Problem Set 3 — Due Tuseday, February 4, 1997

Reminder: *There is an in-class midterm on Thursday, February 6.*

Problem 1. Use the pumping lemma to show that the following languages are not regular.

(a) $L_a = \{0^m 1^n 0^{m+n} : m, n \geq 0\}$.

(b) $L_b = \{0^{2^n} : n \geq 0\}$.

Problem 2. Are the following languages regualar? Support your answers either way.

Part A. $L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^* : w_1 \neq w_2\}$.

Part B. $L = \{w|\ w$ contains an equal number of occurrences of the substrings 01 and 10$\}$.

Problem 3. Are the following propositions true or false? Support your answers with either proofs or counterexamples.

(a) If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.

(b) If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.

(c) If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.

(d) If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.

(e) If L^* is regular then L is regular.

Problem 4. Show that L is regular if and only if L can be generated by a CFG G in which every rule is of the form $A \to aB$ or $A \to \varepsilon$, where a is a terminal and B is a variable.

Problem 5.

Part A. Page 120, Exercise 2.4, part (e).

Part B. Page 120, Exercise 2.5, part (a).

Problem 6. Page 121, Problems 2.15 and 2.16.

Problem EC. (Extra credit.) Page 90, Problem 1.44.