$ theoretical case.

In the cases with non-zero sensing time, we see that while achieved collision probabilities remain the same, C_s decreases due to the time spent sensing the channel. The decreases in C_s correspond to the Δ_s/Δ factor spent for sensing in each time slot. Therefore, we conclude that for non-zero SU slot lengths, performance remains within acceptable limits assuming that the SU slot length is not greater than the PU packet length.

C. SI-2 Policy Performance

We now compare SI-2 performance to the other two cases. Simulation parameters are the same as the previous section. For SI-2 simulations, optimal search is impractical because of heavy computation. Instead, we use a randomly selected (μ_1, μ_2) pair to generate an arbitrary, simplified SI-2 policy. Since the SI-2 policy is dependent on the lost time vector $\boldsymbol{X}_i$, for any (μ_1, μ_2) pair there are an infinite number of policies corresponding to different lost times, meaning that the achieved collision probability of any SI-2 policy is computationally expensive.

To make this more tractable, we divide the possible lost time into intervals of width W. For each interval, a policy is determined in a manner similar to (39). For a 2-PU system with a given (μ_1, μ_2) pair,

$$q(t|\boldsymbol{x}_i, \lceil \frac{x_j}{W} \rceil = k) = \begin{cases} 1, & \text{if } \frac{1-F_{si}(t|x_j=kW)}{\sum_{j=1}^{M} \mu_j \phi_{ij}(t|x_j=kW)} > 1,\ \Phi'(t) = Idle \\ p^*, & \text{if } \frac{1-F_{si}(t|x_j=kW)}{\sum_{j=1}^{M} \mu_j \phi_{ij}(t|x_j=kW)} = 1,\ \Phi'(t) = Idle \\ 0, & \text{otherwise,} \end{cases} \tag{42}$$

where $x_i = 0$, and x_j is the lost time of PU j with $j \neq i$. The modified policy is then

$$q(t) = \begin{cases} q(t|\boldsymbol{x}_i, \lceil \frac{x_j}{W} \rceil = k), & \text{if } \boldsymbol{X}_i = \boldsymbol{x}_i,\ \Phi'(t) = Idle \\ 0, & \text{otherwise.} \end{cases} \tag{43}$$

Although the policy defined in (43) reduces computation greatly, it is still expensive to search for an optimal (μ_1, μ_2) for a given η_1, η_2 pair. Instead, we obtain an arbitrary SI-2 policy using a randomly selected (μ_1, μ_2) pair. Using the collision probabilities obtained under the SI-2 policy, we then perform a search to determine the No-SI and SI-1 optimal policies corresponding to those collision probabilities.

Table IV shows the results of one such comparison. We use a lost time interval width $W = 10$, and obtain a SI-2 policy that yields $\eta_1 = .0101, \eta_2 = .0101$. The optimal search algorithm is run for No-SI and SI-1 using those constraints.

TABLE III
SU PERFORMANCE FOR SI-1 POLICY WITH NONZERO SLOT LENGTH Δ, SENSING TIME Δ_s

Policy Type	$\eta_1 = .01, \eta_2 = .006$			$\eta_1 = .01, \eta_2 = .01$		
	C_s	$\hat{\eta_1}$	$\hat{\eta_2}$	C_s	$\hat{\eta_1}$	$\hat{\eta_2}$
$\Delta = .01, \Delta_s = 0$ (Theoretical)	.2024	.0100	.0060	.2483	.0100	.0100
$\Delta = .1, \Delta_s = 0$	.2028	.0101	.0061	.2484	.0102	.0101
$\Delta = 1, \Delta_s = 0$	.2010	.0101	.0060	.2485	.0103	.0101
$\Delta = 1, \Delta_s = .1$	.1809	.0101	.0060	.2237	.0103	.0101
$\Delta = 1, \Delta_s = .2$	.1608	.0101	.0060	.1988	.0102	.0100

TABLE IV
PERFORMANCE COMPARISON BETWEEN 3 CASES, WITH $\eta_1 = 0.0101$, $\eta_2 = 0.0101$, AND LOST INTERVAL WIDTH $W = 10$.

Policy	η_1 achieved	η_2 achieved	C_s
No-SI	0.0101	0.0060	0.2029
SI-1	0.0101	0.0101	0.2495
SI-2	0.0101	0.0101	0.2724

The SI-2 policy achieves a 9.2% gain over SI-1, and a 34.3% gain over the No-SI.

