Problem Set 4 – Due Tuesday, April 27, 2010, at 4:15 pm

Problem 1. Describe a decision procedure to solve the following problem: given a regular expression α, is α a *shortest* regular expression for $L(\alpha)$? How efficient is your procedure?

Problem 2. Are the following statements true or false? Either prove the statement or give a counter-example to it.

Part 2A. If $L \cup L'$ is regular than L and L' are regular.

Part 2B. If L^* is regular than L is regular.

Part 2C. If LL' is regular than L and L' are regular.

Part 2D. If L and L' agree on all but a finite number of strings, then one is regular iff the other is regular.

Part 2E. If R is regular, L is not regular, and L and R are disjoint, then $L \cup R$ is not regular.

Problem 3. Define $A = \{x \in \{a, b, \sharp\}^* : x$ contains an equal number of a's and b's or x contains consecutive $\sharp$s or consecutive letters$\}$.

Part 3A. Can you use the pumping lemma to prove that L is not regular? Explain.

Part 3B. Prove that A is not regular. *Hint: consider closure under homomorphisms and problem 2E.*

Problem 4. Give a context free grammar for $L = \{a^n b^m : n \neq 2m\}$. Try to make your grammar unambiguous—and explain why it is unambiguous.

Problem 5. A *regular grammar* is a context-free grammar $G = (V, \Sigma, R, S)$ in which every rule is of the form $A \to \varepsilon$ or $A \to aB$, where a is a terminal and A and B are variables. Show that L is regular iff L is generated by a regular grammar.

Problem 6. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) *Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.*

(b) *Is $L(G)$ regular? Prove your answer either way.*

(c) *Is G ambiguous? Prove your answer either way.*

(d) *Is $L(G)$ inherently ambiguous? Give a convincing argument either way.*