Integration Gain of Heterogeneous WiFi/WiMAX Networks

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Abstract

We study the integrated WiFi/WiMAX networks where users are equipped with dual-radio interfaces that can connect to either a WiFi or a WiMAX network. Previous research on integrated heterogeneous networks (e.g., WiFi/cellular) usually consider one network as the main, and the other as the auxiliary. The performance of the integrated network is compared with the "main" network. The gain is apparently due to the additional resources from the auxiliary network. In this study, we are interested in *integration* gain that comes from the better utilization of the resource rather than the increase of the resource. The heterogeneity of the two networks is the fundamental reason for the integration gain. To quantify it, we design a generic framework that supports different performance objectives. We focus on max-min throughput fairness in this work. We first prove that it is NP-hard to achieve integral max-min throughput fairness, then propose a distributed heuristic algorithm, which provides 2-approximation to the optimal fractional solution. Simulation results demonstrate significant integration gain from two sources, namely spatial multiplexing and network diversity. We then analyze the integration gain following the generic framework. We study the impact of different user distributions on the integration gain, and validate the analysis using numerical simulations.

Index Terms

WiFi, WiMAX, Heterogeneous network, Integration gain, NP-hardness, Approximation algorithm.

I. INTRODUCTION

The IEEE 802.16 (WiMAX) is a promising technology due to its high data rate, wide coverage, and built-in support for mobility and security. Given the current vast deployment of WiFi networks, the coexistence between WiFi and WiMAX is inevitable. Major companies such as Intel and Motorola are promoting the integrated WiFi/WiMAX interface to take advantage of such scenario. Users equipped with such interfaces

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can associate with a WiMAX base station (BS) or a nearby WiFi access point (AP). Compared to the scenario where users only connect to WiFi networks, the benefit of the integrated network is obvious: we have additional spectrum resource from the WiMAX network. However, a closer look suggests that we may be able to reap significant gain from the heterogeneity of these two networks in addition to the extra resource. For example, in a typical integrated WiFi/WiMAX network, a WiMAX BS may cover a service area with up to hundreds of WiFi APs. In the WiFi network, users may experience poor quality of service (QoS) in some congested APs, while in some other APs, capacity may not be fully utilized. Similarly, in the WiMAX network, per-user throughput could be low if the number of WiMAX users is large. If users have the flexibility to switch between WiFi and WiMAX networks using the integrated interface, some WiFi users can switch from congested APs to WiMAX, while some WiMAX users can switch to under-utilized WiFi APs. Thus, the QoS in both networks improve. We refer to this as the spatial multiplexing gain, which will be discussed in more detail later. In addition, a user may have a low WiFi link capacity and a high WiMAX link capacity or vice versa. If the user intelligently selects its association, the network capacity improves, which is referred to as network diversity gain. These two types of improvements come from network heterogeneity and better utilization of the resource rather than the increase of the resource. This observation motivates our work. Our objective is to qualitatively and quantitatively identify such an integration gain, which was not addressed in existing work on integrated heterogeneous network (e.g., WiFi/cellular).

Our contributions are as follows. First, we propose a generic framework to identify the integration gain. The framework can serve different objectives. In this study, we focus on the *max-min* throughput fairness. Second, we prove that it is NP-hard to achieve integral max-min throughput fairness. We propose an approximation algorithm which provides 2-approximation to the optimal fractional solution. The algorithm is easy to implement due to its distributed nature, and is shown to achieve significant integration gain. Last but not least, we analyze the integration gain following the framework, and identify the impact of different user distributions on the integration gain.

II. Related Work

In cellular networks, macro/micro cellular architecture is similar to WiFi/WiMAX architecture in terms of spatial heterogeneity. Much work on macro/micro architecture focuses on how to perform optimal handoff [18]. The intuition is to associate low mobility users with micro-cells, and high mobility users with macro-cells to reduce the handoff frequency. The decision is mainly based on signal strength. The association policies considered in our work can also be viewed as handoff decisions. But instead of signal strength, we make the decision based on network performance metrics, such as throughput fairness. Other work focuses on the resource management and capacity analysis [7], [8], [15]. In general, micro-cells do not bring additional spectrum resources into the original "macro" network. The capacity improvement comes from frequency reuse and intelligent allocation of the resource between macro and micro-cells. In our work, we do not have control over spectrum allocation between WiFi and WiMAX networks. Each network has its own spectrum as well as users. Our objective is to study the integration of the two (separate) networks. From the viewpoint of either one of them, the other does bring additional resource as well as its own users. But we are interested in the integration gain, which is independent of the spectrum resource each network has.

Integrated WiFi/cellular network architecture has also been studied. Usually cellular network has a much smaller bandwidth than that of WiFi network. In most of the work, the cellular network is considered as the main network, and WiFi as the auxiliary. Most research efforts are put on the architecture design and QoS support of such network [16], [11], [4]. Usually, the performance of the integrated WiFi/cellular network is compared with the cellular network where the gain is obvious due to additional resources.

There are a few recent works on the handoff and load balancing in integrated WiFi/WiMAX networks [5], [9], [17]. But none of them explicitly studies the performance gain due to the heterogeneity of the two networks.

AP association in WiFi networks has been extensively studied. The association decision could be based on the received signal strength, the existing traffic load on APs, or a combination of several metrics [13], [2]. Bejerano et. al. [3] proved that it is NP-complete to achieve global max-min throughput fairness under integral association control. They proposed approximation algorithms to guarantee the performance ratio to the optimal fractional association which is the fairest association possible. Our problem can be viewed as a special case of theirs (i.e., consider WiMAX BS as a special AP with a much larger transmission range to cover the whole network). However, we exploit the special structure of the integrated WiFi/WiMAX network, and propose an algorithm that is simpler and with better performance. The algorithm is also easier to implement because both its message exchange and the computation are distributed. The heuristic algorithm in [3], on the other hand, requires a central controller to gather global information, perform the computation, and disseminate the decision to each user.