VIII. CONCLUSION

In this paper, we studied the opportunistic access in a system with multiple PUs, where PUs are heterogeneous in terms of idle time distribution and packet collision probability requirement. We determined the form of the optimal policy, and specified two special cases for which the optimal policy is easier to determine: the case where all PUs have exponential idle times, and the case where the PU idle time distributions lead to a time-threshold policy. We then studied the effect that extra information of PU activity has on the SU optimal policy. The first case studied was one in which the SU knows which PU was the last to transmit before the channel becomes idle. In the second, the SU had the additional knowledge of how long each PU had been idle before all PUs became idle. For both cases, optimal policies were also determined. Simulations results demonstrated that the extra PU information significantly increases SU performance.

These results have implications for future work in channel pooling and allocation in SU networks. One such problem is how an SU should optimally access a set of PU channels because there exists a tradeoff between instantaneous data rate and time capacity. Another problem is channel allocation for multiple SUs. In general, previous work on allocation has assumed a static spectral environment, i.e. licensed channels are idle or busy for long periods of time. This work implies new allocation scenarios in which PUs have frequent idle/busy transitions, and SUs must consider PU activity statistics when allocating channels since certain combinations of channels may be better grouped together.

REFERENCES

[1] "Spectrum policy task force report," Federal Communications Commision, Report, Et docket No. 02-135, November 2002.

[2] "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order," Federal Communications Commision, Report, Et docket No. 03-322, December 2003.

[3] Q. Zhao and B. Sadler, "A survey of dynamic spectrum access: Signal processing, networking, and regulatory policy," *IEEE Signal Processing Magazine*, vol. 55, no. 5, pp. 2294–2309, 2007.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, 2005, 0733-8716.

[5] J. Mitola and J. Maguire, G. Q., "Cognitive radio: making software radios more personal," *Personal Communications, IEEE [see also IEEE Wireless Communications]*, vol. 6, no. 4, pp. 13–18, 1999, 1070-9916.

[6] Shared spectrum company: DARPA XG program information. [Online]. Available: http://www.sharedspectrum.com/technology/darpaxg.html

[7] F. W. Seelig, "A description of the august 2006 xg demonstrations at fort a.p. hill," in *Second IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2007, pp. 1–12.

[8] S. Huang, X. Liu, and Z. Ding, "Opportunistic spectrum access in cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, April*, 2008. [Online]. Available: "http://www.ece.ucdavis.edu/˜senhua/OSAinfocom08.pdf"

[9] Q. Zhao, S. Geirhofer, L. Tong, and B. M. Sadler, "Optimal dynamic spectrum access via periodic channel sensing," in *Proc. Wireless Communications and Networking Conference (WCNC)*, 2007.

[10] T. Shu, S. Cui, and M. Krunz, "Medium access control for multi-channel parallel transmission in cognitive radio networks," *Proceedings of the IEEE GLOBECOM 2006 Conference, San Francisco, CA, Dec.*, 2006.

[11] R. Urgaonkar and M. J. Neely, "Opportunistic scheduling with reliability guarantees in cognitive radio networks," *IEEE INFOCOM 2008, Phoenix, AZ, USA, Apr.*, 2008.

[12] W.-Y. Lee and I. F. Akylidiz, "Joint spectrum and power allocation for inter-cell spectrum sharing in cognitive radio networks," pp. 1–12, 2008.

[13] W. Wang and X. Liu, "List-coloring based channel allocation for open-spectrum wireless networks," *Proceedings of the IEEE Vehicular Tech. Conference*, pp. 690–694, 2005.

[14] H. Zheng and C. Peng, "Collaboration and fairness in opportunistic spectrum access," *Proceedings of the IEEE ICC Conference*, 2005.

[15] L. Cao and H. Zheng, "Understanding the power of distributed coordination for dynamic spectrum management," *ACM/Springer Mobile Networks and Applications (MONET),*, vol. 13, pp. 477–497.

[16] L. Yang, L. Cao, and H. Zheng, "Physical interference driven dynamic spectrum management," *Proceedings of IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks(DySPAN)*.

[17] Y. S. Y. Thomas Hou and H. D. Sherali, "Spectrum sharing for multi-hop networking with cognitive radios," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, 2008.

[18] T. Shu and M. Krunz, "Coordinated channel access in cognitive radio networks: A multi-level spectrum opportunity perspective," *IEEE INFOCOM 2009, Rio De Janeiro, Brazil, Br*, 2009.

[19] D. I. Kim, L. B. Le, and E. Hoossain, "Joint rate and power allocation for cognitive radios in dynamic spectrum access environment," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 5517–5527, 2008.

[20] Y. Chen, G. Yu, Z. Zhang, H.-H. Chen, and P. Qiu, "On cognitive radio networks with opportunistic power control strategies in fading channels," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2751–2761, 2008.

