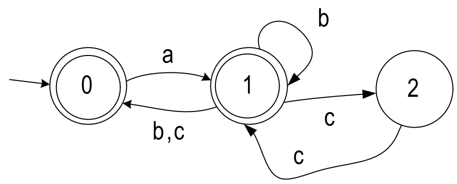
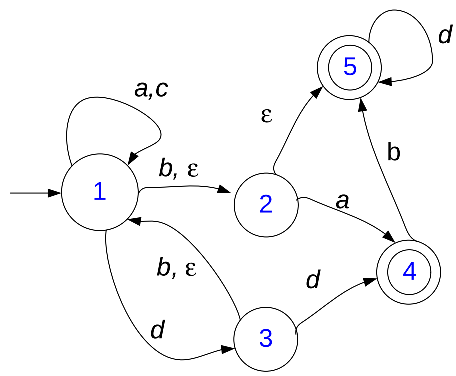
# Problem Set 3 – Due Friday, April 18, 2014

**Problem 1.** Using the procedure shown in class, convert the following NFA into a DFA for the same language. Show all work.




**Problem 2.** Using the procedure shown in class, eliminate all $\varepsilon$-arrows from the following NFA.




**Problem 3.** Let $L_1, L_2, L_3 \subseteq \Sigma^*$ be langauges and let $\mathbf{maj}(L_1, L_2, L_3)$ be the set of all $x \in \Sigma^*$ that are in at least two of $L_1, L_2, L_3$. Prove: if $L_1$, $L_2$, and $L_3$ are DFA-acceptable then so is $\mathbf{maj}(L_1, L_2, L_3)$.

**Problem 4** Let $\mathcal{Z}(L) = \{a_1 0 a_2 0 \cdots a_n 0 \in \Sigma^* : a_1 a_2 \cdots a_n \in L\}$. Prove that the DFA-acceptable languages are closed under $\mathcal{Z}$. Having proved it once: can you think of another, different proof?

**Problem 5.** How many states are in the smallest possible DFA for $\{0,1\}^*\{1^{10}\}$? Prove your result.

**Problem 6** Let $L_n$ (for $n \geq 1$) be $\{0,1\}^*\{1\}\{0,1\}^n$. Prove that there is an NFA for $L_n$ having $n+2$ states, but that there is no DFA for $L_n$ having $2^n - 1$ or fewer states. In a well written English sentence or two, give give a high-level interpretation of your result.