III. Network Model

We consider a service area large enough to contain multiple access points (APs). For example, In Chicago, up to 256 APs can be found in 1/2 square mile suburban area [12]. All APs directly connect to the Internet. Each AP has a limited transmission range, and only serves users within its range. We assume that there exists one WiMAX BS that covers the whole service area. It is a reasonable assumption since a WiMAX BS can typically reach a distance up to tens of miles. Each user is equipped with one WiFi radio and one WiMAX radio. It can choose to connect to a nearby AP, or the WiMAX BS, or even both by utilizing two radios at the same time. We use *integral* association to denote the first two cases because all traffic is sent on a single radio. We use *fractional* association to denote the last case because the user has to split its traffic on two active radios. In principle, fractional association provides better performance due to its flexibility. However, technical difficulties exist in practice. First, the requirement for the carrying device increases due to the excessive power consumption and heat when two radios are active at the same time. Second, the interference between the two co-located radios cannot be ignored even if they are operating on non-overlapping channels [19]. Last, the complexity of the MAC layer is largely increased because it has to deal with splitting traffic and sort out-of-order packets such that upper-layer protocols such as TCP will not be affected. Thus, we use the performance under the fractional association as a benchmark and study the integral association in practice.

We focus on the association with WiFi and WiMAX networks for each user. When a user can associate with multiple APs in its vicinity, we assume there exists a rule to pre-determine an AP. The pre-determination rule could be any load balancing algorithm in WLAN [13], [2] or based on the signal strength. Whenever the user decides to switch to the WiFi network, the WiFi radio always associates with the pre-determined AP. Given a set of users, we can determine the corresponding AP for each user following the pre-determination rule. We call such pre-determined user-AP mapping the *virtual AP association*, which is independent of the actual association. We will use it later to help simplify the presentation. We use (i, j) to denote the jth user associated with AP i in the virtual AP association. Since a user can only associate with a single AP, (i, j) can be used to uniquely identify a user. We use x_{ij} to denote the fraction of user (i, j)'s traffic to be sent through its WiFi radio, and $1 - x_{ij}$ as the fraction of the traffic through its WiMAX radio. We have $x_{ij} \in [0, 1]$ in fractional association, and $x_{ij} \in \{0, 1\}$ in integral association. So the *actual* association of user (i, j) is determined by x_{ij}.

We assume that the transmission in one AP does not interfere with that in adjacent APs. This can be achieved by assigning non-overlapping channels (e.g., 3 in 802.11b and 12 in 802.11a) to neighboring APs. WiMAX BS does not interfere with APs because it usually operates on a different frequency band. We use r_{ij} to denote the average WiFi rate observed by user (i, j), and R_{ij} to denote the average WiMAX rate if it is associated with the WiMAX BS.

Within each AP and WiMAX BS, we assume the bandwidth is *fairly* shared among all associated users. It could be either throughput fairness or time fairness. We note that WiFi MAC evenly divides the access opportunity among its associated user, which leads to the same throughput for each user. On the other hand, WiMAX MAC can choose to achieve either type of fairness. So in our study, we assume throughput fairness in an AP or the BS. We use T_i to denote the throughput of each user in AP i. For abbreviation, we also call it the throughput of AP i. Assume we have N users in AP i. Denote t_{ij} as the proportion of time for user (i, j) in AP i. We have $r_{ij}t_{ij} = r_{ik}t_{ik} \ \ \forall j \neq k$. Given that $\sum_{j=1}^{N} t_{ij} = 1$, we can obtain the throughput for each user in AP i as,

$$T_i = r_{ij}t_{ij} = \frac{1}{\sum_{j=1}^{N} \frac{1}{r_{ij}}} \tag{1}$$

Similar derivation can be applied on WiMAX BS. So in our problem, the throughput of AP i is $T_i = \frac{1}{\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}}}$. The throughput of WiMAX BS is $T_{wimax} = \frac{1}{\sum_i \sum_{j=1}^{N_i} (1-x_{ij}) \frac{1}{R_{ij}}}$, where N_i is the set of users in AP i under virtual AP association.

We list in Table I the notations used throughout the paper.

IV. FRAMEWORK

In this section, we propose a framework to quantify the integration gain, which has not been considered in prior work. We aim to make it generic so the framework can accommodate various performance metrics. The framework consists of three steps:

1) Create a WiFi-only network with N_{wifi} users. Measure the network performance. We denote the performance as r_{wifi}. In this step, the network can be generated arbitrarily. We do not pose any control on it. For instance, it could be a randomly deployed WiFi network.
2) Create a WiMAX-only network with a controllable number of users N_{wimax} in the same service area. In this step, N_{wimax} is carefully adjusted to make the performance of the WiMAX network also r_{wifi}. For example, if the performance metric is average throughput, and $r_{wifi} = 0.5Mbps$, we can generate a set of WiMAX users and adjust its number so that the average throughput is

close enough to $0.5Mbps$. Note that, under other metrics, it could be more complicated than simply adjusting the number of WiMAX users to achieve the same performance.

3) Integrate the two networks with their corresponding users, i.e., total number of users is $N_{wifi} + N_{wimax}$. Based on the first two steps, if the two networks are simply merged without interactions between them, the performance of the integrated network should still be r_{wifi}. On the other hand, there may exist metric-dependent interaction policies that improve the overall performance. We choose the best policy and denote its performance as r_{opt}. We define the integration gain as $\frac{|r_{opt}-r_{wifi}|}{r_{wifi}}$. If the best policy is impractical to find (e.g., the problem is NP-hard), one may resort to its approximations.

The key concept is as follows: *by ensuring WiMAX has the same performance as WiFi before the integration, we ensure that the gain comes from integration instead of additional resources.*

Before we can calculate the integration gain, we need to choose a performance objective, and derive its optimal policy. In this study, we focus on max-min throughput fairness. We choose max-min fairness because it improves worst-case experience and is achieved by the default WiFi access scheme. In addition, max-min fairness is more mathematically tractable, which enables us to focus on the essence of integration gain. We are aware that in a single-cell scenario, a user with a poor channel condition can deteriorate the performance of other users severely under max-min fairness. In this multi-AP WiFi/WiMAX network, the performance is determined by many factors, including user distributions, number of users in each AP, and network heterogeneity. Therefore, the impact of a single user is much smaller. Last, in our on-going work, we are studying optimal integration policies under proportional fairness and its integration gain (although closed-form analysis is more challenging).

