Problem Set 1 — Due Wednesday, January 14, 1998

Instructions: Write up your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems. Recall that homework is due at 4:00 pm in the box in Eng. II, room #2131.

Problem 1. Show that at a party of 20 people, there are at least 2 people who have the same number of friends present at the party. Assume (however unrealistically) that friendship is symmetric and anti-reflexive. *Hint*: Carefully use the pigeonhole principle.

Problem 2. We say that a string x over an alphabet consisting of left parenthesis, '(', and right parenthesis, ')' is *1-balanced* if

a. x has an equal number of left and right parentheses; and

b. any suffix of x has at least as many right parentheses as left parentheses.

Say that a string x (over the same alphabet) is *2-balanced* if it can be generated by the following rules:

a. The empty string, ϵ, is 2-balanced.

b. If x is 2-balanced, so is (x).

c. If x and y are 2-balanced, so is xy.

d. Nothing else is 2-balanced.

Part A. Prove that if a string x is 1-balanced, then it is 2-balanced. *Hint*: By induction on $|x|$.

Part B. Prove that if a string x is 2-balanced, then it is 1-balanced. *Hint*: By induction on the definition of 2-balanced.

Problem 3. Give DFAs for the following languages. Assume $\Sigma = \{0, 1\}$.

(a) The set of all strings with 010 as a substring.

(b) The set of all strings which do not have 010 as a substring.

(c) The set of all strings which have an even number of 0's or an even number of 1's.

(d) The complement of $(1 \cup 10)^*$.

(e) The binary encodings of numbers divisible by 3: $0^*\{\varepsilon, 11, 110, 1001, 1100, 1111, \ldots\}$.

Problem 4

(a) Show that there is a deterministic finite automaton with $n + 1$ states that recognizes the language $(1^n)^*$. (The alphabet is $\Sigma = \{0, 1\}$.)

(b) Sow that there does *not* exist a smaller deterministic finite automaton for this language. (smaller = fewer states).