[21] "In the matter of establishment of an interference temperature metric to quantify and manage interference and to expand available unlicensed operation in certain fixed, mobile and satellite frequency bands," Federal

Communications Commision/Office of Engineering and Technology, OET Report, FCC 07-78, May 2007.

[22] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive mac for opportunistic spectrum access in ad hoc networks: A pomdp framework," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 589–600, 2007.

[23] Y. Chen, Q. Zhao, and A. Swami, "Joint design and separation principle for opportunistic spectrum access in the presence of sensing errors," *Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers, November,*, 2006.

[24] T. Javidi, B. Krishnamachari, Q. Zhao, and M. Liu, "Optimality of myopic sensing in multi-channel opportunistic access," *Proc. of IEEE International Conference on Communications (ICC), May*, 2008.

[25] S. Huang, X. Liu, and Z. Ding, "Optimal transmission strategies for dynamic spectrum access in cognitive radio networks," *IEEE Transactions on Mobile Communications*, vol. 8, no. 12, 2009.

[26] E. Jung and X. Liu, "Opportunistic spectrum access in heterogeneous user environments," *Third IEEE International Symposium on Dynamic Spectrum Access Networks, DySPAN*, 2008.

[27] D. Willkomm, S. Machiraju, J. Bolot, and A. Wolisz, "Primary users in cellular networks: A large-scale measurement study," *Proceedings of IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks(DySPAN)*.

[28] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "A survey on spectrum management in cognitive radio networks," *IEEE Communications Magazine*, April 2008.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*, 1st ed. New York, NY, U.S.A.: Cambridge University Press, 2004.

Appendix A: Derivation of Unionized PU Idle Time Distribution

From (5), we need to derive p_{si} and $f_{si}(t)$. To determine p_{si}, the activity of the PUs is observed over time interval $[0, T]$, where T is the length of the first N' idle/busy cycles of the unionized PU. I_k^i, B_k^i,I'_k, and B'_k are the kth idle and busy periods of the unionized PU and PU i respectively. Therefore,

$$\begin{aligned} T &= \sum_{k=1}^{N'} I'_k + B'_k \\ &= (\sum_{m=1}^{N_i} I_m^i + B_m^i) + R_i, \quad i \in \{1, ..., M\}, \end{aligned} \tag{A-1}$$

where $R_i < I_{N_i+1}^1 + B_{N_i+1}^1$, and in general $N_i \neq N_j \neq N'$ for $i, j \in 1, ..., M$. Since PU i must end a transmission for every idle/busy cycle, each PU ends N_i transmissions during T. An idle period started by PU i results if all other PUs are idle when PU i transits from busy to idle state. Defining $\mathcal{N}_{si}$ as the number of idle periods started by PU i, we know that as $T \to \infty$,

$$\lim_{T\to\infty} \mathcal{N}_{si} = N_i \prod_{\substack{j=1 \\ j\neq i}}^{M} \alpha_j,$$

with

$$\lim_{T\to\infty} \sum_{i=1}^{M} \mathcal{N}_{si} = N'.$$

With these equations, p_{si} can be shown to be

$$p_{si} = \frac{\frac{1}{v_i}}{\sum_{j=1}^{M} \frac{1}{v_j}}. \tag{A-2}$$

We now determine $f_{si}(t)$. From (8), we need to derive $\phi_{ij}(t)$. If $i = j$, then $\phi_{ij}(t) = \phi_{ii}(t)$ is

$$\begin{aligned} \phi_{ii}(t) &= Pr[\min(V_1^r, V_2^r, ..., V_i, ..., V_M^r) = V_i] \\ &= f_{V_i}(t) \prod_{\substack{j=1 \\ j\neq i}}^{M} [1 - F_{V_j^r}(t)]. \end{aligned} \tag{A-3}$$

If $i \neq j$, $\phi_{ij}(t)$ is the probability that the residual idle time of PU j is t, and that all other users' idle times are greater than t, or

$$\begin{aligned} \phi_{ij}(t) &= Pr[\min(V_1^r, V_2^r, ..., V_j^r, ..., V_i, ..., V_M^r) = V_j^r] \\ &= f_{V_j^r}(t)[1 - F_{V_i}(t)] \prod_{\substack{k=1 \\ k\neq i,j}}^{M} [1 - F_{V_k^r}(t)]. \end{aligned} \tag{A-4}$$

We have now derived all terms from (5) for the full unionized idle time distribution $f_{V'_p}(t)$. From (5) we can obtain v'_p, and combining with (1), we can obtain n'_p as

$$n'_p = v'_p(\frac{1}{\alpha'} - 1). \tag{A-5}$$