V. Max-Min Throughput Fairness and Approximation Algorithm

We first prove it is NP-hard to achieve integral max-min throughput fairness in the integrated WiFi/WiMAX network. Then we propose a distributed approximation algorithm that achieves guaranteed performance.

A. Max-Min Throughput Fairness

Let a throughput vector $\overrightarrow{T} = \{t_1, t_2, \cdots, t_N\}$ denotes the throughput distribution of all users in the network. Informally, max-min throughput fairness means that we *cannot* increase the throughput of one user without decreasing that of another user with equal or less throughput. Formally, it is defined as follows.

Definition 1: **Max-Min Throughput Fair**: A throughput vector $\overrightarrow{T}$ is called max-min fair if it has the highest lexicographical value among all throughput vectors. That is, if $\overrightarrow{T} \neq \overrightarrow{T'}$, there exists a position j such that $t_i = t'_i$ for $i < j$, and $t_j > t'_j$.

Note that "max-min" and "maximize the minimum" are two different concepts. We use the former to describe the bandwidth allocation with the best lexicographical order, and the latter to describe those with the maximum minimum throughput. So the former implies the latter, but the reverse usually does not hold.

B. Proof of NP-hardness

In this section, we prove our problem, to provide max-min fairness in an integrated WiFi/WiMAX network under integral association, is NP-hard. The idea is to reduce the *Partition* problem to our problem.

Definition 2: ***Partition*** (decision) : Can a set of numbers, S, be divided into two disjoint subsets S_1 and S_2, such that the sum of both subsets equals?

We consider a special case of our problem: each user's WiMAX rate is the same as its WiFi rate, while different users may have different rates. We prove that this special case is NP-hard. The general case where each user's WiMAX and WiFi rates are different should also be NP-hard. Assuming there are M APs, N users, and one WiMAX BS, we consider a related problem.

Definition 3: $S(M, N, D)$**)**: Can we allocate users between WiFi APs and WiMAX BS, such that each group (i.e., APs or BS) has a load less than or equal to D?

Note that, given the load D, the throughput $T = \frac{1}{D}$. To prove our problem is NP-hard, it is sufficient to show that the subproblem of finding the maximum minimum throughput in the bandwidth vector is NP-hard. To achieve this, we can start from a large T, then use binary search together with $S(M, N, \frac{1}{T})$ to iteratively approach the maximum T we can obtain. We need polynomial number of calls to $S(M, N, \frac{1}{T})$ to find the maximum minimum throughput. So it is sufficient to prove that $S(M, N, D)$ is NP-hard.

We then prove $S(M, N, D)$ can be reduced from *Partition*. Let A be an instance of *Partition*. Each element in A has a weight associated with itself. Let the sum over all weights in A be $2D$. We then construct an instance of $S(M, A + M - 1, D)$ by adding $M - 1$ new elements into A. Each of the new $M - 1$ elements has a weight of D. In our scenario, an element corresponds to a user, the weight corresponds to the load the user contribute to the associated AP or the WiMAX BS.

If A is a "yes" instance of *Partition*, we can divide A into two subsets A_1 and A_2, each has a total weight of D. Thus, $S(M, A+M-1, D)$ is also a "yes" instance. Conversely, if $S(M, A+M-1, D)$ is a "yes" instance, we must have $M-1$ groups where only a single user with the weight of D remain in each group. The remaining two groups must include all elements in the original set A, and each group has a total weight less than or equal to D. Since the sum of all elements in A is $2D$, the sum of each group must be exactly D. Thus, A is also a "yes" instance of *Partition*.

C. Max-Min Fairness under Fractional Association

Optimal fractional association provides the best possible max-min throughput fairness and thus its performance serves as a benchmark for that of integral associations. While it is difficult to achieve max-min fairness under integral association, the optimal fractional association to achieve max-min fairness can be obtained by solving the following simple LP.

$$
\begin{aligned}
&\min \beta \\
&\sum_{j=1}^{N_i} x_{ij} \frac{1}{r_{ij}} \leq \beta \qquad \forall \text{ AP } i \\
&\sum_i \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \leq \beta \\
&0 < x_{ij} < 1 \qquad \forall\ (i, j)
\end{aligned}
\tag{2}
$$

The objective is to minimize the maximum load among all users. If we let $\beta = \frac{1}{\alpha}$, we see that it is equivalent to maximize the minimum throughput.

We denote the solution as χ. Note that (2) does not have assumptions on initial conditions. So we should always get the same output no matter what the initial association the users may have. To ease the presentation, in the following discussion, we imagine all users initially associate with WiFi APs (i.e., following the virtual AP association). We say user (i, j) is *switched* to WiMAX if $x_{ij} < 1$. It includes two cases: the user is in WiMAX entirely ($x_{ij} = 0$) or fractionally ($x_{ij} > 0$).

We use $\vec{T}$ to denote the throughput distribution under χ. Now we prove that, $\vec{T}$ is max-min fair. First, we define the bottleneck group.

Definition 4: **Bottleneck group**: Under χ, WiFi APs with at least one user switched to WiMAX, together with WiMAX BS, are called the bottleneck group G_B.

Lemma 1: In $\vec{T}$, all users in the bottleneck group have the same throughput T, which is the inverse of the objective value of (2).

Proof: Define T as the inverse of the objective value of (2). Thus T is the minimum throughput in $\vec{T}$. Let us consider AP a in the bottleneck group. By definition, it has at least one user switched to WiMAX. We use T_a and T_{wimax} to denote the throughput of AP a and WiMAX, respectively. First we prove that T_{wimax} must equal to the minimum throughput of the network, which is T. Otherwise, there exists an AP with throughput $T_{min} < T_{wimax}$. Then we can switch some users from this AP to WiMAX until both reach the same throughput. Then T_{min} will be improved, which contradicts the objective of (2). Second, we prove $T_a = T_{wimax}$. Otherwise, we must have $T_a > T_{wimax}$. Then WiMAX can "return" some users it previously switched from AP a until T_a and T_{wimax} equal. It improves the minimum throughput of the network, contradicting (2).

The above proof can be applied to each AP in the bottleneck group. Thus, all users in the bottleneck group have the same throughput T. ∎

Theorem 1: χ leads to the max-min throughput fairness under fractional association control.

