Problem Set 5 — Due Tuesday, February 10, 1998

Note the "non-standard" due date! This homework is due next Tuesday, in class, at 6:10 pm. Do not turn this in in the turn-in boxes.

Problem 1. Exhibit a context free grammar for the language

$$L = \{a^n b^m : n \neq 2m\}.$$

Then describe a PDA for the same language. You do not need to be formal about the PDA; explaining how it behaves in English is fine.

Problem 2. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.

(b) Is $L(G)$ regular? Prove your answer either way.

(c) Is G ambiguous? Prove your answer either way.

(d) Is $L(G)$ inherently ambiguous? Give a convincing argument either way.

Problem 3.

a. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

b. Prove that $L_b = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.

Problem 4. A *regular grammar* is a context-free grammar $G = (V, \Sigma, R, S)$ in which every rule is of the form $A \to \varepsilon$ or $A \to aB$, where a is a terminal and A and B are variables. Show that L is regular iff L is generated by a regular grammar.