Midterm Solutions

1 Short Answer

1.1 Draw a **DFA** M for the language

$$L = \{x \in \{a,b,c\}^* : x \text{ contains exactly one } a \text{ and exactly one } b\}.$$

Make your DFA have as few states as possible.

You need five states; see Quiz 1 solutions.

1.2 List the first five strings of this language (Problem 1.1) in lexicographic order. Assume $a < b < c$.

Again from Quiz 1. The answer is ab, ba, abc, acb, bac.

1.3 Write a regular expression for this language (Problem 1.1). Make it as short as possible.

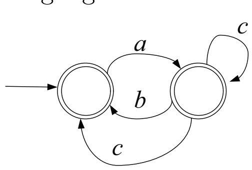
I accepted either $c^*(ac^*b)c^* \cup c^*(bc^*a)c^*$ *or* $c^*(ac^*b \cup bc^*a)c^*$.

1.4 *Give a CFG for* $L = (ab \cup aaa)^*\, baa$. *Make your grammar use as few rules as possible.*

$S \to ab\,S \mid aaaS \mid baa$

1.5 *Let* $M = (Q, \Sigma, \delta, q_0, F)$ *be an NFA with no* ϵ*-arrows. We can convert* M *into a DFA* $M' = (Q', \Sigma, \delta', \{q_0\}, F')$ *whose language is* $L(M)$ *by setting* $Q' = \boxed{\mathcal{P}(Q)}$ and $\delta'(S,a) = \boxed{\{\delta(q,a) : q \in S\}}$ and $F' = \{T \subseteq Q : T \cap F \neq \emptyset\}$.

1.6 Using the procedure shown in class, convert the following NFA into a regular expression for the same language.



I am lazy to typeset the step-by-step solution, but most people knew the procedure.

2 Justified True or False

Put an **X** through the **correct** box. Then provide a brief justification. **Where appropriate, make the justification a counter-example.**

2.1 Every regular language can be accepted by an **NFA** with only a single final state.

True. Connect all of the old final states to a new final state by ϵ*-arrows. Definalize all the old final states.*

2.2 The complement of a regular language is context free.

True. The complement is regular and therefore context free.

2.3 Let $h : \Sigma \to \Sigma^*$ be a function and define $h(a_1 \cdots a_n) = h(a_1) \cdots h(a_n)$ and $h(L) = \{h(x) : x \in L\}$. Suppose $h(L)$ is not regular. Then L is not regular.

True. See the hint of problem set 5, problem 4. This statement is the contrapositive.

2.4 There is a language L for which $L = L^*$.

True. For example, $L = \{\varepsilon\}$ *or* $L = 1^*$.

2.5 Every nonempty regular language L is generated by some ambiguous CFG.

True. Since L *is regular it is context free and has a CFG* G*. If* G *is not already ambiguous, add in a rule* $S \to S$ *where* S *is the start symbol. It will now be ambiguous.*

3 Classify

3.1. Let $L = \{ww : w \in \{0,1\}^*\}$. Is L regular? Prove your answer.

Not regular. Suppose for contradiction that L *were regular. Let* N *be the pumping length, as guaranteed by the pumping lemma. Let* $s = 0^N 1 0^N 1$*. By the "semi-strong" form of the pumping lemma, there exists an* x, y, z *such that* $xyz = s$ *and* $|xy| \le N$ *and* $xy^i z \in L$ *for any* $i \ge 0$*. So* y *must fall within the first run of* 0*'s and so, letting* $i = 0$*, so* $xz = 0^n 1 0^N 1$ *for some* $n < N$*. But this string is not of the form* ww*; the string is not in* L*. Thus* L *is not regular.*

3.2. Let $L = \{w \in \{0,1\}^* : w \text{ contains an equal number of 01's and 10's}\}$. Is L regular? Prove your answer.

Regular. See Problem Set 5, part 1B.