Proof: We prove by contradiction. Assume we can find a better association χ', which leads to a better (in terms of lexicographical order) throughput distribution. Let $\vec{T}'$ denote the throughput vector under χ'. We have $\vec{T}' > \vec{T}$. From Lemma 1, the lowest throughput in $\vec{T}'$ must also be T, which is the best minimum throughput. Following the same proof as in Lemma 1, WiMAX must also have the lowest throughput under χ' . Let G denote the group of APs whose throughput are smaller than T before the load balancing. Under χ, each AP in G must have some users switched to WiMAX and thus belongs to the bottleneck group G_B. Under χ', each AP in G must have equal or larger throughput than T since $\vec{T}' > \vec{T}$. So these APs also have users switched to WiMAX. We argue that AP in G under χ' cannot have a throughput larger than T. Otherwise, such AP can increase the WiMAX throughput by recalling some original users from it. Then WiMAX can in turn help each AP with throughput of T a little bit by increasing the fraction of users it switches from these APs. The minimum throughput will be larger than T, which is not possible. Thus, each AP in G, and the WiMAX must also have the throughput of T under χ'. This suggests that exactly the same set of users with the same fraction are switched from G to WiMAX under both χ and χ'.

The only way $\vec{T}'$ can be better than $\vec{T}$ is therefore χ' may switch some users from APs with original throughput larger than T. These APs, denoted as G', do not have users switched to WiMAX under χ, thus remain the original throughput. So in $\vec{T}'$, at lease one AP in G' has users switched to WiMAX. Then

WiMAX must have a throughput less than T, which leads to $\vec{T}' < \vec{T}$. ■

D. Approximation Algorithm

Since our problem is NP-hard, we have to use an approximation algorithm to provide integral association in practice. We want the approximation algorithm to guarantee the performance relative to the optimal *fractional* solution, which is the fairest among all possible throughput distributions.

1) Algorithm Description: The algorithm is shown in Fig. 1. It works on the integrated network with an arbitrary initial association. Each user queries the WiFi and WiMAX data rates from both radios (The WiFi rate is from the virtually associated AP). After each user reports its rate information to the corresponding virtual AP, each AP sorts all associated virtual users based on their WiMAX-WiFi rate ratio in a decreasing order. It then starts a loop. Inside the loop, the AP with the smallest throughput is selected, and its first user is marked to be switched to WiMAX. Each AP and the WiMAX BS then update their virtual throughput, and start the loop again until the minimum throughput of the integrated network stops increasing.

The algorithm is distributed. In practice, each AP can report its virtual throughput to WiMAX BS. The BS is then responsible for selecting the right AP in each iteration. The computation is distributed among APs and the BS. We have two layers of information exchange (i.e., user-AP and AP-BS) with limited message overhead in each layer. Due to the special structure of the integrated WiFi/WiMAX network, such a simple algorithm can still provide performance guarantee. In the following, we focus on the users switched to WiMAX under optimal fractional association χ. Among them, we define *integral* users as the users with $x_{ij} = 0$, and *fractional* users as the users with $0 < x_{ij} < 1$ in χ.

2) Proof of the Performance Bound:

Theorem 2: Algorithm 1 provides 2-approximation to the optimal fractional solution.

Proof: In the following, we outline the proof before we provide the details.

1) Prove that an intermediate algorithm (Algorithm 2) results in a throughput distribution with an equal or higher lexicographical order than an existing algorithm (Algorithm 3), which is shown to provide 2-approximation to the optimal fractional association.
2) Prove that our approximation algorithm (Algorithm 1) results in a throughput distribution with an equal or higher lexicographical order than the intermediate algorithm (Algorithm 2).

■

*a) **Step 1**:* Reference [3] presents a 2-approximation algorithm with threshold, as shown in Fig. 3. In the algorithm, $Fractional\_Load\_Balancing(A, U)$ consists of two LPs and a simple graph coloring procedure. It gives the optimal fractional user association to provide max-min throughput fairness. The rounding method [14] constructs a bipartite graph based on the optimal fractional association, then uses maximal matching to determine the integral association. Since our problem can be viewed a special case of theirs in [3], this algorithm can also be applied on our problem after we replace $Fractional\_Load\_Balancing(A, U)$ with the LP defined in (2). Though applicable to the same problem, our algorithm is better than theirs because of two reasons. First, our algorithm can also provide 2-approximation to the optimal fractional solution. Numerical simulation (later in this section) shows that our algorithm outperforms theirs in practice. Second, our algorithm is distributed, which has a better scalability than their centralized algorithm. One important property of Algorithm 3 is that it switches all the integral users and a subset of fractional users to WiMAX. We need it for the following proof.

We design an intermediate algorithm shown in Fig. 2. Algorithm 2 takes χ as input. It first switches all integral users in χ to WiMAX. Then it performs a similar loop as in Algorithm 1. But it only looks at fractional users in each AP inside the loop. We now show that Algorithm 2 performs better than Algorithm 3 in terms of lexicographical value.

Lemma 2: Under χ, within each AP, a user switched with WiMAX has a higher or equal WiMAX-WiFi rate ratio than any user remaining in that AP.

Proof: The Lagrangian function of the LP defined in (2) is

$$L(\beta, \chi) \;=\; \beta - \sum_{i=1}^{|A|} \theta_i \left(\beta - \sum_{j \in N_i} x_{ij} \frac{1}{r_{ij}} \right) - \sum_i \sum_{j=1}^{N_i} \lambda_{ij} x_{ij} - \sum_i \sum_{j=1}^{N_i} \omega_{ij} (1 - x_{ij}) - \psi \left(\beta - \sum_i \sum_{j=1}^{N_i} (1 - x_{ij}) \frac{1}{R_{ij}} \right)$$

where θ, λ, ω, and ψ are slack variables. According to Lagrangian Multiplier method and complementary slackness, we have the following equations,

$$\frac{\partial L}{\partial \beta} = 1 - \sum_{i=1}^{|A|} \theta_i - \psi = 0 \tag{3}$$

$$\frac{\partial L}{\partial x_{ij}} = \theta_i \frac{1}{r_{ij}} - \lambda_{ij} + \omega_{ij} - \psi \frac{1}{R_{ij}} = 0$$

$$\lambda_{ij} x_{ij} = 0 \quad \forall \;\; \text{user } (i, j)$$

$$\omega_{ij} (1 - x_{ij}) = 0 \quad \forall \;\; \text{user } (i, j)$$

Within AP i, if user j has been switched, i.e., $x_{ij} < 1$, then we have

$$\omega_{ij} = 0, \lambda_{ij} \geq 0 \Rightarrow \theta_i \frac{1}{r_{ij}} \geq \psi \frac{1}{R_{ij}}$$

Similarly, if user k has not been switched, i.e., $x_{ik} = 1$, then we have

$$\omega_{ik} \geq 0, \lambda_{ik} = 0 \Rightarrow \theta_i \frac{1}{r_{ik}} \leq \psi \frac{1}{R_{ik}}$$

Then we have $\frac{R_{ij}}{r_{ij}} \geq \frac{R_{ik}}{r_{ik}}$. ■

Corollary 1: Given χ, there exists an association χ' with the same performance where an AP can have at most one fractional user.

