Midterm Exam

Instructions: This is a closed book, closed notes exam. Do all **4** problems. Do your best to communicate your ideas *clearly* and *succinctly*. Ask me if you don't understand what a problem means. Draw a picture if it is useful to explain something.

— Phil Rogaway and John Black

Name:

E-mail:

On problem	you got	out of
1		35
2		28
3		17
4		20
Σ		105

1 Short Answer [35 points]

(A) Carefully define what it ***means*** when we say that the regular languages are closed under intersection. (Don't explain if this statement is true or false; just give a "mathematical translation" of the sentence).

(B) Carefully state the ***pumping lemma*** for **context-free** languages. (Please don't use the word "pumps," at least not without defining it!) Be sure that your quantifiers are precise.

(C) *(5 points.)* Draw a DFA which accepts

$$L = \{w \in \{0,1\}^* : w \text{ is the binary representation of a number divisible by } 3\}\ .$$

That is, $L = 0^* \circ \{\varepsilon, 11, 110, 1001, \ldots\}$

(D) Complete the definition: A context free grammar is a 4-tuple $G = (\quad,\quad,\quad,\quad)$, where

(E) Give a CFG for $L = \{w \in \{a,b\}^* : \ w \text{ is a palindrome}\}$. (Recall that a string w is a palindrome if $w = w^R$ — the string reads the same forwards or backwards.)

(F) Prove that the language of Part (E), $L = \{w \in \{a,b\}^* : \ w \text{ is a palindrome}\}$, is **not** regular.

(G) (*5 points.*) Give a decision procedure which, given a regular expression α, decides if the smallest NFA for $L(\alpha)$ has fewer states than the smallest DFA for $L(\alpha)$.

2 Justified True or False [28 points]

Put an **X** through the **correct** box. When it says "Explain" provide a **brief** (but convincing) justification.

---

1. If F is finite and $L - F$ is regular, then L is regular, too. **True** **False**

---

2. If L^* is regular, then L is regular, too. **True** **False**

Explain:

---

3. Every regular language can be accepted by an NFA which has one and only one final state. **True** **False**

Explain:

---

4. The context-free languages are closed under complement. **True** **False**

Explain:

---

5. Suppose L is context free. Then there exists positive numbers a and b such that L contains some string of length $ai + b$ for every $i \geq 0$.

True **False**

Explain:

---

6. There is a language L for which $\overline{L} = L^*$.

True **False**

Explain:

---

7. Every regular language L is generated by some non-ambiguous CFG.

True **False**

Explain:

---

3 Minimal-Size DFA [17 points]

Let $L_4 = \{\varepsilon, 0, 1, 00, \ldots, 1111\}$ be the set of all binary strings of length 4 or less. Show that that there is no 4-state DFA that recognizes L.

4 A Closure Property of Regular Languages [20 points]

Let Σ and Γ be alphabets and let $h : \Sigma \to \Gamma^*$ be a function. For $w = a_1 \dots a_n$ a string, $a_i \in \Sigma$, define $h(w) = h(a_1) \cdots h(a_n)$. Define $h(\varepsilon) = \varepsilon$. For a language L over Σ define $h(L) = \{h(w) : w \in L\}$.

For example, if $L = \{a, bb, acb\}$ and $h(a) = bba$, $h(b) = cc$, $h(c) = \varepsilon$ then $h(L) = \{bba, cccc, bbacc\}$.

Part A. Prove or disprove: if L is regular, then $h(L)$ is regular.

Part B. Prove or disprove: if L is *not* regular, then $h(L)$ is not regular.