

## Problem Set 3—Due Thursday , May. 26, 5PM

**NOTE: please explain and justify your solutions clearly. Give a high-level overview before plunging into details (consider the lecture formats as a model)**

### (30) Problem 1.

1. Consider a PRAM variant that reflects some of the characteristics of a GPU. We assume that shared memory is divided into banks, and each bank is divided into blocks (for example, memory is of size  $2^{30}$  divided into  $2^{10}$  banks of size  $2^{20}$ . Each bank is divided into  $2^7$  blocks of size  $2^{13}$ . Assume the basic unit we are discussing is 4 byte integers, so the memory has  $2^{32}$  bytes.

A processor reads a block of memory in a read step, and writes a block in a write step. In a parallel read step, multiple processors can read from the same memory bank only if they all read the same block. However, each processor reading a different memory bank (from the others), can read an arbitrary block in their bank in a single read step. Similarly, multiple processors can write to the same memory bank only if they all are writing the same block (an arbitrary one succeeds if multiple writers), but multiple memory banks can be written to in a single write step.

Consider a system with memory as in the example above (so of size  $2^{30}$  ) and  $p = 2^{10}$ . Our goal for this system is to design algorithms that minimize the number of parallel read/write steps. Unlike a GPU, you may also assume each processor has a large local memory.

(10) a) Give a good algorithm to sum all the ( $2^{30}$ ) values in memory. How many R/W steps do you use?

(20) b) Find the maximum of an array A of  $2^{20}$  integers

i) If the entire array is stored in one memory bank

ii) if it is “striped” so the first  $2^{10}$  elements are in memory bank one, the next  $2^{10}$  in memory bank 2, . . . and the last  $2^{10}$  are in the final memory bank.

Give the number of R/W steps for your solutions.

(c) extra credit: Give a good algorithm for multiplying two  $2^{10}$  by  $2^{10}$  matrices in this model. Discuss a good way to store the matrices to speed up the process.

### (20) Problem 2.

Exercise 27 in the notes

### (20) Problem 3. Exercise 28 in the notes,

### (20) Problem 4. Exercise 38, , .

Note that this is asking for an example family of graphs where  $\Omega(\log n)$  steps are required.

Also, give the flip side. For the **normal** hooking algorithm give a best case number of iterations (that is, give a family of graphs that take rather less than  $\log n$  iterations to solve.

### (15) Problem 5. Exercise 41, , .