Proof: Under χ, if AP i has two fractional user (i, j) and (i, k), we have $0 < x_{ij}, x_{ik} < 1$. Thus $\frac{R_{ij}}{r_{ij}} = \frac{R_{ik}}{r_{ik}}$ based on (3). Similar idea applies to the case with multiple fractional users. So all fractional users in the same AP must have the same WiMAX-WiFi rate ratio. In this case, we can always "aggregate" multiple fractional users into some integral users and at most one fractional user without changing the performance. We start from a simple case by assuming AP a has 2 fractional users under χ. Their fractions are x_1 and x_2, WiFi rates are r_1 and r_2, and WiMAX rates are R_1 and R_2, respectively. The throughput of this AP is

$$T_a = \frac{1}{L_a + x_1 \frac{1}{r_1} + x_2 \frac{1}{r_2}} \tag{4}$$

where L_a is the load contributed by other users in this AP. Similarly, the throughput of the WiMAX is

$$T_{wimax} = \frac{1}{L_{wimax} + (1 - x_1)\frac{1}{R_1} + (1 - x_2)\frac{1}{R_2}} \tag{5}$$

where L_{wimax} is the load from other users in WiMAX. We have $\frac{R_1}{r_1} = \frac{R_2}{r_2}$. We consider two cases:

Case 1: $\frac{r_2}{r_1} x_1 + x_2 \leq 1$. We consider a new association χ' where both fractional users have associations of $x'_1 = 0, x'_2 = \frac{r_2}{r_1} x_1 + x_2$, and associations of other users remain unchanged. We can verify that the throughput of AP a and WiMAX is the same in χ and χ'. Since the throughput of other users remain the same, χ and χ' lead to the same throughput distribution.

Case 2: $\frac{r_2}{r_1} x_1 + x_2 > 1$. We consider a new association χ' where both fractional users have associations of $x'_1 = x_1 - (1 - x_2)\frac{r_1}{r_2}, x'_2 = 1$. We have $0 < x'_1 \leq 1$. Similarly, χ and χ' lead to the same throughput distribution.

If we have multiple fractional users, we iteratively apply the same approach on two fractional users until at most one fractional user remains. ■

In the following, we assume each AP under χ has at most one fractional user. Otherwise, we can always use the corresponding χ' to replace χ.

Theorem 3: Algorithm 2 results in a throughput distribution with an equal or higher lexicographical order than Algorithm 3.

Proof: Note that Algorithm 3 switches all integral users and a subset of fractional users to WiMAX. From Corollary 1, Algorithm 2 also switches all integral users and a subset of fractional users from χ to WiMAX. We use Γ to denote all algorithms which switch all integral users and a subset of fractional users to WiMAX. It suffices to prove that Algorithm 2 is the best in Γ in terms of max-min fairness. Suppose there exists Algorithm $2'$ in Γ with a better performance. Since they share the same set of integral users, they must differ in fractional users. Let Δ and Δ' be the set of fractional users switched in Algorithm 2 and Algorithm $2'$, respectively. It is trivial that Δ cannot be a subset of Δ'. Otherwise, the minimum throughput under Algorithm $2'$ will be lower than that of Algorithm 2. So let us focus on the case where some fractional users switched in Δ are not switched in Δ'. Consider one of these users, a, and the corresponding AP i it originally associated with. Let T_i^{a-} denote the throughput of AP i before a is switched with WiMAX. In Algorithm 2, the minimum throughput is strictly larger than T_i^{a-} because it keeps switching fractional users until the minimum throughput stops increasing. On the other hand, the minimum throughput of Algorithm $2'$ is at most T_i^{a-} because user a is not switched. Thus, Algorithm 2 actually performs better than Algorithm $2'$, which contradicts the assumption. ■

*b) **Step 2**:* Now we show that Algorithm 1 performs better than Algorithm 2. We first prove the following lemma.

Lemma 3: The set of users switched in Algorithm 1 is a subset of that under χ.

Proof: We prove by contradiction, and only need to consider the case where Algorithm 1 switches at least one user which is not switched under χ. We consider one of such user, a. We have two scenarios:

Scenario 1: a is from AP i which has no user switched by χ. So AP i must have an original throughput T_i larger than the bottleneck throughput T. If Algorithm 1 switches user a at some iteration, the minimum throughput in Algorithm 1 is larger than T_i. Thus, Algorithm 1 leads to a better max-min throughput distribution than χ, which is not possible.

Scenario 2: a is from AP i which has users switched by χ. Thus, the throughput of AP i under χ is T. From Lemma 2, we know that users switched under χ have a larger WiMAX-WiFi rate ratio than users stay associated with the original AP. Since Algorithm 1 sorts users based on their rate ratio, user a must have a lower ratio than users switched under χ. Thus, switching user a suggests that users switched under χ should have already been switched in Algorithm 1. Therefore AP i already has a throughput at least T before a is switched. Then after a is switched in Algorithm 1, the minimum throughput will be larger than T. We face the same contradiction as in the first scenario. ■

Theorem 4: Algorithm 1 results in a throughput distribution with an equal or higher lexicographical order than Algorithm 2.

Proof: Let Ω denote the set of users Algorithm 1 switches. By Lemma 3, Ω is a subset of all users switched under χ. We have two scenarios.

Scenario 1: Ω includes all integral users and a subset of fractional users from χ. In this case, Algorithm 1 and Algorithm 2 perform exactly the same.

