Problem Set 7 – Due Tuesday, May 18, 2010, at 4:15 pm

Problem 1. Formally specify (draw a transition diagram for) a TM $M = (Q, \{a, b\}, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ that decides the language $L = \{ww : w \in \{a, b\}^*\}$. You may assume a one-way or two-way infinite tape, whichever you prefer. (Do indicate your choice.) Work to make your machine as simple as possible. Realizing that a problem like this is hard to grade, give your states mnemonic names, if that seems helpful, and compose a careful paragraph to explain how your machine works.

Problem 2. Recall that $L = \{ww : w \in \{a, b\}^*\}$ is not context free. Exhibit an unrestricted grammar for it. An unrestricted grammar is like a CFG except that rules can look like $aBCb \to accbDe$, for example: left-hand sides can contain any string of terminals and nonterminals with at least one nonterminal.

Problem 3. Here is another TM-variant for you. A *three-pebble Turing-machine* (3PTM) is like an ordinary TM except that the input x is presented on a read-only tape, surrounded by delimiters, $[x]$, and there is a auxiliary tape to which the machine cannot write but on which machine can place three pebbles on. The machine can move these pebbles around, picking up a pebble and moving to a neighboring tape square. Formalize the syntax of 3PTM. Then formalize how a 3PTM runs, and what is ones language. *(I won't ask you to prove it here, but 3PTM's accept exactly the r.e. languages.)*

Problem 4.

Part A. Suppose that L is recursive. Show that L^* is recursive.

Part B. Suppose that L is r.e. Show that L^* is r.e.