Problem Set 3—Due Friday, February 16, 3PM

(30) Problem 1. The *yes/no clique*— problem is: given an undirected graph G=(V,E) and a target integer k, is there a clique of size k? A *clique* is a set of vertices C in V such that each pair of vertices (u, v) in C, is also an edge in E (thus every pair of vertices in a clique are connected by an edge).

a) Show that the yes/no clique problem is in NP (note, this problem is NP-C but you are NOT being asked to prove that).

b) Show that you can use a program which solves the yes/no clique problem to actually find a clique of size k (when one exists). You should find the clique using a polynomial number of calls to the yes/no clique routine, plus polynomial additional work. Thus you are showing that the problem of finding a clique of a given size is polynomialy reducible to yes/no clique.

c) Give a polynomial-time algorithm for *yes/no clique*— when $k < c$ for a constant c. Would your algorithm still run in polynomial time if we restrict k so $k < \log n$, with n the number of vertices in the graph?

(27) Problem 2. Consider the directed *Hamilton Path* problem (DHP) described on page 979.

a) Show that DHP $\leq_p$ s, t Hamilton Path
where s, t Hamilton Path takes as input a directed graph G=(V,E) and two designated vertices in V, and returns yes when there is a Hamilton path starting at s and ending at t.

b) Consider the problem of finding a shortest **simple** path in a directed graph (with negative cycles) from s to t (thus you are given a directed graph G, edge weights, possibly negative, and two designated vertices). Show that this problem is NP-hard (hint: Use your result of part a)).

(18) Problem 3. Suppose we are given a *Traveling Sales Man* (TSP) problem where cities are points in the plane and distances are actual Euclidean distances (Call this the Euclidean TSP).

Show that the *proof* of theorem 34.14 p. 1013 in the text does **not** prove that the Euclidean TSP is NP-hard (this problem is in fact still NP-hard, but you are not asked to prove this).