Scenario 2: Ω omits at least one integral users from χ, and includes the other integral users and a subset of fractional users. We consider one of the omitted integral user a from AP i. Let T_i^{a-} denote the throughput of AP i before a is switched in Algorithm 1. We assume T_i^{a-} is the smallest throughput among all APs containing omitted integral users. There must exist at least one fractional user which is switched to WiMAX. Otherwise, Algorithm 1 will not stop because WiMAX still has a higher throughput than the bottleneck throughput T (no fractional user in WiMAX yet). It can therefore switch user a to WiMAX to achieve a better throughput distribution. We look at the *last* fractional user b to be switched to WiMAX under Algorithm 1. Assume the corresponding AP is j. According to Algorithm 1, we have $T_j^{b-} < T_i^{a-}$. We prove by contradiction. Suppose Algorithm 2 performs better. We argue that the fractional user b should also be switched in Algorithm 2. Otherwise the minimum throughput under Algorithm 2 is at most T_j^{b-} while the minimum throughput is strictly larger than T_j^{b-} under Algorithm 1. We can argue the same on all the fractional users switched by Algorithm 1 before b because the corresponding APs have throughput less than T_j^{b-} before their fractional users are switched. These users should also be switched in Algorithm 2. Since Algorithm 2 switches an extra user a to WiMAX, its WiMAX throughput should be lower than that of Algorithm 1.

If WiMAX throughput is the lowest in both algorithms, we have a contradiction that Algorithm 1 actually performs better than Algorithm 2. Otherwise, there must be an AP k which has the lowest throughput under Algorithm 1, while under Algorithm 2, AP k has equal or better throughput. But it cannot be better because it means the fractional user is switched to WiMAX under Algorithm 2. Since

WiMAX throughput under Algorithm 1 is higher, it can also switch the fractional user in AP k before the algorithm stops. We argue the same for each AP except for AP i because it has a lower throughput in Algorithm 1 than in Algorithm 2. But WiMAX throughput under Algorithm 1 has to be lower than T_i^{a-}, otherwise it can always switch a to WiMAX. Thus, if WiMAX is not the lowest, the throughput vectors before WiMAX must be equal under both algorithms, and they differ from WiMAX. Thus, we have the same contradiction. ■

E. Simulations

We use simulations to compare the performance of the optimal fractional solution, our heuristic algorithm (i.e., Algorithm 1), and the algorithm in [3] (i.e., Algorithm 3). We implement a realistic wireless channel model that considers the transmission power, path loss, and different modulation schemes of both WiFi and WiMAX. We adopt the value suggested in the standard and measured by other researchers [1].

We consider a service area of 1500x1500. Nine APs form a regular grid in the service area, while users are randomly and uniformly distributed. We assume that users determine the initial AP association based on the received signal strength. WiMAX BS is placed at the center of the service area. We change the number of users from 50 to 250. For a given number of clients, we average the results over 20 random instances.

In each instance, we follow the three steps in the generic framework. We use the minimum throughput as the performance metric because better minimum throughput is sufficient to indicate a better max-min throughput distribution. We plot the throughput under three algorithms in the integrated network and that of the original WiFi network. We consider two cases. In the first case, we assume users have uniform WiFi and WiMAX rates. Uniform rate means that all users have the same rate, e.g., 54Mbps in WiFi and 70Mbps in WiMAX. In the second case, users have non-uniform data rates where the rate is determined by the received SNR, the target bit error rate (BER) and the corresponding modulation scheme. In general, the rate decreases with the distance between the sender and the receiver.

Under non-uniform rate, we have difficulties to generate the WiMAX network in step two of the framework. In this case, the minimum throughput is determined by both the number of users in the group and the minimum data rate among users. It is difficult to create the WiMAX network with the same minimum throughput because of the variations in the data rate. So we make a slight modification to the framework. For both cases, we generate the WiMAX network based on the average throughput. But the integration gain is still calculated based on the minimum throughput. We plot both minimum throughput and average throughput performance for the two cases. Fig. 4(a) shows the minimum throughput under uniform rate. In this case, Algorithm 1 performs slightly better than Algorithm 3. Both of them have close to optimal performance. We observe about 65% integration gain. It mainly comes from the load balancing. Users in congested WiFi APs will be switched to WiMAX to improve the minimum throughput. But as shown in Fig. 4(b), the total capacity of the network remains the same, i.e., the integrated network has the same average throughput as the original WiFi network. We also use *spatial multiplexing gain* to denote this type of integration gain.

Fig. 4(c) plots the minimum throughput under non-uniform rate. The gain is about 300%, which is much larger than that under uniform rate. In the non-uniform rate case, users with low WiFi rates may have high WiMAX rates or vice versa. By switching these users from where it has low rate to where it has high rate, in addition to the load balancing, the network capacity also improves, which is indicated by higher average throughput in the integrated network than that of WiFi-only network (Fig. 4(d)). That is the reason for the large improvement we observe. We refer it as *network diversity gain*.

In summary, we observe significant integration gain, coming from spatial multiplexing and network diversity. Our proposed algorithm performs close to the optimal fractional scheme, and outperforms the algorithm in [3], especially in the presence of network diversity.

VI. Analysis of the Integration Gain

In the previous section, we proposed a simple algorithm with 2-approximation ratio that can decide network association. Numerical results show good performance. In this section, we analyze the integration gain assuming users have uniform WiFi and WiMAX rates defined in the previous section. That is, we focus on the spatial multiplexing gain. This can simplify the analysis while still providing sufficient insights. We will analyze the performance of non-uniform rate case in the future.

We study different user distributions and their impact on the integration gain. The performance metric is minimum throughput. We use C_{wifi} and C_{wimax} to denote the WiFi rate and WiMAX rate, respectively. For WiFi-only network, assume we have N_{wifi} users and M APs. Assume APs are regularly deployed in the service area. Let N_i denote the number of users associated with AP i.

