Problem Set 5 — Due Wednesday, May 3, 1999

Problem 1 Let L be an infinite language with arbitrarily large "gaps": for every $n \in \mathcal{N}$ there exists some number $\alpha(n)$ such that L contains no strings of length ℓ for any number ℓ between $\alpha(n)$ and $\alpha(n)+n$. Prove that L is not regular.

Problem 2.

Part A. Exhibit a context free grammar for the language

$$L = \{a^n b^m : n \neq 2m\}.$$

Then describe a PDA for the same language. You do not need to be formal about the PDA; explaining how it behaves in English is fine.

Part B. Prove that

$$L = \{x \neq y : \ x \text{ and } y \text{ are binary strings and } x \neq y$$

is context free.

Problem 3. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.

(b) Is $L(G)$ regular? Prove your answer either way.

(c) Is G ambiguous? Prove your answer either way.

(d) Is $L(G)$ inherently ambiguous? Give a convincing argument either way.

Problem 4. A *regular grammar* is a context-free grammar $G = (V, \Sigma, R, S)$ in which every rule is of the form $A \to \varepsilon$ or $A \to aB$, where a is a terminal and A and B are variables. Show that L is regular iff L is generated by a regular grammar.

Problem 5. Prove that the context-free languages are closed under reversal.