Problem Set 2 — Due Wednesday, April 14, 1999

Problem 1. We say that a string x over an alphabet consisting of left parenthesis, '(', and right parenthesis, ')' is *1-balanced* if

a. x has an equal number of left and right parentheses; and

b. any suffix of x has at least as many right parentheses as left parentheses.

Say that a string x (over the same alphabet) is *2-balanced* if it can be generated by the following rules:

a. The empty string, ϵ, is 2-balanced.

b. If x is 2-balanced, so is (x).

c. If x and y are 2-balanced, so is xy.

d. Nothing else is 2-balanced.

Part A. Prove that if a string x is 1-balanced, then it is 2-balanced. *Hint*: By induction on $|x|$.

Part B. Prove that if a string x is 2-balanced, then it is 1-balanced. *Hint*: By induction on the definition of 2-balanced.

Problem 2. Let $L = \{w : w$ is a binary string of length at least 1 that starts and ends with the same character$\}$. Show that 5 states is necessary and sufficient for a DFA to recognize L.