A. Uniform User Distribution

Assuming each user is randomly and uniformly distributed in the service area, N_i follows a Binomial distribution:

$$\Pr(N_i = k) = \binom{N_{wifi}}{k} \left(1 - \frac{1}{M}\right)^{N_{wifi}-k} \left(\frac{1}{M}\right)^k . \tag{6}$$

Let $Y = \max N_i$, the minimum throughput of the WiFi network, r_{wifi}, is thus:

$$r_{wifi} = \frac{C_{wifi}}{Y} \tag{7}$$

The number of WiMAX users is calculated as $N_{wimax} = C_{wimax}/r_{wifi}$. For the integrated network, we have $N = N_{wifi} + N_{wimax}$ users. We assume all users associate with WiFi APs initially. We use N_i' to denote the number of users associated with AP i. It also follows Binomial distribution. Under the uniform rate assumption, the approximation algorithm we derived from the previous section is simplified as switching a certain number of users from each AP to WiMAX. We define a threshold N_T. The idea is that N_T is carefully chosen such that the throughput of the WiMAX network is the same as the minimum throughput of WiFi network in the integrated network. Under integral association, we want both sides to be as close as possible. If $N_i' > N_T$, we switch $N_i' - N_T$ users from AP i to WiMAX. Otherwise, no users from this AP will be switched. Then we will have the following equation.

$$\frac{C_{wifi}}{N_T} = \frac{C_{wimax}}{\sum_{i=1}^{M} \left(\max\left\{0, N_i' - N_T\right\}\right)} \tag{8}$$

The minimum throughput of the integrated network is thus $r_{opt} = \frac{C_{wifi}}{N_T}$. The integration gain $G = \frac{r_{opt} - r_{wifi}}{r_{wifi}}$

Since N_i and N_i' are both Binomial random variables, Y and N_T are also random variables. We will use the results from *Order Statistics* to obtain their expectations [6]. Order Statistics deals with the distributions of a series of *sorted* random variables of the same probability distribution function f. Specifically, for M sorted random variables, the expected value of the ith largest variable can be approximated as $F^{-1}\left(\frac{M+1-i}{M+1}\right)$, where $F^{-1}(x)$ is the inverse of the cumulative distribution function F. In our problem, f is the Binomial distribution, and F is known as a *regularized incomplete beta function*. Thus, we take expectations on both sides of the above formulations. For example, $E[Y] = F^{-1}\left(\frac{M}{M+1}\right)$. Similarly, we can obtain expectations of other random variables and calculate the integration gain.

B. Clustering User Distribution

In practice, users may tend to gather in some specific locations rather than uniformly distributed in the service area. For example, it is more likely for people to stay in the meeting room and office room than the corridor inside a building. We use Preferential Attachment (PA) to model such a clustering-like user distribution. PA means the more users an AP has, the more likely it can attract additional users. In our implementation, the probability of an AP to attract a user is proportional to the ratio between its current number of users and all users currently in the network. That is, a new user associates with AP i with probability $p = d_i / \sum_{i=1}^{M} d_i$, where d_i is the number of users in AP i. We assume that each AP has one user at the beginning.

We follow the same idea as in uniform user distribution case. The only difference is that f is not the Binomial distribution under the PA assumption. We assume users join the network one at a time. When the tth user joins the network, taking into account the initial M users, $p = d_i/(t + M)$. Let $P_{k,t}$ denote the probability density function that an AP has k users after t users have joined the network. We have,

$$P_{k,t+1} = P_{k,t}\left(1 - \frac{k}{t+M}\right) + P_{k-1,t}\frac{k-1}{t+M} \tag{9}$$

With the initial condition $P_{1,0} = 1$ and $P_{k,0} = 0 \ \ \forall k > 1$, we have the closed-form solution for $P_{k,t}$

$$P_{k,t} = \frac{(M-1)\prod_{i=0}^{k-2}(t-i)}{\prod_{i=1}^{k}(M+t-i)} \tag{10}$$

We then use the same approach as in the uniform distribution case to calculate the gain.

C. Validation of the Analysis

We use numerical simulations to validate our analysis in the previous section. Fig. 5 shows the integration gain under uniform user distribution. Under a given number of APs, integration gain decreases with the number of users. On the other hand, under a given number of users, integration gain increases with the number of APs. In general, the larger the ratio between the number of users and the number of APs, the smaller the integration gain. The idea is, larger ratio leads to smaller “relative” difference in the number of associated users among APs, which leaves less room for load balancing.

Note that, the difference between the analysis and simulation becomes smaller when the number of APs is larger (i.e., 100 APs). In the analysis, the expected value of the ith variable is an approximation. This approximation is more accurate when the number of sorted variables (number of APs in our scenario) is larger.

Fig. 6 shows the performance gain under clustering user distribution. The integration gain is about 65%, much higher than that in the uniform user distribution. This is reasonable because we have some congested “hotspots” under clustering user distribution, the benefit of load balancing is more significant than the uniform distribution. Furthermore, the gain is almost independent of the number of users and number of APs. Note that PA approximates a scale-free network. In such a network, the relative difference among APs is stable, which leads to the flat gain.

VII. Conclusions

In this paper, we study the integration gain of integrated WiFi/WiMAX network. Previous work on integrated heterogeneous networks usually assumes one of the networks is the main, and compare the performance of the integrated network with the main network. Thus the performance gain comes from the additional resources brought by the auxiliary network as well as the network integration. To our knowledge, we are the first to propose a framework to explicitly identify the integration gain, which is separated from the impact of additional resources. In other words, we quantify the gain from the network heterogeneity and better resource utilization. The framework supports different performance metrics. In this study, we focus on max-min throughput fairness.

We prove that it is NP-hard to achieve integral max-min fairness. We propose a heuristic algorithm that provides 2-approximation to the optimal fractional association policy. The algorithm is simple and intuitive. It is also easy to implement due to its distributed nature. Numerical simulations show significant gain under both uniform and non-uniform rate scenarios. We identify two sources of integration gain, namely the spatial multiplexing and network diversity.

To provide more insights, we analyze the integration gain from spatial multiplexing following the framework. We study the gain under different user distributions, the uniform and clustering user distribution. The analysis is validated by numerical simulations.

