Problem Set 4 — Due Wednesday, April 28, 1999

Problem 1. Use the pumping lemma to show that the following languages are not regular.

(a) $L_a = \{0^m 1^n 0^{m+n} : m, n \geq 0\}$.

(b) $L_b = \{0^{2^n} : n \geq 0\}$.

(c) $L_c = \{x = y + z : x, y, \text{ and } z \text{ are binary numbers and } x \text{ is the sum of } y \text{ and } z\}$.

Problem 2. *Are the following propositions true or false? Support your answers with either proofs or counterexamples.*

(a) If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.

(b) If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.

(c) If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.

(d) If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.

(e) If L^* is regular then L is regular.

Problem 3. Exhibit decision procedures (algorithms) which answer the following questions. (Your algorithms need not be "efficient.")

(a) Given a DFA M, does $L(M)$ contain a string of the form $(babb \cup aa^*b)^*$?

(b) Given NFAs M_1 and M_2, is $|L(M_1)| = |L(M_2)| < \infty$?

(c) Given a regular expression α, is there a shorter regular expression which denotes the same language?

(d) Given an NFA M, does $L(M)$ contain a palindrome? (A palindrome is a string w such that $w^R = w$.)

Problem 4. Use closure properties and the fact that $\{a^n b^n : n \geq 0\}$ is not regular to show that the following languages are not regular.

(a) $L_a = \{a^i b^j c^k : i = j \text{ or } j = k\}$.

(b) $L_b = \{w \in \{a, b\}^* : w \text{ has more } a\text{'s than } b\text{'s}\}$.