Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **4** problems. Do your best to communicate your ideas *clearly* and *succinctly*. Draw a picture if it is useful to explain something. Show all work on these pages. There are two blank sheets at the end for you to use.

NAME:

On problem	you got	out of
1		30
2		20
3		30
4		20
Σ		100

1 Justified True or False [30 points]

Put an **X** through the **correct** box and provide a **brief** (but convincing) **justification**. Where appropriate, make the justification a counterexample. No credit will be given for correct answers which lacks an acceptable explanation.

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1. For every i, the languge $L_i = \{0^i 1^i\}$ is regular. **True** **False**

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2. Assume L is a regular language and let L_e be the subset of L which contains the even length strings of L. Then L_e is regular.

True **False**

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3. If L^* is context free then L is context free. **True** **False**

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4. Every regular language L is accepted by an NFA in which, on input of any string $x \in L$, no state is ever visited twice. **True** **False**

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5. Some context free language has a context free complement. **True** **False**

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6. Every context free language is generated by a grammar in which all the rules are of the form $A \to BC$ or $A \to \varepsilon$ or $A \to a$. (A, B and C are variables and a is a terminal). **True** **False**

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7. Suppose an infinite language L has arbitrarilly long "gaps" — for every n there is an interval $[LEFT_n..RIGHT_n]$ such that $RIGHT_n - LEFT_n > n$ and L has **no** strings of **any** length between $LEFT_n$ and $RIGHT_n]$. Then L is **not** context free.

True **False**

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8. The emptystring, ε, is a special string which belongs to every regular language L.

True **False**

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9. If $L_1 \subseteq L \subseteq L_2$ where L_1 and L_2 are reguar, then L must be regular, too.

True **False**

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10. Let $L \subseteq \{a, b\}$ be a regular language. Let L' be all the strings x' for which x' is obtained from some string $x \in L$ by replacing each a in x with a ba, and each b in x with an aab. Then L' is regular.

True **False**

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11. If L is inherently ambiguous, then L should take a class in Rhetoric Department.

True **False**

2 NFA $\Longrightarrow$ DFA **[20 points]**

Using the procedure described in class, convert the following NFA into an equivalent DFA. Show all your work. (*Reminder*: Step 1— Eliminate ε-moves. Step 2— Subset construction.)

3 Regular / Context Free [30 points]

(A) Carefully state the "pumping lemma" for **regular** languges.

(B) Prove that language

$$L = \{ww^R : \; w \in \{0,1\}^*\}$$

is **not** regular.

(C) Give a context free grammar for L.

(D) Is your grammar ambiguous? Why or why not?

(E) Is you said that your grammar was ambiguous in Part (D), then provide an unambiguous grammar for L. Is you said that your grammar was unambiguous in Part (D), then provide an ambiguous grammar for L.

(F) Is $\overline{L}$ context free? Yes or No, and why.

4 A Decision Procedure [20 points]

Show that the language

$$L = \{\langle r, R\rangle : r \text{ and } R \text{ are regular expressions over and } L(r) \subseteq L(R)\}$$

is decidable. (That is, give a decision procedure which, given regular expressions r and R, decides if $L(r) \subseteq L(R)$.