
Midterm Exam Section 1 (11:00)

NAME:

Instructions: This is an open book, open notes exam. Do all 4 problems. Communicate your ideas *clearly* and *succinctly*. Show all work.

On problem	you got	out of
1		83
2		17
Σ		100

1 Short answers**[83 points]**

Part A. (11) On the Homework we suggested splitting the strip into four sets of points instead of just two. Suppose now that we split the strip into 6 sets of points (each of width $d/3$). Let the x -coordinate of the dividing line be X . Let Y'_{L2} be the points (x, y) on the left side of the dividing line such that $2d/3 \geq X - x > d/3$ (so their x -coordinate is more than $d/3$ from the dividing line, but is at most $2d/3$ from it). Similarly, Y'_{R2} are the points on the right so their x -coordinate is more than $d/3$ from the dividing line, but is at most $2d/3$ from it.

For a point (a, b) in Y'_{L2} , what is the maximum y coordinate of a point in Y'_{R2} for which we need to compute the distance from (a, b) ? (drawing a picture may be helpful).

Part B. (15) Solve the following recurrences by finding a $\Theta()$ bound for $T(n)$. In each case, assume that $T(n)$ is 1 for $n \leq 1$, and otherwise is defined by the recurrence.

1. $T(n) = T(n/8) + 3T(n/6) + 5n^2$

2. $T(n) = 6T(n/2) + 5n^2$

Part C.(14) You are given n activities with start times s_1, s_2, \dots, s_n and finish times $f_1 \leq f_2 \leq \dots \leq f_n$. As we discussed you can't schedule two activities which overlap, and your goal is to schedule as many activities as possible.

However, activity k is very important, so you have to schedule it. Describe how to modify the activity selection algorithm we discussed to find the maximum size set of activities which includes activity k .

Give the running time of your algorithm.

Part D. (17) Suppose we need to make change for the new country of Fubar which uses coins of value 1, 5, 7 and 28 cents. As usual, we want to make change using as few coins as possible.

i) Show that the greedy strategy of always using the largest coin is NOT optimal.

ii) Let $C(n)$ denote the minimum number of coins to make change on n cents. Write a recurrence relation for $C(n)$. You may assume $C(n) = n + 1$ when $n < 0$.

Part E. (9) Suppose we have a hash table of size m , using chaining, with n items already inserted. If we insert a new item x into this table, what is the expected number of items in the chain x hashes to (assume uniform hashing)? Briefly justify your answer.

Part F. (17) Suppose that we have an unsorted list L of $100 * n$ numbers where there are 20 copies of each item in the list (so $5n$ distinct values).

i) Suppose that we wanted to create a new list of the $5n$ distinct values in L . Describe an efficient algorithm to do this and discuss the run time.

ii) Suppose we want to find the k th element of the original list L where $k = 50 * n$ (note: this is unrelated to part i). What is the **worst case** number of calls to the partition routine used by the randomized select algorithm? Assume we use the version of the partition routine which splits the list into three groups at each partition step (those less than, equal to, and greater than the split element). Briefly justify your answer.

2 Dynamic Programming

[17 points]

We consider a variant of the subset sum problem. As usual we have n integers x_1, x_2, \dots, x_n and a target b . Suppose that x_1 is a negative value and the rest are all positive (as usual). As before we want to find a set of items whose sum is as close to b as possible but the sum cannot exceed b .

Part A. Describe how to modify the dynamic programming formulation we used to find the best sum in this setting. Be clear on the size and contents of the table, how to fill in initial values, and how to compute a new row from the old row(s).

Part B. Justify your part A) answer and give its running time.