Problem Set 4 — Due Wednesday, February 4, 1998

Problem 1. Exhibit decision procedures (algorithms) which answer the following questions. (Your algorithms need not be "efficient.")

(a) Given a DFA M, does $L(M)$ contain a string of the form $(babb + aa^*b)^*$?

(b) Given NFAs M_1 and M_2, is $|L(M_1)| = |L(M_2)| < \infty$?

(c) Given a regular expression α, is there a shorter regular expression which denotes the same language?

(d) Given an NFA M, is $L(M)$ a language containing only palindromes? (A palindrome is a string w such that $w^R = w$.)

Problem 2. Use closure properties and the fact that $\{a^n b^n : n \geq 0\}$ is not regular to show that the following languages are not regular.

(a) $L_a = \{a^i b^j c^k : i = j \text{ or } j = k\}$.

(b) $L_b = \{w \in \{a, b\}^* : w \text{ has more } a\text{'s than } b\text{'s}\}$.

Problem 3. Use the pumping lemma to show that the following languages are not regular.

(a) $L_a = \{0^m 1^n 0^{m+n} : m, n \geq 0\}$.

(b) $L_b = \{0^{2^n} : n \geq 0\}$.

Problem 4. Are the following languages regular or not? Prove your answer either way.

(a) $L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^* : w_1 \neq w_2\}$.

(b) $L = \{w \in \{0, 1, 2\}^* : w \text{ has an equal number of 01's and 10's}\}$.

Problem 5. Are the following propositions true or false? Give proofs or counterexamples.

(a) If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.

(b) If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.

(c) If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.

(d) If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.

(e) If L^* is regular then L is regular.