Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **4** problems. Do your best to communicate your ideas *clearly* and *succinctly*. Ask me if you don't understand what a problem means. Draw a picture if it is useful to explain something. There's extra scratch paper if you need it.

Good luck — Have fun! (It's not *too* hard.)

— Phil Rogaway

Name:

E-mail:

On problem	you got	out of
1		30
2		40
3		20
4		20
Σ		110

1 Short Answer [30 points]

(A) Complete the definition: A ***language*** over an alphabet Σ is

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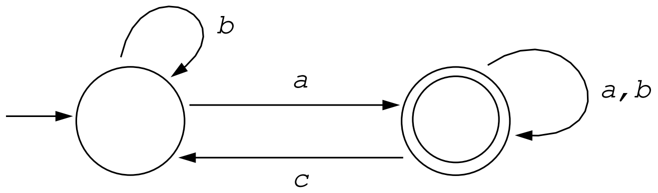
(B) Carefully state the ***pumping lemma*** for regular languages. (Don't use the word "pumps" without defining it.)

(C) You are given the regular expression $\alpha = 00^* \cup 11^*$. Using the constructions given in class and in your text, imagine converting α into a **DFA** M for which $L(M) = L(\alpha)$. How many states will M have? Justify your answer.

(D) Using the procedure given in class and in your text, convert the following NFA M into a regular expression α such that $L(M) = L(\alpha)$.



(E) Complete the definition: A CFL L is ***inherently ambiguous*** if

2 Justified True or False [40 points]

Put an **X** through the **correct** box. When it says "Explain" provide a **brief** (but convincing) justification. Where appropriate, make this justification a counterexample. For some problem, no credit will be given for correct answers which lacks an acceptable explanation.

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1. For every n, the language $L_n = \{0^i 1^i : \ i \le n\}$ is regular. **True** **False**

Explain:

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2. If L^* is regular, then L is regular, too. **True** **False**

Explain:

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3. Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA and let $M' = (Q, \Sigma, \delta, q_0, F')$, where $F' = Q - F$. Then $L(M) = \overline{L(M')}$. **True** **False**

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4. Let $L = \{a^n b^n : n \ge 0\}$. Then $\overline{L}$ is regular. **True** **False**

Explain:

5. Let NOPREFIX$(L) = \{w \in L |$ no proper prefix of w is in $L\}$. If L is regular, then so is NOPREFIX(L). $\boxed{\textbf{True}}$ $\boxed{\textbf{False}}$

Explain:

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6. The pumping lemma is a useful technique to show that a language is regular.

$\boxed{\textbf{True}}$ $\boxed{\textbf{False}}$

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7. There is a language L for which $L = L^*$. $\boxed{\textbf{True}}$ $\boxed{\textbf{False}}$

Explain:

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8. Every regular language is accepted by an **NFA** with **no** self-loops. (A self loop is pair (q, a) for which $\delta(q, a) \ni q$). $\boxed{\textbf{True}}$ $\boxed{\textbf{False}}$

Explain:

3 Pigeonhole Principle **[20 points]**

Let $L = \{111\}$ be the language over $\{1\}$ which contains only the string 111.

(A) Show that L can be recognized by a 5-state DFA.

(B) Prove that L can not be recognized by a 4-state DFA.

4 A Closure Property [20 points]

For L a language, let

$$\mathit{Stutter}(L) \quad = \quad \{a_1 a_1 a_2 a_2 \cdots a_n a_n : \; a_1 a_2 \cdots a_n \in L\}$$

(Above, each a_i is a character.) Prove that if L is regular, then $\mathit{Stutter}(L)$ is regular, too.