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TABLE I
NOTATIONS

Notations	Comments
A	Set of WiFi APs in the service area
U	Set of all users
N_i	Set of users virtually associated with AP i
T_i	Throughput of both AP i and its member
T_{wimax}	Throughput of WiMAX and its member
r_{ij}	WiFi rate of user (i, j)
R_{ij}	WiMAX rate of user (i, j)
x_{ij}	Fraction of user (i, j) with its AP
χ	Optimal fractional association from the LP

Algorithm 1 Approximation

```
Each user queries WiFi and WiMAX rates from its two radios, and report to its virtual AP
AP i sorts virtual users based on their WiMAX-WiFi rate ratio R_ij/r_ij in decreasing order
while The minimum virtual throughput improves do
   Find the AP with the minimum virtual throughput
   Re-associate the first user to WiMAX
   Check the virtual throughput of WiMAX and APs
end while
Output the association
```

Fig. 1. Proposed approximation algorithm

Algorithm 2 Intermediate

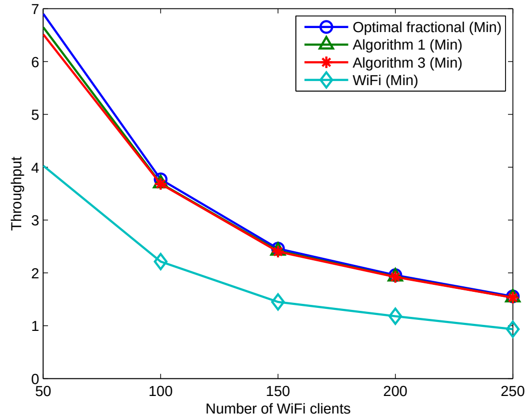
```
χ ← Solve LP (5)
Include integral users in χ into WiMAX
For fractional users in χ
while The minimum throughput improves do
   Find the AP with the minimum throughput
   switch its fractional user to WiMAX
   Check the throughput of WiMAX and APs
end while
```

Fig. 2. An intermediate algorithm

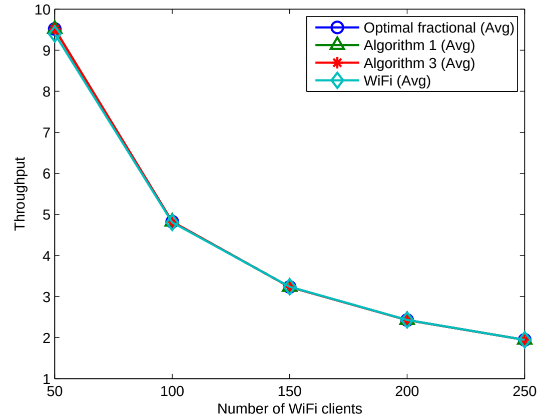
Algorithm 3 Reference paper

$\chi^{frac} \leftarrow Fractional\_Load\_Balancing(A, U)$
$\chi^{int} \leftarrow Rounding(\chi^{frac})$
return χ^{int}

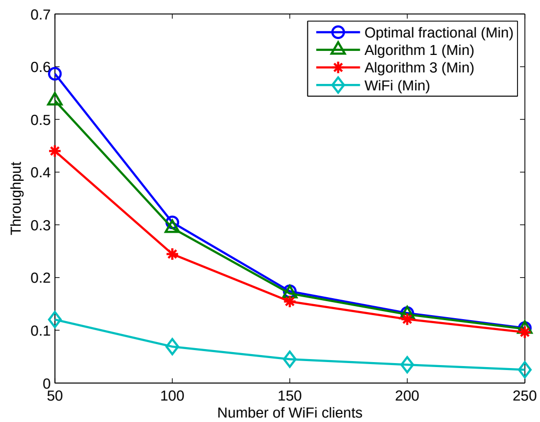
Fig. 3. Approximation algorithm in [3]



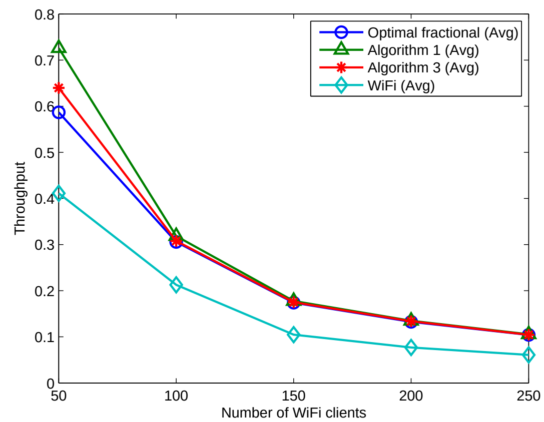
(a) Uniform rate: Minimum throughput



(b) Uniform rate: Average throughput



(c) Non-uniform rate: Minimum throughput



(d) Non-uniform rate: Average throughput

Fig. 4. Performance under uniform and non-uniform rates

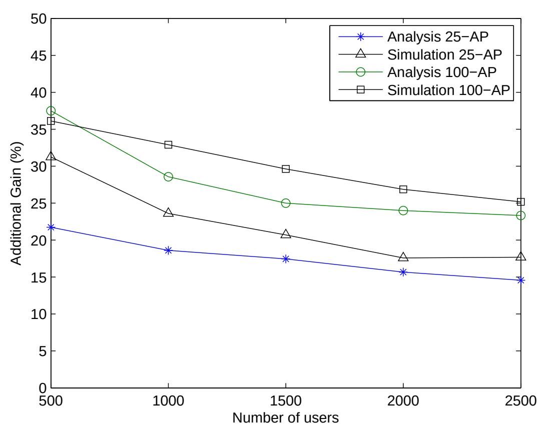


Fig. 5. Integration gain under uniform user distribution.

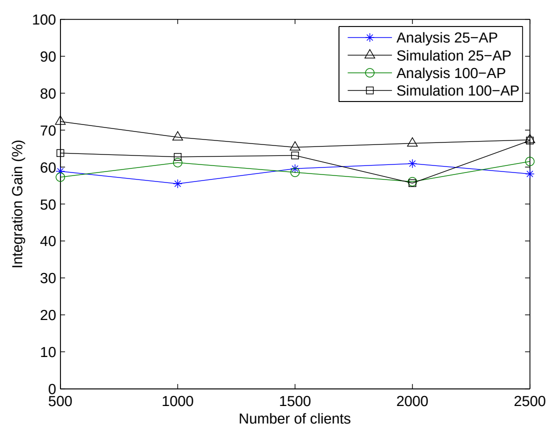


Fig. 6. Integration gain under clustering user distribution.