Problem Set 1 — Due Tuesday, January 14, 1997

Instructions: Write up your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems.

Problem 1. Show that at a party of 20 people, there are at least 2 people who have the same number of friends present at the party. Assume (however unrealistically) that friendship is symmetric and anti-reflexive. *Hint*: Carefully use the pigeonhole principle.

Problem 2. We say that a string x over an alphabet consisting of left parenthesis, '(', and right parenthesis, ')' is *1-balanced* if

a. x has an equal number of left and right parenthesis; and
b. any suffix of x has at least as many right parenthesis as left parenthesis.

Say that a string x (over the same alphabet) is *2-balanced* if it can be generated by the following rules:

a. The empty string, ϵ, is 2-balanced.
b. If x is 2-balanced, so is (x).
c. If x and y are 2-balanced, so is xy.
d. Nothing else is 2-balanced.

Part A. Prove that if a string x is 1-balanced, then it is 2-balanced. *Hint*: By induction on $|x|$.

Part B. Prove that if a string x is 2-balanced, then it is 1-balanced. *Hint*: By induction on the definition of 2-balanced.

Problem 3. Page 84, Exercise 1.4, parts c, e, f.

Problem 4.

Part A. Page 88, Exercise 1.22.

Part B. Page 87, Exercise 1.20.

Problem 5. Page 89, Problem 1.30.